From: International Journal for Numerical Methods in Engineering, Vol. 11, 683-708 (1977)

A STRATEGY FOR THE SOLUTION OF PROBLEMS INVOLVING LARGE DEFLECTIONS, PLASTICITY AND CREEP

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(This is an abridged version. See the full-length paper for more: bosor5.papers/1977.strategy.pdf)

SUMMARY

A strategy for solving problems involving simultaneously occurring large deflections, elastic-plastic material behaviour, and primary creep is described. The incremental procedure involves a double iteration loop at each load level or time. In the inner loop the material properties are held constant and the non-linear equilibrium equations are solved by the Newton-Raphson method. These equations are formulated in terms of the tangent stiffness. In the outer loop the plastic and creep strains are determined and the tangent stiffness properties are updated with use of a subincremental algorithm. The magnitude of each time subincrement is determined such that the change in effective stress is less than a preset percentage of the effective stress. The strategy is implemented in a computer program, BOSOR5, for the analysis of shells of revolution. Examples are given of elastic-plastic deformations of a centrally loaded flat plate and elastic-plastic-creep deformations of a beam in bending. The major benefits of the subincremental technique are the increased reliability with which problems involving non-linear plastic and time-dependent material behaviour can be solved and the greatly relaxed requirement on the number of load or time increments needed for satisfactory results.

INTRODUCTION

The high speed digital computer has enabled analysts to construct elaborate models of structures, including large deflection effects and material non-linearity. There are several recent excellent surveys of the various approaches: Tillerson et al [1] review numerical methods used to solve non-linear equations; Armen [2] describes several analytical models of multi-axial plasticity; Nickel1[3] gives a survey of techniques for treatment of creep and reviews many widely used computer programs in which creep is included; Hunsaker et al [4] present comparisons between test and theory for currently used models of elastic-plastic material behaviour. Therefore a review of methods will not be included here.

The purpose of this paper is to explain in detail a ‘subincremental’ numerical strategy for the solution of problems in which large deflections, plasticity, and primary creep are simultaneously present. This strategy is an extension of a procedure described in Reference 5. It includes modifications for the solution of problems involving primary creep without the occurrence of numerical instability. The method has been incorporated into the BOSOR5 computer program for analysis of shells of revolution [6], and it can be used for more general configurations. Huffington [7] was the first to point out the advantage of using a subincremental method. Nayak and Zienkiewicz [8] and Stricklin et al [9] have incorporated versions of it into their computer programs.

THE SUBINCREMENTAL METHOD

Before a detailed description of the analysis is presented a brief explanation will be given of what the ‘subincremental’ technique is and why it is needed.

In practically all non-linear analyses the load is applied incrementally and the response is determined for each value of the load. Each load level involves the solution of a system of simultaneous algebraic equations, the rank of this system being equal to the number of degrees-of-freedom in the discretized mathematical model. Let us henceforth refer to this system of simultaneous equations as ‘System A’. In most analyses in which material non-linearity is included, the iteration loop for the solution of System A contains calculations for determination of the plastic strain components. Usually these quantities are obtained in a one-step process in which the total increments of strain accumulated from one load level to the next are allocated among elastic, plastic, and possibly creep components. The relative magnitudes of the various components are known, at least as the load step begins, because the analysis contains a flow theory and the position of each material point in stress space is known from the converged results associated with the previous load level. The direction of plastic flow for each material point is generally considered to be constant for the entire load increment. For example, it may be assumed that this direction is parallel to the normal to the yield surface at a location in stress space determined by the converged result at the previous load level. Determination of the plastic strain components requires in the general three-dimensional case solution of a set of six simultaneous equations at each material point and in the case of axisymmetric deformations of thin shells the solution of two simultaneous equations at each material point. We shall henceforth refer to this small system of simultaneous equations as ‘System B’.

The analysis presented here differs from most other analyses in two respects. The calculation of the plastic and creep strain components is removed from the iteration loop in which System A is solved, and a subincremental approach is used for calculation of the plastic and creep strain components so that the direction of flow is permitted to change continuously within a single load interval.

The removal of the calculations involving plastic flow from the iteration loop for the solution of System A removes an objection pointed out by Tillerson et al [1] to the use of the Newton-Raphson method for problems involving elastic-plastic material. They found that the ‘Newton-Raphson’ procedure failed to converge if they used the tangent stiffness approach because of indications of alternative loading and unloading from iteration to iteration. Since the coefficients of their System A changed in a discontinuous manner in successive iterations, their strategy could not really be called a Newton method. In the present analysis the Newton-Raphson method is used with success.

