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BIFURCATION BUCKLING OF SHELLS OF REVOLUTION INCLUDING LARGE DEFLECTIONS, PLASTICITY AND CREEP

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ABSTRACT

A summary is first presented of the conceptual difficulties and paradoxes surrounding plastic bifurcation buckling analysis. Briefly discussed are nonconservativeness, loading rate during buckling, and the discrepancy of buckling predictions with use of J2 flow theory vs J2 deformation theory. The axisymmetric prebuckling analysis, including large deflections, elastic-plastic material behavior and creep is summarized. Details are given on the analysis of nonsymmetric bifurcation from the deformed axisymmetric state. Both J2 flow theory and J2 deformation theory are described. The treatment, based on the finite-difference energy method, applies to layered segmented and branched shells of arbitrary meridional shape composed of a number of different elastic-plastic materials. Numerical results generated with a computer program based on the analysis are presented for an externally pressurized cylinder with conical heads.

INTRODUCTION

Summary

To date bifurcation buckling analyses involving plasticity have been applied to simple structures with uniform prestress. Basic conceptual difficulties have been cleared up and paradoxes resolved. It is now understood that the nonconservative nature of plastic flow does not prevent the use of bifurcation buckling analysis to predict instability failure of practical structures; the concept of consistent loading of the material in the transition from prebifurcation state to adjacent postbifurcation state permits the use of instantaneous prebifurcation material properties in the stability equations; and an investigation of the effect of very small initial imperfections on the collapse loads of cruciform columns indicates that the reduced shear modulus G obtained from deformation theory should be used in the stability equations even if there is no history of shear along the prebifurcation path. With the high speed electronic computer it is now feasible to calculate elastic-plastic bifurcation buckling loads of rather complex structures. The purpose of this paper is to present the theory for nonsymmetric bifurcation buckling of axisymmetrically loaded branched shells of revolution, including large axisymmetric prebuckling deflections and elastic-plastic effects.

The problem of nonconservativeness

Systems involving plastic flow are nonconservative. The energy required to bring a structure from its prebifurcation state to an adjacent buckled state depends upon the path of transition if any of the material is loading into the plastic region. Hill [l, 21 has shown, however, that as long as the infinitesimal path is reasonably direct the variation in infinitesimal energy dissipation from one path to another consists of higher order terms only.

The problem of loading rate during buckling

Analysis of the bifurcation buckling of elastic-plastic structures dates back to 1889 when Engesser[3] presented his tangent modulus theory for columns and Considere[4] set forth the "effective" or "double" modulus theory based on the assumption that the column unloads elastically on the concave side during incipient buckling at a given load. In 1895 Engesser, who had assumed that the total load on the column remains constant during buckling, acknowledged error in his original theory and determined the general expression for the reduced modulus. In 1910 von Karman [5] presented the Considere-Engesser theory again, with actual evaluation of the reduced modulus for rectangular and idealized H-sections and comparisons with tests. Until Shanley's paper appeared in 1947 [6], the reduced modulus or "double modulus" model was accepted as the exact theory of column action, even though the tangent modulus model gave better agreement with tests. Shanley[6] resolved the paradox in 1947, when he stated:

"…in the derivation of the reduced-modulus theory a questionable assumption was made. It was assumed, by implication at least, that the column remains straight while the axial load is increased to the predicted critical value, ***after which*** the column bends, or tries to bend, Actually, the column is free to 'try to bend ' at any time. There is nothing to prevent it from bending ***simultaneously*** with increasing axial load. Under such a condition it is possible to obtain a nonuniform strain distribution without any strain reversal taking place."

...
In a discussion appended to Shanley's paper, von Karman further clarified the theory stating,

" Both Engesser's and my own analyses of the problem were based on the assumption that the equilibrium of the straight column becomes unstable when there are equilibrium positions infinitesimally near to the straight equilibrium position ***under the same axial load.*** . . . Mr. Shanley's analysis represents a generalization of the question. . . . What is the smallest value of the axial load at which a bifurcation of the equilibrium positions can occur, regardless of whether or not the transition to the bent position requires an increase of the axial load?"

