

POSSIBLE PITFALLS AND RECOMMENDED SOLUTIONS

The following is a compilation of items which may cause the user of the BOSOR4 program some difficulty. Suggestions are given for overcoming the difficulties.

Provision of Consistent Input Data

In the initial use of a complex program such as BOSOR4 it is possible that the input data may not be consistent. The user is urged to check carefully the list and plot output for errors in the input data. In particular, boundary conditions, position of discrete ring stiffeners, meridian shape, line loads, and surface loads should be checked. Often the best way the user can familiarize himself with the input procedures is to run cases for which he knows the answers beforehand. A check of the mode shapes and stress distributions often reveals possible errors in input. It is emphasized that the user should check the sample cases in the BOSOR4 manual to see if they might help him to set up a new case.

Finding the Minimum Minimum Buckling Load:
Appropriate Choices for NOB, NMINB, NMAXB, INCRB

The theory on which BOSOR4 is based does not exclude the possibility that several values of circumferential wave number N may be associated with minimum buckling loads. One must always find the minimum minimum. This problem frequently arises in the calculation of buckling loads for complex shells or ring stiffened shells. A ring stiffened conical shell under external pressure is such a case (Fig. A15). Here there could be a minimum buckling load corresponding to general instability and additional minima (at higher values of N) corresponding to the local failure of each conical frustum (the bays between the rings). Physical intuition is invaluable as a guide for finding the absolute minimum load. One may idealize each bay of a ring stiffened shell by assuming that the bay is simply supported, calculate corresponding "panel" buckling loads with certain appropriate ranges of N , and then use the critical loads and values of N as starting points in an investigation of the assembled structure.

It is not necessary always to increase the circumferential wave number N by one. In the search for the minimum buckling load, for example, one may only be certain that the N corresponding to the minimum buckling load, N (critical), lies in the range $2 \leq N \leq 100$. One might, therefore, choose INCRB = 10 and "zero in" on a more accurate value in a subsequent run. The user should ordinarily set INCRB = $0.05 * (NMINB + NMAXB)$.

Experimental evidence is of course very useful in determining a good choice of NOB, NMINB, and NMAXB. If none is available the user is advised to try the following formulas:

(1) "Square" buckles for short shells or panel buckling

$N = \pi r/L$, where L is the shell meridional arc length between nodes of the buckling mode.

(2) For monocoque deep shells, axial compression:

$$N = [(\text{Nominal circumferential rad. of curve})/t]^{1/2} (1 - \nu^2)$$

(3) For shallow spherical caps supported rigidly at their edges; external pressure:

$$N = 1.8 * \alpha_2 * (R/t)^{1/2} - 5$$

(4) For axially compressed conical shells and frustrums:

Use formula 2 where the circumferential radius of curvature, R , is the average of the radii at the ends.

(5) Spherical segments of any depth under axial tension

$$N = 1.8 * (R/t)^{1/2} \sin [\alpha_1 + 4.2 (t/R)^{1/2}]$$

where α_1 and α_2 are the meridional angles at the segment beginning and end, respectively.

The above list of formulas is by no means complete. However, notice that $(R/t)^{1/2}$ is a significant parameter. If N is known for a shell of a given geometry loaded in a certain way, a new value can be predicted for a new R/t through the knowledge that N often seems to vary as $(R/t)^{1/2}$. (R is the circumferential radius of curvature.)

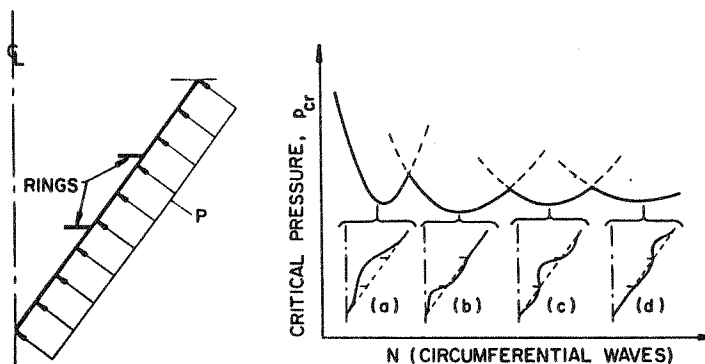


Fig. A15 Several buckling modes for ring stiffened conical shell
(a) General instability (b) 1st bay buckling (c) 2nd bay buckling
(d) 3rd bay buckling

Experience in the use of the program will lead to further competence in the selection of appropriate values for NOB, the initial guess at N. Again, the user must be sure that the input range $N_{MINB} \leq N \leq N_{MAXB}$ includes the minimum minimum buckling load.