In the subincremental process the total increments of strain accumulated from one load level to the next are divided into subincrements of a certain magnitude. For each subincrement the direction of plastic flow is considered to be constant, given by the normal to the yield surface at a location in stress space determined by the result at a previous subincrement. For each strain subincrement the stress subincrements are determined from the flow law and the given relationship between effective stress subincrement and effective plastic strain subincrement (the uniaxial stress-strain curve). Thus, the equation System B is solved for each subincrement and each material point.

THE NEED FOR THE SUBINCREMENTAL METHOD

Why is the subincremental method needed? This question can perhaps be best answered with reference to the equations which form the simultaneous System B (creep neglected), etc., etc. (See the paper for more text and for the references. See the figures below and the paper for discussions of them.)

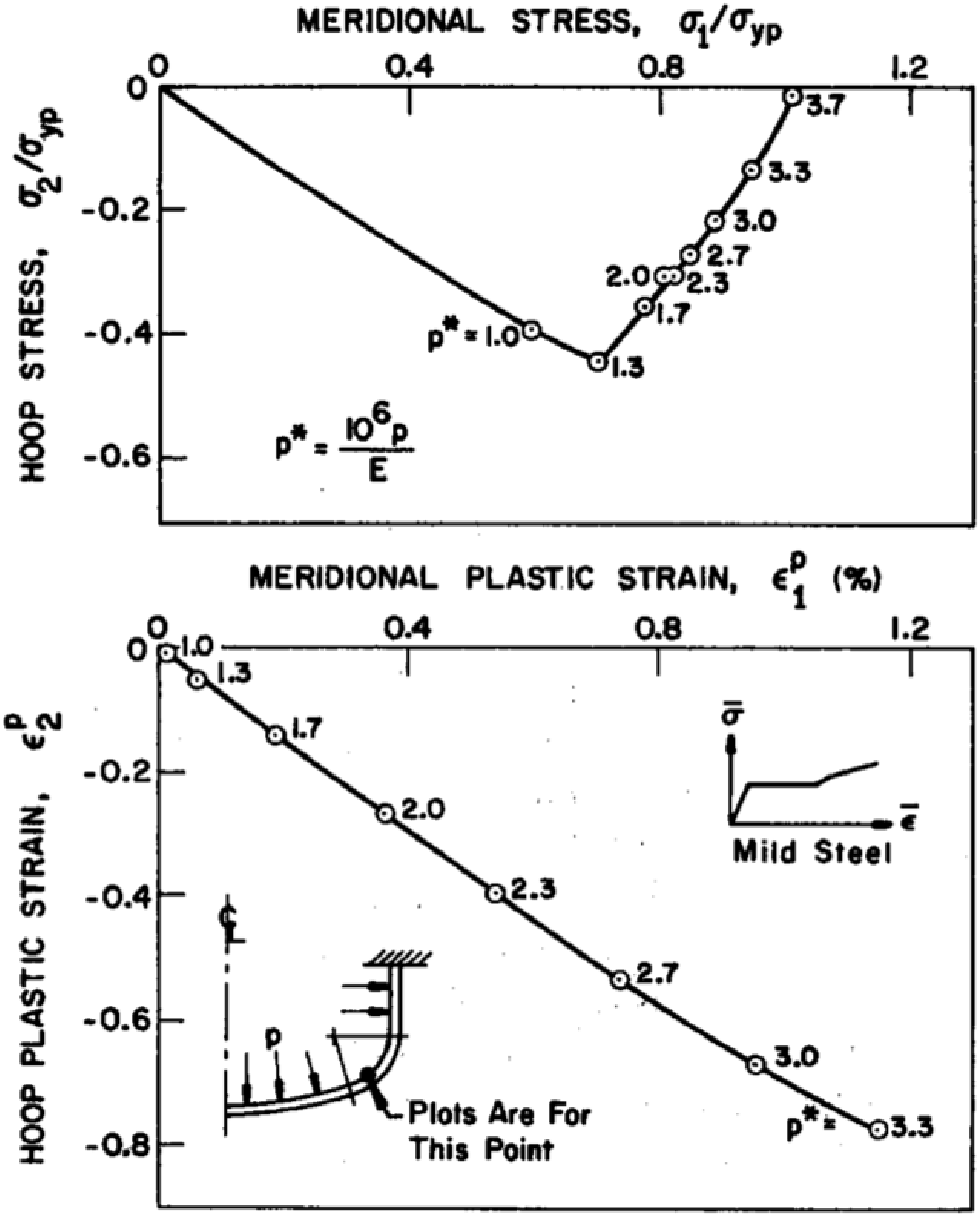

Fig. 2 Paths followed in stress space and strain space by a material point in an internally pressurized torispherical vessel head of mild steel. (from International Journal for Numerical Methods in Engineering, Vol. 11, 683-708, 1977)

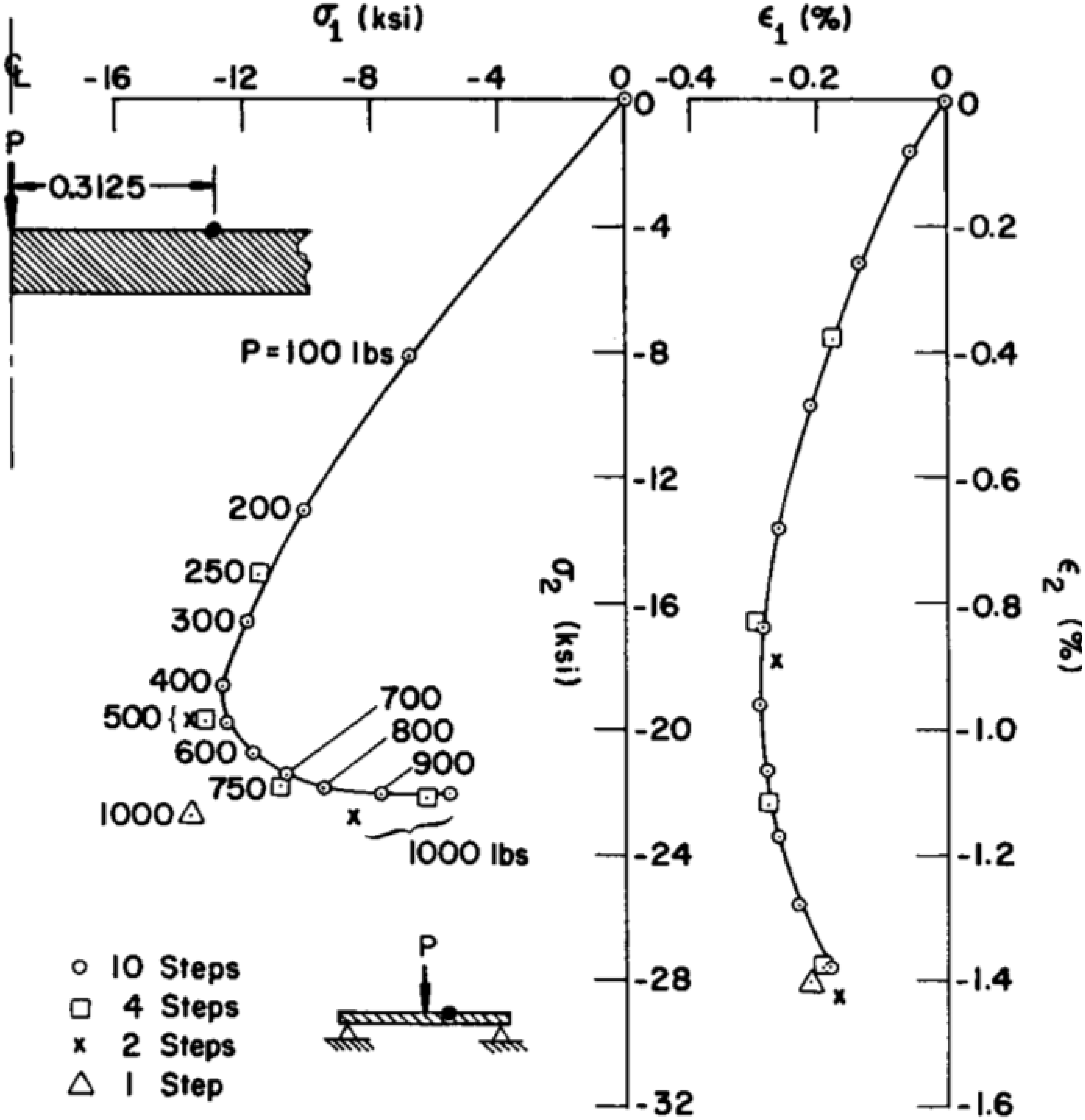

Fig. 3 Paths followed in stress space and strain space by a material point in a circular flat plate with a concentrated load, P. (from International Journal for Numerical Methods in Engineering, Vol. 11, 683-708, 1977)

