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MINIMUM WEIGHT DESIGN OF IMPERFECT ISOGRID-STIFFENED ELLIPSOIDAL SHELLS UNDER UNIFORM EXTERNAL PRESSURE

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(This is an abridged version. See the full-length paper for more: genopt.papers/2009.ellipsoid.pdf)

ABSTRACT

GENOPT, a program that can be used to optimize anything, and BIGBOSOR4, a program for stress, buckling, and vibration analysis of segmented, branched, stiffened, elastic shells of revolution, are combined to create a capability to optimize a specific kind of shell of revolution: an internally isogrid-stiffened elastic ellipsoidal shell subjected to uniform external pressure. Optimum designs are obtained for isogrid-stiffened and unstiffened axisymmetrically imperfect and perfect titanium 2:1 ellipsoidal shells. The decision variables are the shell skin thickness at several user-selected meridional stations, the height of the isogrid stiffeners at the same meridional stations, the spacing of the isogrid stiffeners (constant over the entire shell), and the thickness of the isogrid stiffeners (also constant over the entire shell). The design constraints involve maximum stress in the isogrid stiffeners, maximum stress in the shell skin, local buckling of an isogrid stiffener, local buckling of the shell skin between isogrid stiffeners, general nonlinear bifurcation buckling, nonlinear axisymmetric collapse, and maximum normal displacement at the apex of the dome. Optimum designs first obtained by GENOPT are subsequently evaluated by the use of STAGS, a general-purpose finite element computer program. It is found that in order to obtain reasonably good agreement between predictions from BIGBOSOR4 and STAGS it is necessary to model the ellipsoidal shell as an "equivalent" ellipsoidal shell consisting of a spherical cap and a series of toroidal shell segments that closely approximates the true ellipsoidal meridional shape. The equivalent ellipsoidal shell is optimized with up to four axisymmetric buckling modal imperfections, each imperfection shape assumed to be present by itself. Computations include both plus and minus axisymmetric buckling modal imperfection shapes. At each design cycle and for the plus and minus version of each axisymmetric imperfection shape the following analyses are conducted: 1. linear general axisymmetric bifurcation buckling analysis (in order to obtain the axisymmetric linear buckling modal imperfection shapes), 2. nonlinear axisymmetric stress analysis at the design pressure, 3. nonlinear axisymmetric collapse analysis, and 4. nonlinear non-axisymmetric bifurcation buckling analysis. For each axisymmetric imperfection shape the design margins include an axisymmetric collapse margin, a general buckling margin, a margin involving the normal displacement of the apex of the shell, and local skin and stiffener stress margins and local skin and stiffener buckling margins within two approximately equal meridional regions of the equivalent ellipsoidal shell. There is generally good agreement of the predictions from STAGS and from BIGBOSOR4 for the elastic behavior of the perfect stiffened and unstiffened optimized shells and for the behavior of the imperfect stiffened optimized shells with axisymmetric buckling modal imperfections. Optimization with the use of only axisymmetric buckling modal imperfections has a disadvantage in the case of the unstiffened imperfect shell under certain conditions: the optimum design of the axisymmetrically imperfect unstiffened shell evolves in such a way that, according to predictions from STAGS, a non-axisymmetric buckling modal imperfection with the

same amplitude as an axisymmetric buckling modal imperfection causes collapse of the shell at an external pressure far below the design pressure. This disadvantage is easily overcome if, during optimization cycles, the unstiffened shell wall in the neighborhood of the apex is forced to remain thick enough so that local axisymmetric buckling does not occur primarily at and near the apex but instead occurs primarily in the remainder of the shell. An extensive study of some of the previously optimized elastic shells is conducted with STAGS including elastic-plastic material properties. The effect on collapse pressure of initial imperfections in the form of off-center residual dents produced by load cycles applied before application of the uniform external pressure is determined and compared with the effect on collapse pressure of imperfections in the form of non-axisymmetric and axisymmetric linear buckling modes, especially the non-axisymmetric linear buckling modal imperfection with n=1 circumferential wave, which seems to be the most harmful imperfection shape for optimized externally pressurized ellipsoidal shells. For the optimized unstiffened shell it is found that a residual dent that locally resembles the n=1 linear buckling modal imperfection shape is just as harmful as the entire n=1 linear buckling modal imperfection shape.