Stress Resultant or Stress Discontinuities at Juncutres and Boundaries

Stress resultants and stresses need not be continuous at segment juncutres in all cases, of course. However, the user of BOSOR4 will notice that for some cases in which these quantities should be continuous there exist small discontinuities right at juncutres. These discontinuities arise from the fact that the finite-difference energy method leads to larger truncation errors at boundaries than inside domains. If the user is particularly interested in stress at a juncutre or boundary, it is urged that he concentrate mesh points in these areas to minimize truncation error. In any case, the BOSOR4 program is written so as to minimize the effect of boundary truncation error. The stress resultants are "corrected" as described on pages of 172 and 173 of [6]. In addition, "extra" mesh points are automatically inserted near the points on juncutres and boundaries in order to make the truncation error as small as is feasible without encountering difficulties associated with precision round-off error. This feature is more completely described in the input data section.

Correct Modeling of Discrete Rings

It has been common in past analyses to neglect out-of-plane bending stiffness (terms involving I_x) which is called "RIX" in the manual and torsional stiffness (terms involving GJ) in the analysis of shells with discrete rings. The user is cautioned not to neglect these terms, in particular not to neglect the out-of-plane bending stiffness of the discrete rings (I_x). Such neglect may lead to very low estimates of the buckling loads, particularly in cases in which the ring is prestressed in compression and in which its centroid is located at several shell thicknesses away from the reference surface. Note also, that if the web of the ring is very thin in comparison with its length (height), the composite shell-ring structure may fail by crippling or "sidesway" of the web. These failure modes can be predicted by treatment of the webs as shell branches rather than as parts of the discrete rings, as described in previous sections. If a discrete ring occurs at a plane of symmetry, and this plane is used as a boundary in the analytical model, the user should set the ring modulus E, torsional rigidity GJ, and density RM, equal to 1/2 their actual values. All other quantities remain unchanged.

Also see the note on the page where the discrete ring input is defined. This note has to do with the use of discrete rings to simulate a large mass.

Rigid Body Displacement

For $n = 0$ and $n = 1$ circumferential waves, rigid body motion is possible if the shell is not sufficiently constrained by the boundary conditions. The six possible rigid body modes, three translational and three rotational, can be prevented by choosing a meridional station at which to restrain the axial displacement u^* and the circumferential displacement v . The BOSOR4 manual describes an input variable, IRIGID, through which rigid body constraints are introduced in order to prevent $n = 0$ and $n = 1$ rigid body displacements. For $n > 1$ these constraints are automatically released and replaced by whatever the user has specified for IU^* , IV , IW^* , IX at the segment number and mesh point number corresponding to the meridional location at which the rigid body constraints have been applied. In this way rigid body displacements are prevented without introduction of spurious stresses.

Behavior at Apex of Shell

Certain regularity conditions exist at the apex of shells the meridians of which intersect the axis of revolution. These conditions have been satisfied to the extent which the finite difference model permits. Because of the "half-station" spacing of u and v , however, all of the regularity conditions are not satisfied exactly at the apex. This truncation error leads to errors in the local values of the stress resultants in the immediate neighborhood of the apex. The actual stress resultants at the apex can be obtained simply by extrapolating the solution from a region slightly away from the apex in which it is regular.

Buckling and Vibration of Structures with Planes of Symmetry

A fairly common oversight on the part of a program user is the failure to run a case in which buckling and vibration modes are sought which are antisymmetric with respect to a plane of symmetry. If half a shell or a part of a shell is being analyzed because of the existence of planes of symmetry, then the analyst should check for buckling and vibration both symmetrical and antisymmetrical with respect to the planes of symmetry. See the paragraph on "Correct Modeling of Discrete Rings" for how to model a discrete ring at a plane of symmetry.