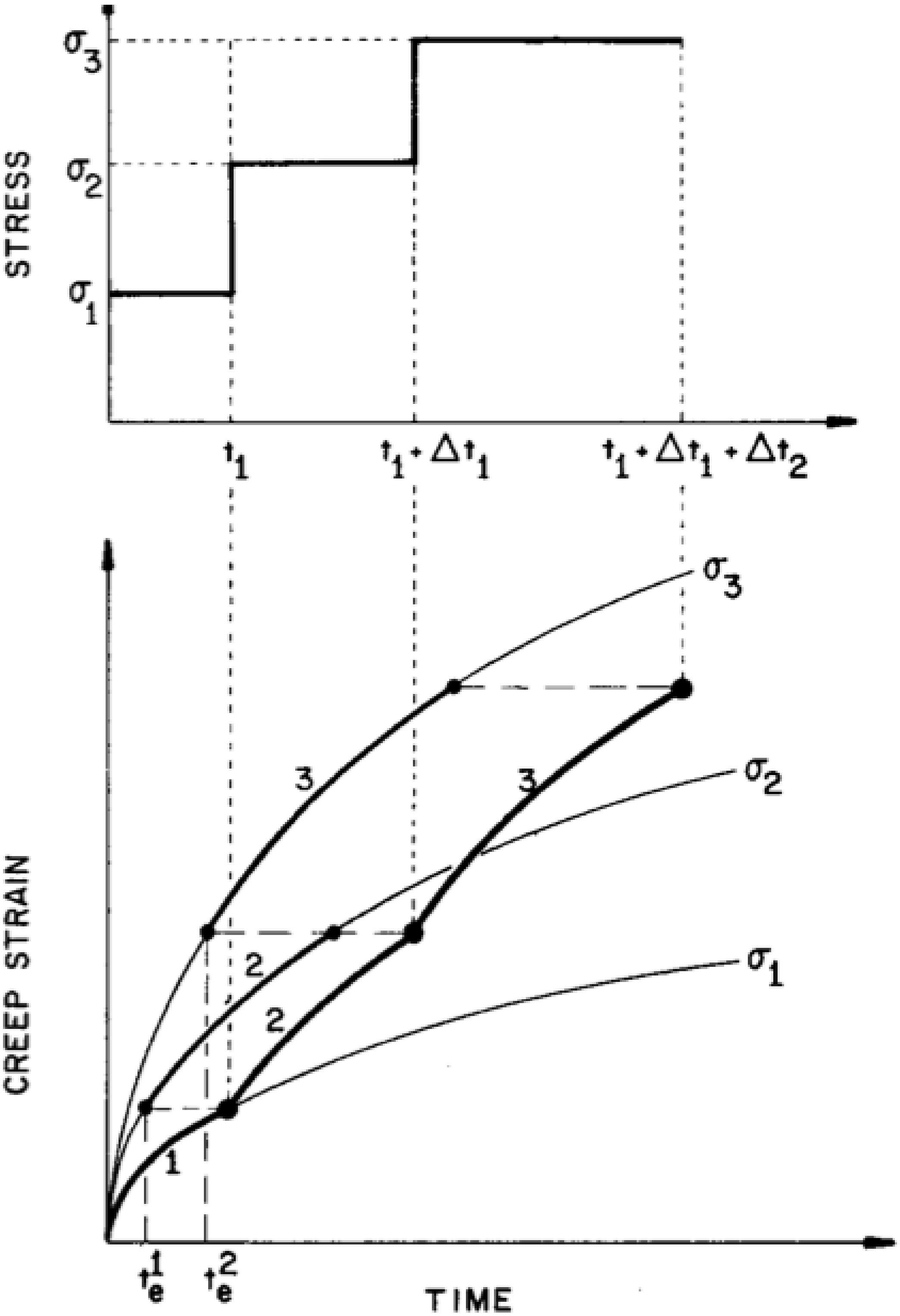

Fig. 4 Accumulated creep strain predicted with use of the strain hardening model. (from International Journal for Numerical Methods in Engineering, Vol. 11, 683-708, 1977)