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1.0 INTRODUCTION

Cohen and Haftka [1] were the first to create a capability that could be used for the automated design of imperfect shells of revolution. In this report a further step is taken by combination of GENOPT [2-7], which incorporates the optimizer ADS written by Vanderplaats and his colleagues [8,9], with a version of BOSOR4 [10-12] called "BIGBOSOR4" [7] to permit optimization of a certain class of shells of revolution: an elastic isogrid-stiffened ellipsoidal shell of revolution subjected to uniform external pressure. BIGBOSOR4 requires the same input data and performs the same analyses as BOSOR4, but BIGBOSOR4 will handle shells of revolution with more segments and more degrees of freedom than can be handled by BOSOR4.

GENOPT is described in detail in [2]. It has previously been used to obtain optimum designs of various systems [3-7]. Optimum designs are obtained via the optimizer, ADS, created many years ago by Vanderplaats and his colleagues [8,9]. In GENOPT the optimizer, ADS, is "hardwired" in the "0-5-7" mode, which is a modified method of steepest descent. This optimization method requires as input the gradients of the design constraints (stress, buckling, collapse, displacement) with respect to each of the decision variables.

As described in [2] and [7], GENOPT creates a system of processors called "BEGIN", "DECIDE",

"MAINSETUP", "OPTIMIZE", "SUPEROPT", "CHANGE", "CHOOSEPLOT", "DIPLOT", by means of which optimum designs can be obtained. The architecture of this system of processors is analogous to that previously generated for specific applications [13,14]. "SUPEROPT" is a script by means of which "global" optimum designs can be obtained, as described in [7] and [15] and briefly on the second page of Table 34. “Global” is enclosed in quotation marks because SUPEROPT does not actually find the global optimum design but, with repeated executions, will find a design that is very likely to have an objective that is very close to the global objective.

2.0 PURPOSE AND SUMMARY OF THIS REPORT

2.1 Purpose of the report

The author was motivated to create a program to optimize an elastic shell of revolution **the behavior of which is significantly nonlinear**. In this application the nonlinearity is entirely caused by moderately large axisymmetric prebuckling meridional rotations of the elastic externally pressurized 2:1 ellipsoidal (or "equivalent" ellipsoidal) shell. (NOTE: some of the STAGS models used to evaluate the optimum designs produced by GENOPT include plastic flow.)

The author wanted to generate another application of GENOPT. **He hopes that in the future GENOPT will be used by others to obtain optimum designs of entirely different systems**. In this long version of this document (called “report” in the four file folders and names for text, figures, tables, and appendix), the text, the figures, the tables, and the appendix are stored electronically in four separate folders:

text = sdm50.report.pdf;
figures = sdm50pdf.report.figures;
tables = sdm50pdf.report.tables;
appendix = sdm50pdf.report.appendix.

This electronic report is long. In particular there are many, many figures and tables. Some of the tables are very long. **They are all included here in order that a researcher or designer will be able to use this information to obtain, by analogy, the optimum design of any different system.**

The captions of the figures and tables are unusually long. The intention is to minimize the need for the reader to flip back and forth from text to figure or from text to table to learn the meaning of the data presented there. A significant amount of the textual material in this report is contained in those captions. A nomenclature section is not included in this report because there are few equations, the equations are simple, and the meanings of the symbols in them are explained with them.

2.2 Summary of this report

GENOPT [2, 7] is used to obtain optimum designs of externally pressurized, perfect or imperfect, isogrid-stiffened or unstiffened, elastic titanium 2:1 ellipsoidal shells subjected to uniform external pressure, p = 460 psi (called the “design pressure”).