In 1950 Duberg and Wilder [7] provided further insight into the problem by showing that for small, finite imperfections bending will take place immediately as the load is applied but that local unloading of the material will not occur until the column is subjected to a relatively large bending moment. For vanishingly small initial imperfections, finite bending of the column will start at the bifurcation load predicted by the tangent modulus theory. Elastic unloading will not occur, however, until a higher load at which the column has deformed a finite amount along the post-bifurcation load-deflection curve. Duberg and Wilder show that for practical engineering materials the maximum load carrying capability of the column is only slightly above the tangent modulus bifurcation point.

It is physically reasonable to extend the concept of "tangent modulus bifurcation" to buckling of two-dimensional structures-plates and shells. Experiments and analyses have been conducted for simple plates and shells in which the prebuckling state is characterized by uniform compressive stress (see, e.g. [8 -16]). The analyses just cited are based on the tangent modulus method. Sewell[l7] gives a more extensive bibliography.

In 1972 Hutchinson [18] calculated axisymmetric collapse pressures of an elastic-plastic spherical shell with various axisymmetric imperfections. As the imperfection amplitude approaches zero the collapse load approaches a value very slightly above the tangent modulus bifurcation load calculated from J_2 flow theory for a perfect shell. In justifying the use of the tangent modulus approach to bifurcation problems in general, Hutchinson [19] in 1974 wrote:

"The bifurcation solution is a linear sum of the fundamental solution increment and the eigenmode. We can always include a sufficiently large amount of the fundamental solution increment relative to the eigenmode such that the bifurcation mode satisfies the total loading restriction. . . . The confusion in bifurcation applications apparently stems from the misconception that when bifurcation occurs total loading will be violated. On the contrary, it is the total loading condition itself which supplies the constraint on the combination of fundamental solution increment and eigenmode which must pertain."

The "total loading" condition cited above justifies the use of the "tangent modulus" approach to bifurcation buckling problems of elastic-plastic shells. The fact that the collapse load is only slightly above the bifurcation load for vanishingly small imperfections makes an elastic-plastic bifurcation stability analysis in principle just as suitable for design purposes as an elastic bifurcation stability analysis. For bifurcation buckling of general shells under combined loading, in which the stresses are nonuniform and in which the prebuckling solution may be characterized by regions which are elastic or unloading and other regions which are loading into the plastic range, the "total loading" condition enunciated by Hutchinson may be generalized by the statement that the rate of change of material properties or "tangent properties " in the prebifurcation analysis governs the eigenvalue analysis also.

The flow theory versus deformation theory paradox

During the years when plastic buckling of uniformly stressed plates and shells was first being investigated, a perplexing paradox became apparent: Theoretical considerations and direct experimental evidence indicates that for general load paths flow theory is correct while deformation theory is not. However, bifurcation buckling analyses based on deformation theory conform better to experimentally determined buckling loads than do such analyses based on flow theory. The discrepancy may have to do with whether or not the instantaneous yield surface has corners. Experimental evidence on this point is contradictory. Experiments by Smith and Almroth [20] indicate that the yield surface may develop a region of very high curvature which "smooths" out with time.

The discrepancy in the prediction of bifurcation buckling loads is most pronounced in the case of an axially compressed cruciform column, discussed by Drucker [21], Cicala [22], Bijlaard [23] and Onat and Drucker [12]. In this example the prebifurcation stress state is uniform compression while the bifurcation mode involves pure shear. In a flow theory involving a smooth yield surface the shear modulus remains elastic as the material of the column is uniformly compressed into the plastic range. Use of the deformation theory gives the instantaneous shear modulus

$$\boldsymbol{Gbar = G/[1 + 3G(1/E_s - 1/E)]} \qquad (1)$$

where $\boldsymbol{E_s}$ is the secant modulus. Since the predicted buckling stress is proportional to the effective shear modulus, the discrepancy in predicted bifurcation loads is governed by the difference between *G* and *Gbar*. Onat and Drucker resolved the paradox by showing that cruciform columns with very small initial twist distributions collapse at loads slightly above the bifurcation loads predicted with deformation theory. Apparently a very small amount of shearing strain in the prebifurcation solution suffices to reduce the effective shear modulus from the elastic value *G* to a value near that predicted by deformation theory.