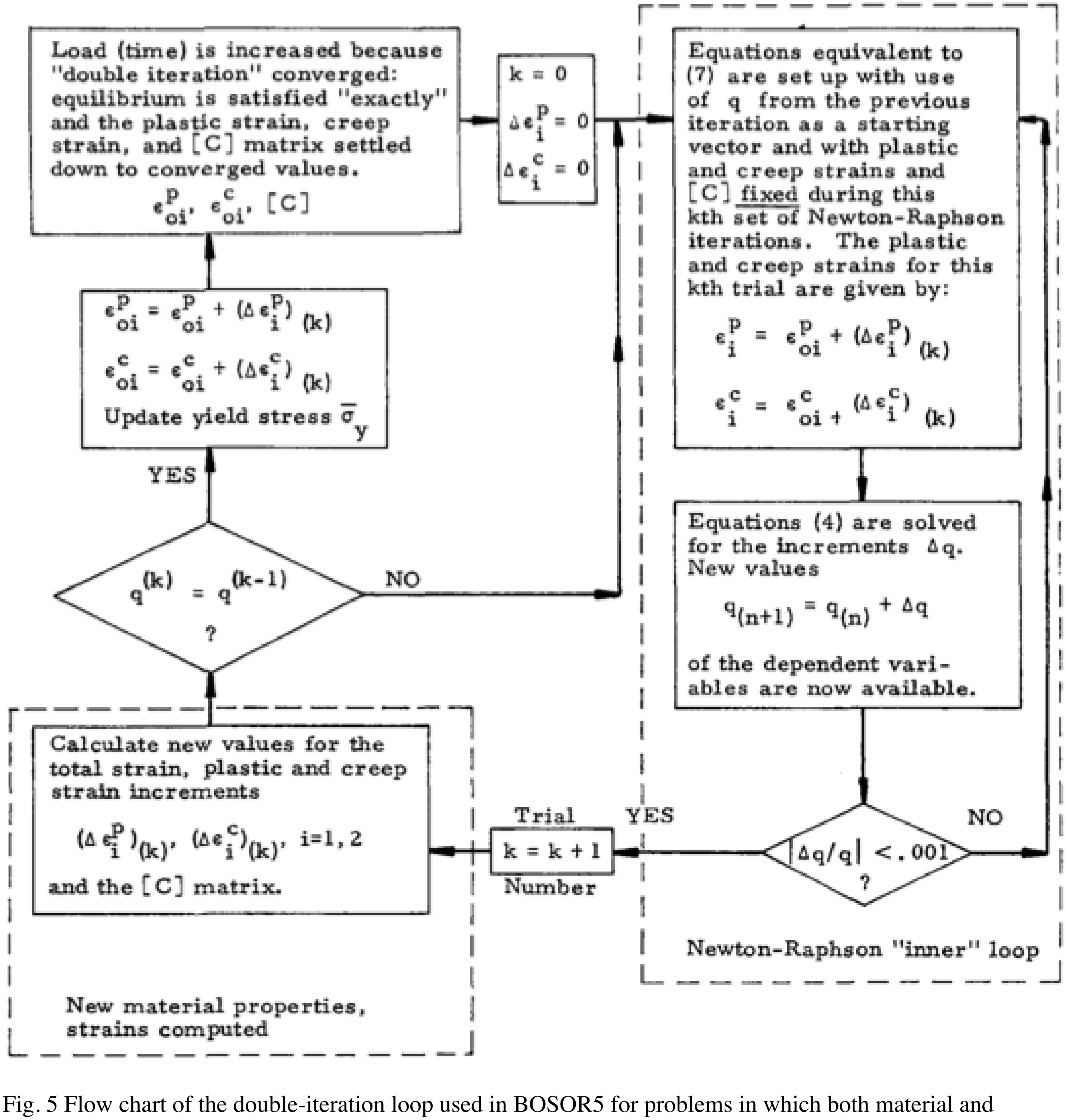

Fig. 5 Flow chart of the double-iteration loop used in BOSOR5 for problems in which both material and geometrical nonlinearities exist. (from International Journal for Numerical Methods in Engineering, Vol. 11, 683-708, 1977)