GENOPT [2, 7] is described in Section 3. **BIGBOSOR4** [7] and **STAGS** [20 – 24] are described in Section 4.

The necessity to use an "**equivalent**" ellipsoidal shell profile rather than a **true** ellipsoidal shell profile is explained in Section 5.

Minimum-weight optimum designs are obtained in the presence of **stress, collapse, bifurcation buckling and displacement design constraints** derived from the various analyses described in Section 6.

Numerical results for **optimized isogrid-stiffened and unstiffened perfect and imperfect elastic titanium 2:1 equivalent ellipsoidal shells** are presented in Sections 7 and 8. The shells are first optimized by GENOPT in the presence of **axisymmetric buckling modal imperfections**. Then the optimum designs are evaluated by means of STAGS models that include both axisymmetric and non-axisymmetric buckling modal imperfection shapes.

It turns out that the pressure-carrying capacity of the optimum design of the **unstiffened, imperfect** shell is extremely sensitive to non-axisymmetric buckling modal imperfections, a type of imperfection that cannot be accounted for in the GENOPT model, which can only handle axisymmetric buckling modal imperfections because the BIGBOSOR4 computer program can only handle axisymmetric imperfections. Therefore, the optimized unstiffened shell is severely under-designed. **This deficiency is avoided by a simple reformulation of the optimization problem in which a higher lower bound is specified for the shell wall thickness in the neighborhood of the apex of the shell.**

Improved optimum designs of **unstiffened** axisymmetrically imperfect equivalent ellipsoidal shells are derived in Section 9. These improved optimum designs, the so-called "**thick-apex**" optimum designs, are evaluated by means of STAGS [20 – 24]. The STAGS models account for elastic-plastic material, axisymmetric and non-axisymmetric buckling modal imperfections, and imperfections in the form of **off-center residual dents** produced via a "Load Set B" load cycle involving concentrated normal inward-directed loads or imposed displacements. By "**off-center**" is meant dents located at some radius from the axis of revolution of the shell. Collapse of the dented shells is determined after completion of the Load Set B load cycle by subsequent application of uniform external pressure in Load Set A.

A "**thick-apex**" optimum design of the unstiffened, imperfect shell is found which survives the design pressure in the presence of either axisymmetric or non-axisymmetric buckling modal imperfections or off-center residual dents.

The optimum design of the **isogrid-stiffened** shell derived in Section 8.1 is evaluated in Section 10 by means of STAGS models that include elastic-plastic material, axisymmetric and non-axisymmetric buckling modal imperfections, and imperfections in the form of off-center residual dents.

It is emphasized that STAGS is not used within the optimization "loop", but only AFTER the optimum design has been obtained by GENOPT.