Calculation of Same Eigenvalue Twice, Eigenvalues Out of Order

In problems for which the user requires more than about 5 eigenvalues for a given circumferential wave number N, the eigenvalue extraction routine occasionally computes the same eigenvalue and eigenvector more than once. It is also possible on occasion that eigenvalues will be calculated out of order or that an eigenvalue will be missed. Unfortunately, there is no way to make an eigenvalue finder based on

equations of the type used in the BOSOR4 program 100% reliable. The calculated eigenvalues are always eigenvalues of the system, but occasionally some eigenvalues may be repeated or missed. If it is suspected that an eigenvalue has been missed, it may help to run the case with a different number of mesh points, or to run the same case with a higher value of NVEC.

Multiple or Closely-Space Eigenvalues

In the case of ring stiffened shells it may turn out that eigenvalues corresponding to vibration frequencies or buckling loads are close together. This is particularly true of ring stiffened cylinders where the rings are equally spaced and rather stiff in bending compared to the shell bending stiffness. With such a configuration there are many modes in which the motion of the rings is of small amplitude compared to that of the shell. The bays between the rings vibrate at frequencies or buckle at loads which may approximate those corresponding to a simply-supported cylinder of the same geometry as the bay. Multiple or close-spaced eigenvalues correspond to modes in which one or more of the bays is vibrating or buckling while others are unaffected. True multiple eigenvalues are generally eliminated by use of symmetry and antisymmetry conditions at planes of symmetry in the shell. In eigenvalue problems the user should always analyze as small a segment of shell as possible in order to avoid numerical difficulties associated with multiple eigenvalues.

Block Sizes Too Large

On occasion the user will encounter the diagnostic "Block size of Segment No. exceeds maximum allowable

What is a block? In BOSOR4 the stiffness, mass, and load-geometric matrices are stored on disk or drum in blocks. The logic in the program is set up such that a given block must contain the information relevant to assembly of complete shell segments. The lowest possible number of segments per block is one, of course. Figure A16 shows a stiffness matrix configuration. Only the elements inside the "skyline" - the heavy line enclosing all non-zero elements below and including the main diagonal - are stored. The block size is equal to the number of little squares. In prebuckling problems the maximum block size is 2850; in stability, vibration, and non-symmetric stress problems the maximum block size is 3333. The program checks at the end of each segment to see if the elements corresponding to the next segment will cause the block to overflow. If they do, a new block is started.

It occasionally happens that the number of elements within the "skyline" corresponding to a single segment exceeds one or both of the allowable limits of 2850 or 3333. For example, referring to Figure A16, one can imagine that if the horizontal "skyscrapers" corresponding to the juncture conditions in segment ② were very long, or if there were very many of them, the number of little squares within the skyline from Equation 30 to Equation 64 (Segment ②) might exceed the allowable limits. It is this situation that causes the message "Block size ... exceeds maximum allowable...." to be printed and the run to be aborted. The user can almost always find a way around this problem by reordering the segments or dividing up the segment with many branch conditions into more than one segment.

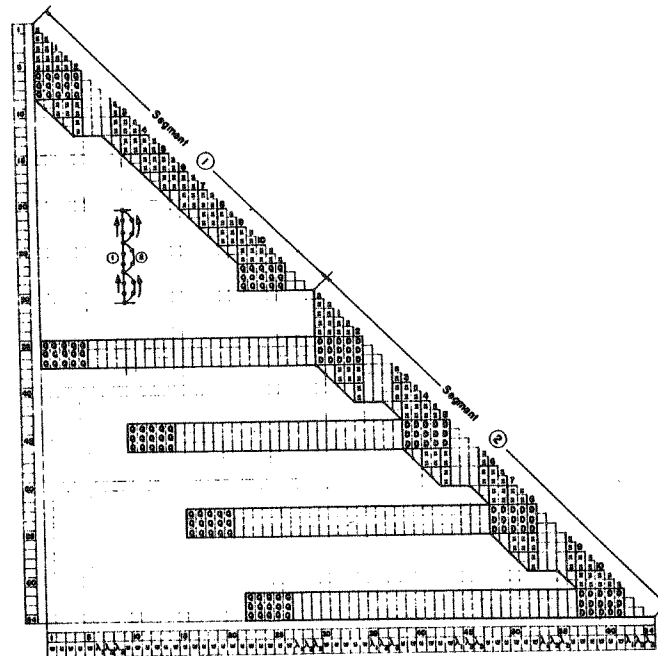


Fig. A16 Stiffness matrix configuration for double-walled shell fastened intermittently.