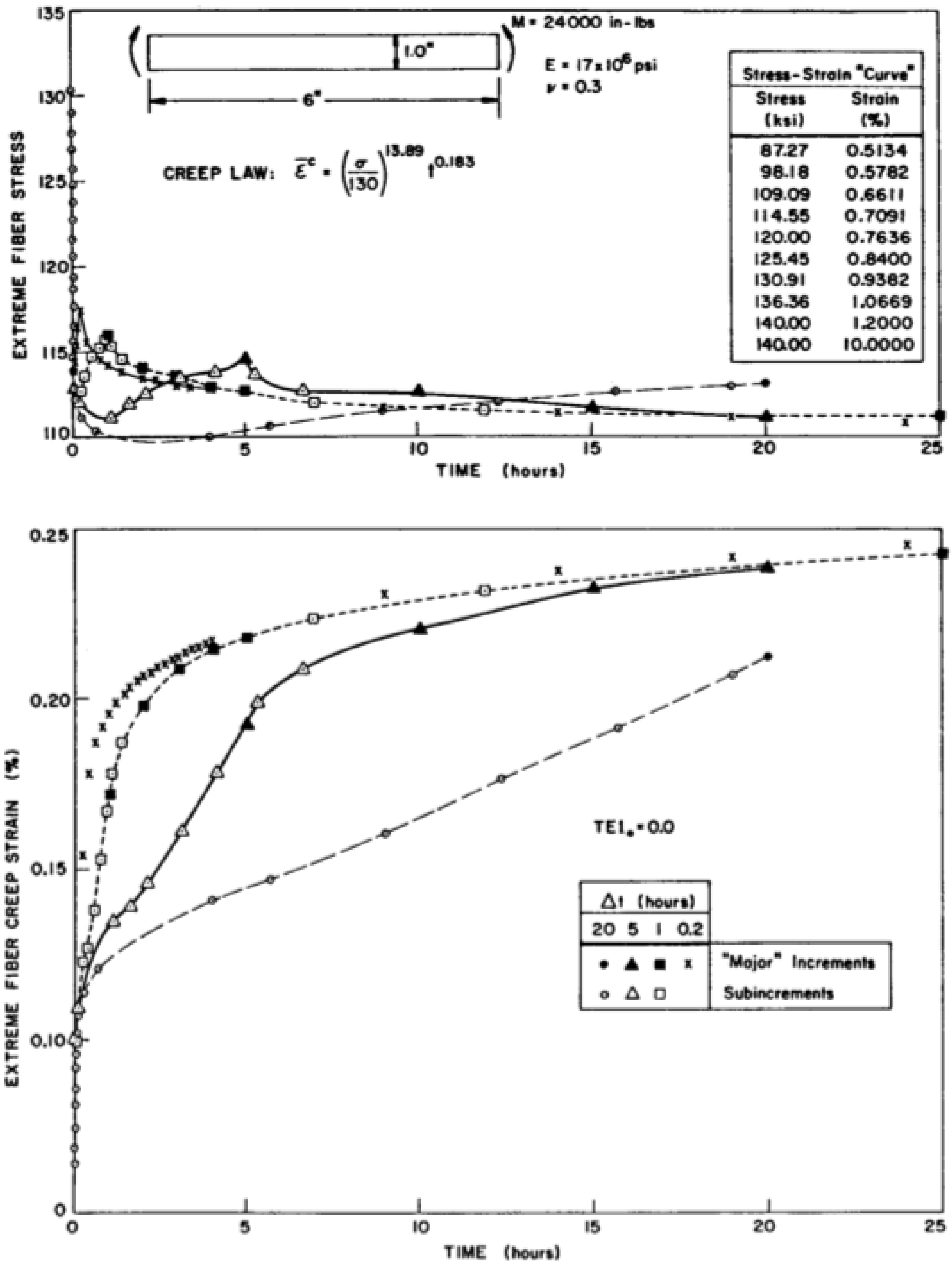

Fig. 13 Extreme fiber stress and creep strain in the titanium beam as functions of time predicted with the use of various time increments. . (from International Journal for Numerical Methods in Engineering, Vol. 11, 683-708, 1977)