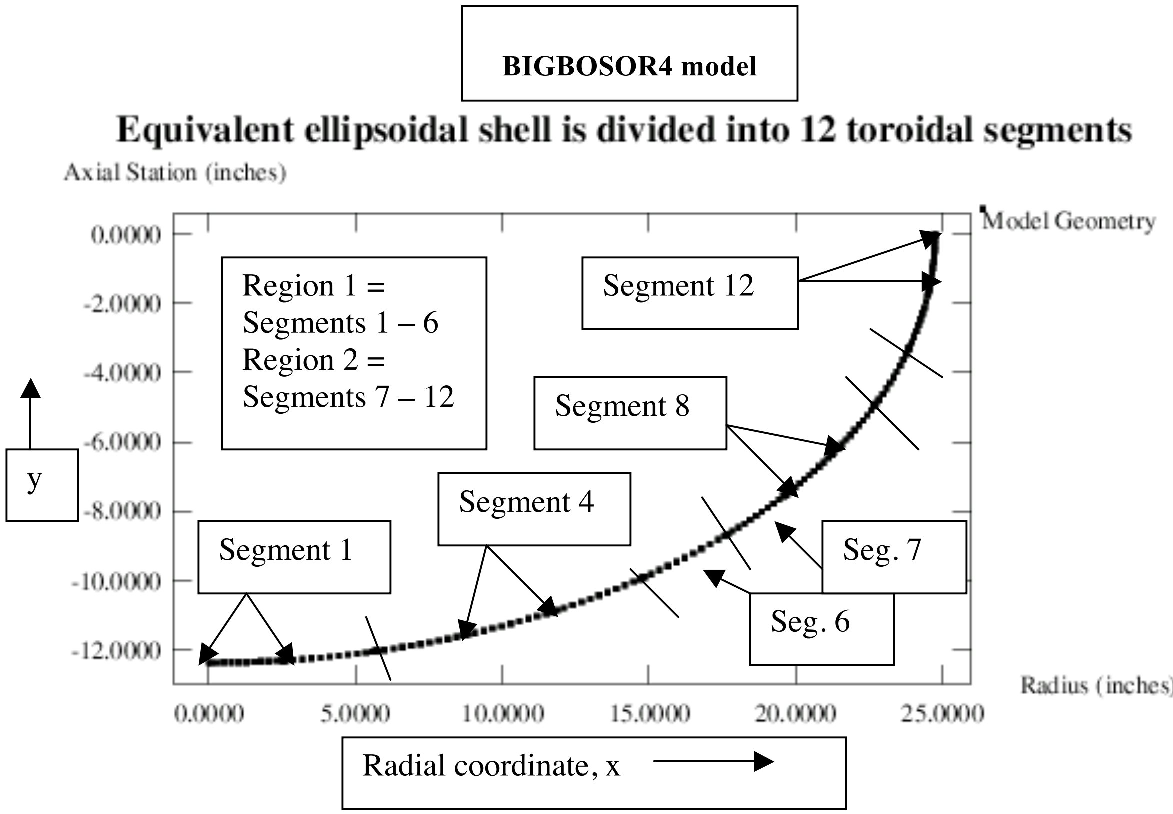

Fig. 2 This is a **BIGBOSOR4** model of the **EQUIVALENT** ellipsoidal shell. The equivalent ellipsoidal shell consists of 12 shell segments: one spherical cap (Segment 1) and 11 toroidal shell segments with end points that fall on the profile of the **TRUE** ellipsoidal shell and that match as closely as possible the local profile of the **TRUE** ellipsoidal shell. Finite element "lockup" is avoided because the meridional radius of curvature within each segment of the perfect **EQUIVALENT** ellipsoidal shell is constant. The (r,z) = (x,y) = (x_3,y_3) = (radius, axial station) location of the center of meridional curvature of each toroidal shell segment is computed as set forth in Table 29. Maximum local shell skin extreme fiber effective stress and minimum local skin buckling load factor and maximum local meridional isogrid member extreme fiber stress and minimum local meridional isogrid member buckling load factor are computed for each of the two regions: Region 1 and Region 2. The corresponding design margins are listed in Tables 31 and 32, for example. The 360-degree STAGS finite element model shown in Fig. a1 of the appendix is analogous to this BIGBOSOR4 model. The 360-degree STAGS finite element model has fewer nodal points along the meridian than the BIGBOSOR4 model shown here. (from AIAA 50th Structures, Structural Dynamics, and Materials Conference, Palm Springs, California, 2009, AIAA Paper 2009-2702)