Because of this extreme sensitivity of the shear modulus to small. imperfection-related shearing forces applied while the material is being stressed, nominally without shear, into the plastic range, the value of G_{eff} predicted by deformation theory is used in the J_2 flow theory bifurcation analysis presented herein. The purpose of this strategy is to eliminate much of the flow theory versus deformation theory discrepancy in buckling predictions while retaining a realistic model of the material submitted to reasonably general axisymmetric prebifurcation loading histories.

However, in cases involving no in-plane shear either in the prebuckling phase or in the buckling process, J_2 deformation theory still predicts lower bifurcation buckling loads than does J_2 flow theory. The axisymmetric buckling analysis of a spherical shell presented by Hutchinson [18] is a good example. Since J_2 deformation theory has given better agreement with test results than has J_2 flow theory, and since the discrepancy is not entirely related to the difference in effective shear modulus, it is prudent to perform stability analyses using both theories in order to establish the sensitivity of the predictions to the two models. Therefore, a J_2 deformation theory option is included in the analysis presented here.

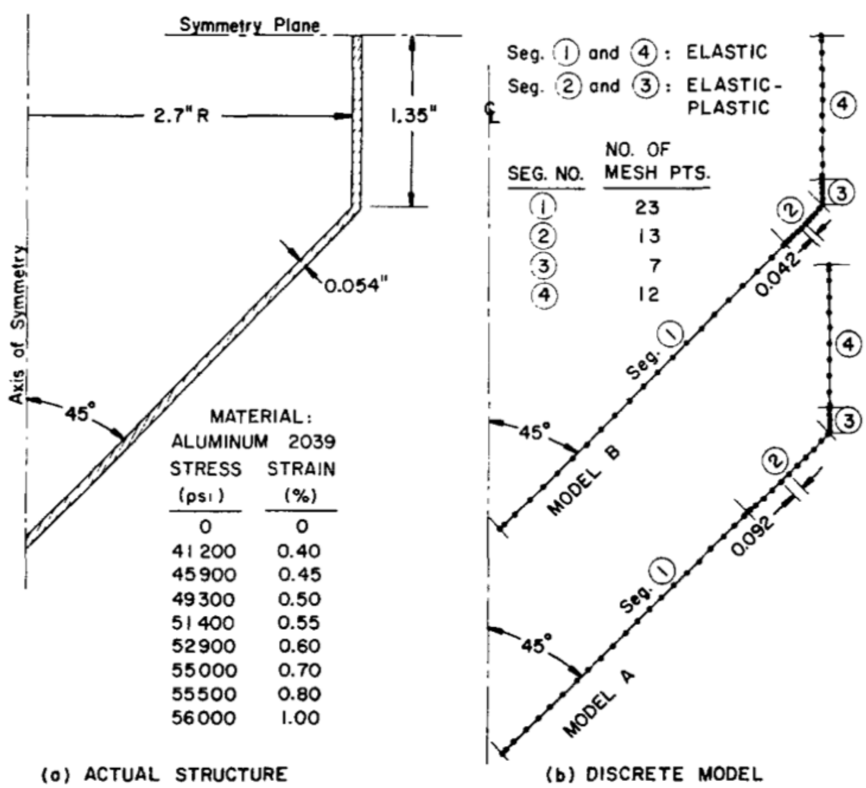

Fig. 3 Aluminum pressure vessel (external pressure) tested by Galletly at the University of Liverpool and discretized models for BOSOR5. (from International Journal of Solids and Structures, Vol. 10, pp. 1287-1305, 1974)