Moment Resultants and Reference Surface Location

More than one BOSOR4 user has indicated concern about the values obtained for the moment resultants M_{10} , M_{20} or M_1 , M_2 . In this paragraph it is emphasized that these moment resultants are the values with respect to the reference surface, which may not necessarily be the middle surface. The magnitude of the moment resultants depends, therefore, on the location of the reference surface relative to the shell wall material. For example, in a uniformly loaded monocoque cylinder, if the inner or outer surface is used as the reference surface, the moments M_1 , M_2 will approach the values

$$|M_1| = |N_1|t/2; \quad |M_2| = |N_2|t/2$$

far away from the edges. (t = thickness; N_1 , N_2 = stress resultants) Note, however, that the extreme fiber stresses are of course not dependent on the location of the reference surface. In this connection please recall that the commonly used formula for extreme fiber stress

$$\sigma = \frac{N}{t} + \frac{6M}{t^2}$$

only applies if the shell is monocoque and if the middle surface is used as the reference surface.

Remarks on the Hemisphere Vibration

In this sample case the control integer IRIGID is set equal to unity. While this case does illustrate the proper mechanical use of the IRIGID $\neq 0$ option to prevent rigid body motion associated with $n = 0$ and $n = 1$ circumferential waves, the choice of a vibration analysis for the demonstration is a poor one, since the frequencies corresponding to $n = 0$ and $n = 1$ will depend upon the location of the constraints. The frequencies and modes will correspond to the actual free-free hemisphere vibrations only if the constraints are imposed such that the center of mass of the structures does not move during vibration in the $n = 0$ or $n = 1$ modes. Actually, in vibration analyses it is never necessary to set IRIGID $\neq 0$. To put it more clearly, IRIGID should be zero in vibration analyses. Note, however, that this case does illustrate the proper way to handle the problem of rigid body motion, which must be handled in stress and stability analyses.

Modeling Global Moments and Shear Forces

The user may wish to determine local stresses in a shell structure caused by certain known global moments and shear forces. Figure A17 shows one way in which the global forces might be converted into equivalent line loads. A cylinder with an end ring is loaded by a net shear force and moment (a). The shear force is assumed to act uniformly around the circumference as shown in (b). At every circumferential station θ , the shear force in (b) is resolved into components

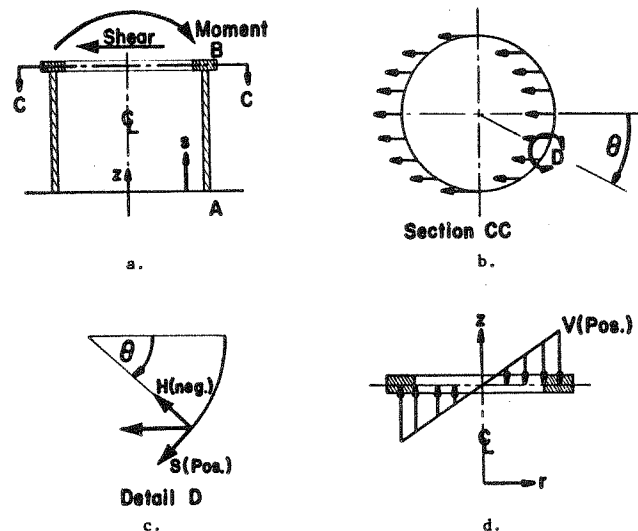


Fig. A17 Modeling global moments and shear forces

normal (H) and tangential (S) to the ring centroidal axis (c). The "global" moment M is modeled as an axial load which varies around the circumference as shown in (d). With the coordinate system shown it is clear that

$$V = V_0 \cos \theta, \quad S = S_0 \sin \theta, \quad H = H_0 \cos \theta$$

with V_0 and S_0 positive and H_0 negative. Referring to Table A2 we see that for this circumferential distribution of line loads we must use $n = -1$ as input to BOSOR4. (NSTART = NFIN = -1, INCR = 1 or -1)

Shear Line Loads, Concentrated or Otherwise

BOSOR4 users have had difficulty providing the correct input for shear line loads. This paragraph should help to clear up the trouble. Figure A18 shows an example of a ring with equal concentrated loads S applied at $\theta = 90^\circ$ and $\theta = 270^\circ$. In BOSOR4 concentrated