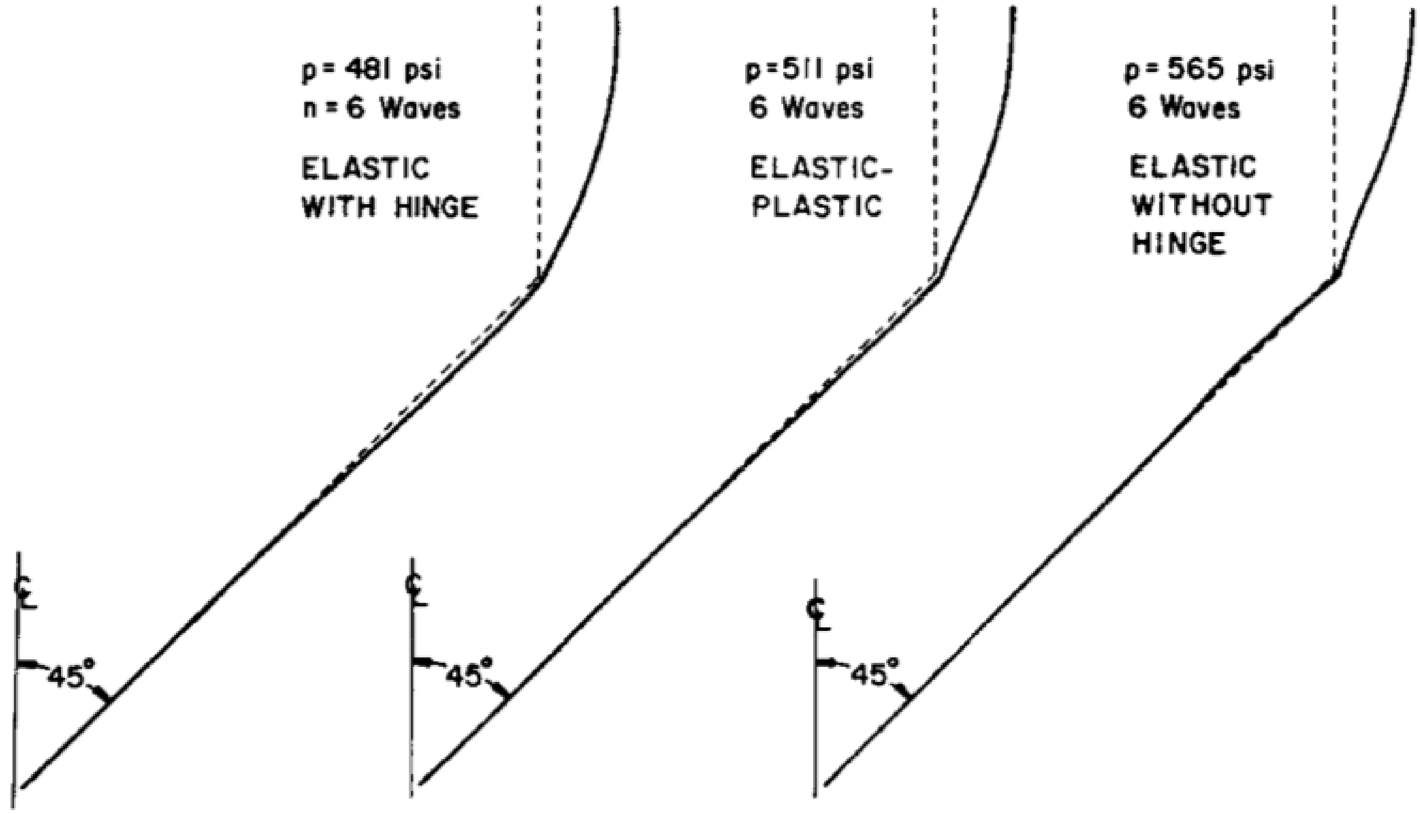

Fig. 6 Buckling modes with various models. (from International Journal of Solids and Structures, Vol. 10, pp. 1287-1305, 1974)