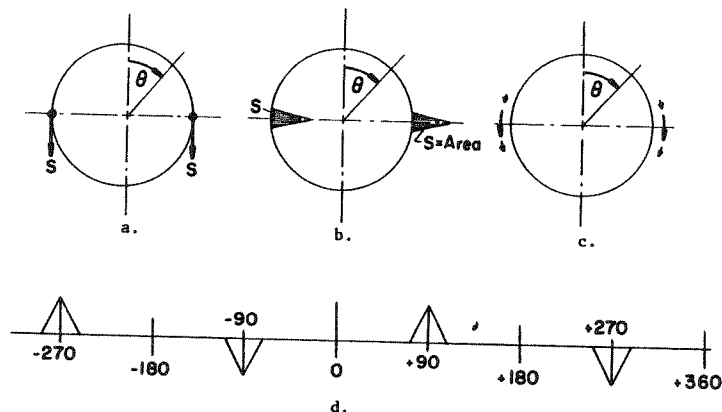


Fig. A18 Concentrated shear loads

loads are handled by spreading them out over a finite angle, say 5° or 10° , as shown in Fig. A18. The sign convention for shear loads is such that positive S is in the direction of increasing circumferential coordinate θ . Thus, if we plotted the triangular peaks shown in Fig. A18b on a rectilinear scale, we would obtain the odd function shown in Fig. A18d. It is emphasized that even though the shear loading is symmetrical about $\theta=0$, as seen from Fig. A18a, it is described by an odd function in the interval $-180^\circ < \theta < 180^\circ$. In this example, therefore, THETAM would be 180° , NODD would be 2, and the Fourier Harmonics -1 through -39 in steps of -2 would be used, since negative harmonics imply that the shear force varies as $\sin|N|\theta$. Also note that the entire range $-180^\circ < \theta < 180^\circ$ must be used, since the function repeats every 360° . If in Fig. A18a the S at 270° pointed upward, then the function would be even, THETAM would be 90° , and the Fourier harmonics would be $+0, +2, +4, \dots, +38$.

Best Way to Run Cases with the INDIC = 1 Option

The BOSOR4 User's Manual says that with INDIC = 1 a linear buckling analysis is performed. Actually, this is not strictly so. With INDIC = 1 BOSOR4 performs a nonlinear prebuckling analysis for the "fixed" or "initial" loads, $P, V(\cdot)$, etc., and then another nonlinear prebuckling analysis for $P + DP, V(\cdot) + DV(\cdot)$, etc. The prestresses and shape change (meridional rotation distribution X_0) corresponding to the initial loads $P, V(\cdot)$, etc., modify the stability

stiffness matrix. The changes in meridional and hoop stress resultants N_{10} and N_{20} due to the load increments $DP, DV(\cdot)$, etc., contribute terms to the so-called "load-geometric" matrix or "Lambda-matrix." The critical loads are then

$$P_{cr} = (P + (\text{Eigenvalue}) \cdot DP) \cdot PDIST$$

$$V(\cdot)_{cr} = V(\cdot) + (\text{eigenvalue}) \cdot DV(\cdot); \text{ etc.,}$$

where PDIST represents the meridional distribution of pressure. It is best, when doing an INDIC = 1 type of buckling analysis, to observe the following two rules:

1. Never have both non-zero initial load and non-zero increment for the same type of load.

EXAMPLE: $P = 0.0, DP = 1.0$ is O.K.

$P = 50.0, DP = 1.0$ is inadvisable, mainly because the user could easily err in interpreting the eigenvalue.

ANOTHER EXAMPLE: $P = 0.0, DP = -1.0$

$V(1) = 75.0, DV(1) = 0.0$ is O.K. because P and $V(1)$ are different kinds of loads.

2. Always choose loads that are small compared to the design load of the structure. In other words, choose magnitudes of the loads for which the prebuckling behavior really is linear. It is generally advisable to set $DP = -1.0$, for example, since the eigenvalue then represents the critical pressure directly. Remember that the actual pressure is $DP \cdot f(s)$, where $f(s)$ is the meridional distribution. (In the examples it is tacitly assumed that $f(s) = 1.0$.)

Miscellaneous Suggestions

It is often advisable in buckling analyses to use INDIC = 1 with a rather wide range for N for the first run through the computer (linear buckling analysis). With this choice NVEC buckling loads are obtained for circumferential wavenumbers from $N = NOB$ to $N = NMAXB$ in steps of INCRB. The user can obtain multiple buckling loads at a given N only with INDIC = 1 and 4. Computer time is often saved in this manner, since the wavenumber corresponding to the minimum load is often not known a priori, even approximately. Also, there are cases for which two minima exist, and the user must find the absolute minimum. With

INDIC = -1, only the relative minimum will be found unless more than one case is run, each case with its own range of N.

The capability of finding more than one buckling load at a given N is particularly useful to the designer who wishes to find the allowable buckling of a complex shell such as that shown in Fig. 1. The lowest buckling pressure might correspond to buckling of the cylinder, but at a few psi higher the ogive might buckle. Thus, the designer would not greatly improve the overall structure by strengthening just the cylinder. He must know the loads for which each of the segments buckles when these segments are analyzed as part of a larger structure.

In cases for which two eigenvalues are close together or for which bifurcation buckling loads are close to axisymmetric collapse loads, it is occasionally advisable to use INDIC = -2. In this way the first vanishing point of the stability determinant is approached gradually, and if axisymmetric collapse occurs at higher loads than nonsymmetric buckling, the stability determinant will change sign and the bifurcation buckling load will be determined.

With INDIC = 4 there are two possible flows of calculations:
If IPRE = 0 the prebuckling stress resultants N_{10} and N_{20} and the prebuckling meridional rotation X_0 are read in directly for a certain number, NSTRES, of meridional stations. Linear interpolation is performed internally for calculation of these prebuckling quantities at all of the mesh stations of each segment. Buckling loads (NVEC eigenvalues for each circumferential wave number N) are then calculated for the range NOB to NMAXB in steps of INCRB.
If IPRE \neq 0 the prebuckling quantities are calculated from the linear theory for nonsymmetrically loaded shells, just as if INDIC were equal to 3. The user preselects the meridian (value of θ , called THETAS, which he feels represents the "worst" prestress from the point of view of stability. For example, a cylinder submitted to external pressure which varies around the circumference will generally buckle where the pressure has the highest amplitude. The BOSOR4 program will use the meridional stress distribution at $\theta = \text{THETAS}$ in the stability calculations. In the stability analysis the flow of calculations for both cases IPRE = 0 and IPRE \neq 0 is the same as that for INDIC = 1.

BOSOR4 OUTPUT

Nomenclature of the BOSOR4 Output (Sample Units)

(Units do not have to be in in. and lb)

ALPHA1	angle from axis to beginning of spherical segment (degrees)
ALPHA2	angle from axis to end of spherical segment
ALPHAT	distance from axis to center of curvature of spherical segment
AREA	discrete ring cross-sectional area (in. ²)
BETA	meridional rotation, denoted X in analysis (radians)
CHIO	prebuckling rotation X_0 (radians)
CURL	meridional curvature, $1/R_1$ (in. ⁻¹)
CUR2	normal circumferential curvature, $1/R_2$ (in. ⁻¹)
CURLD	s derivative of meridional curvature, $(1/R_1)$ (in. ⁻²)
DET	stability determinant "mantissa": Determinant = DET*10 ^{NEX}
DH	eigenvalue radial line load or radial line load increment (lb/in.)
DM	eigenvalue meridional moment, or meridional moment increment (in.-lb/in.)
DP	eigenvalue pressure multiplier, or pressure increment multiplier (psi)
DTEMP	eigenvalue temperature rise multiplier, or temperature rise increment multiplier
DV	eigenvalue axial line load or axial line load increment (lbs/in.)
EIGENVALUE	Meaning depends upon case. (See following section)
ER	discrete ring modulus of elasticity (psi)
E1	discrete ring radial eccentricity (in.)
E2	discrete ring axial eccentricity (in.)
GJ	discrete ring torsional rigidity (lb-in. ²)
H	"fixed" or initial radial line load (lb/in.)
ITER	number of Newton-Raphson iterations for convergence of nonlinear axisymmetric stress analysis to within 0.1%
IX	discrete ring moment of inertia about x axis (in. ⁴)
IY	discrete ring moment of inertia about y axis (in. ⁴)
IXY	discrete ring product of inertia (in. ⁴)