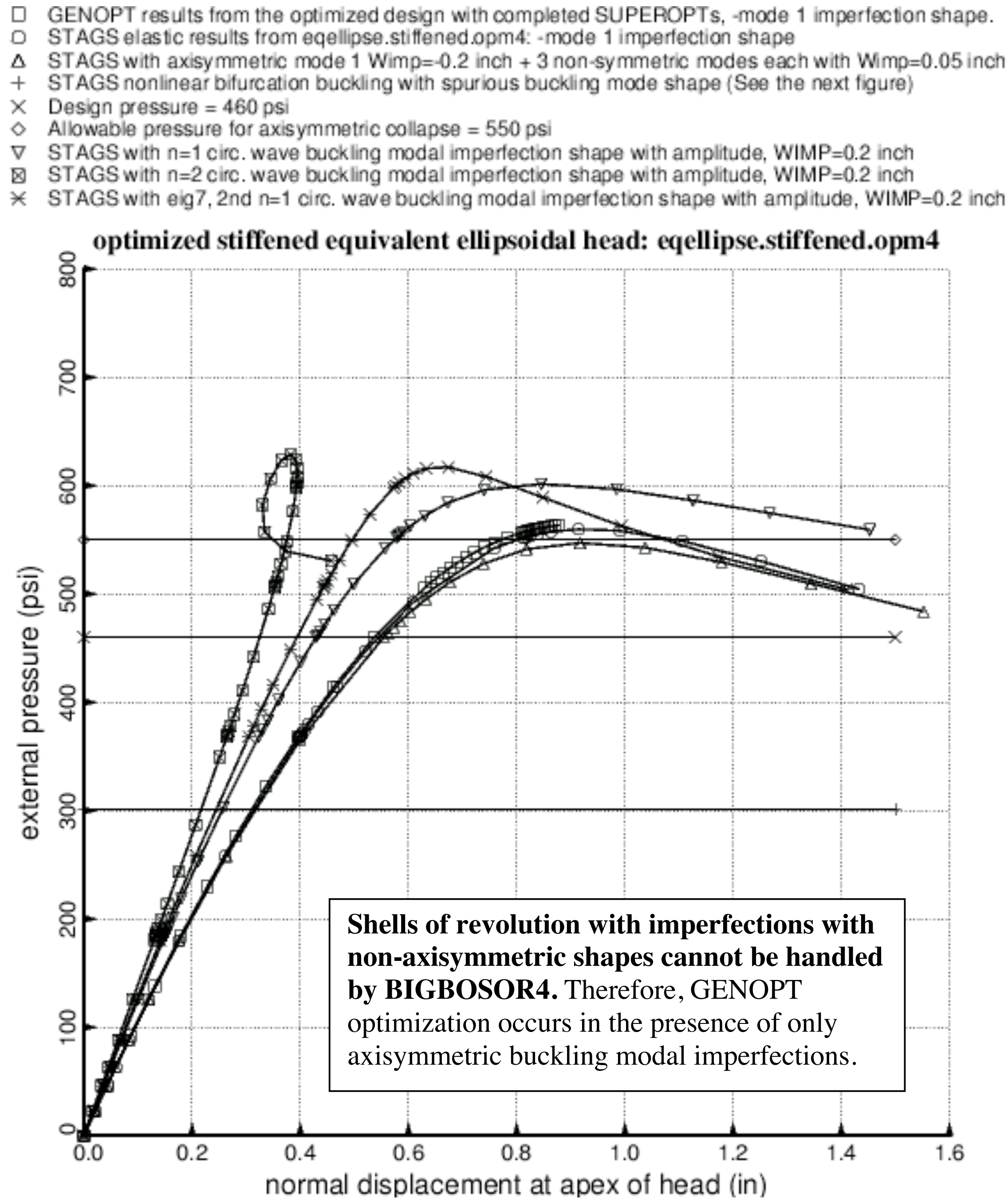

Fig. 17 Axisymmetric and non-axisymmetric collapse of optimized **imperfect isogrid-stiffened** equivalent ellipsoidal shells. The linear bifurcation buckling modal imperfection shapes corresponding to non-axisymmetric collapse (the last three traces) are displayed in Figs. 7, 8, and 10. In this case the –mode 1 (axisymmetric) buckling modal imperfection is predicted to be more harmful than the non-axisymmetric buckling modal imperfections with shapes given in Figs. 7, 8 and 10. This does not hold for the optimized unstiffened imperfect shell, as seen in Fig. 94. Compare with Fig. 254. (from AIAA 50th Structures, Structural Dynamics, and Materials Conference, Palm Springs, California, 2009, AIAA Paper 2009-2702)

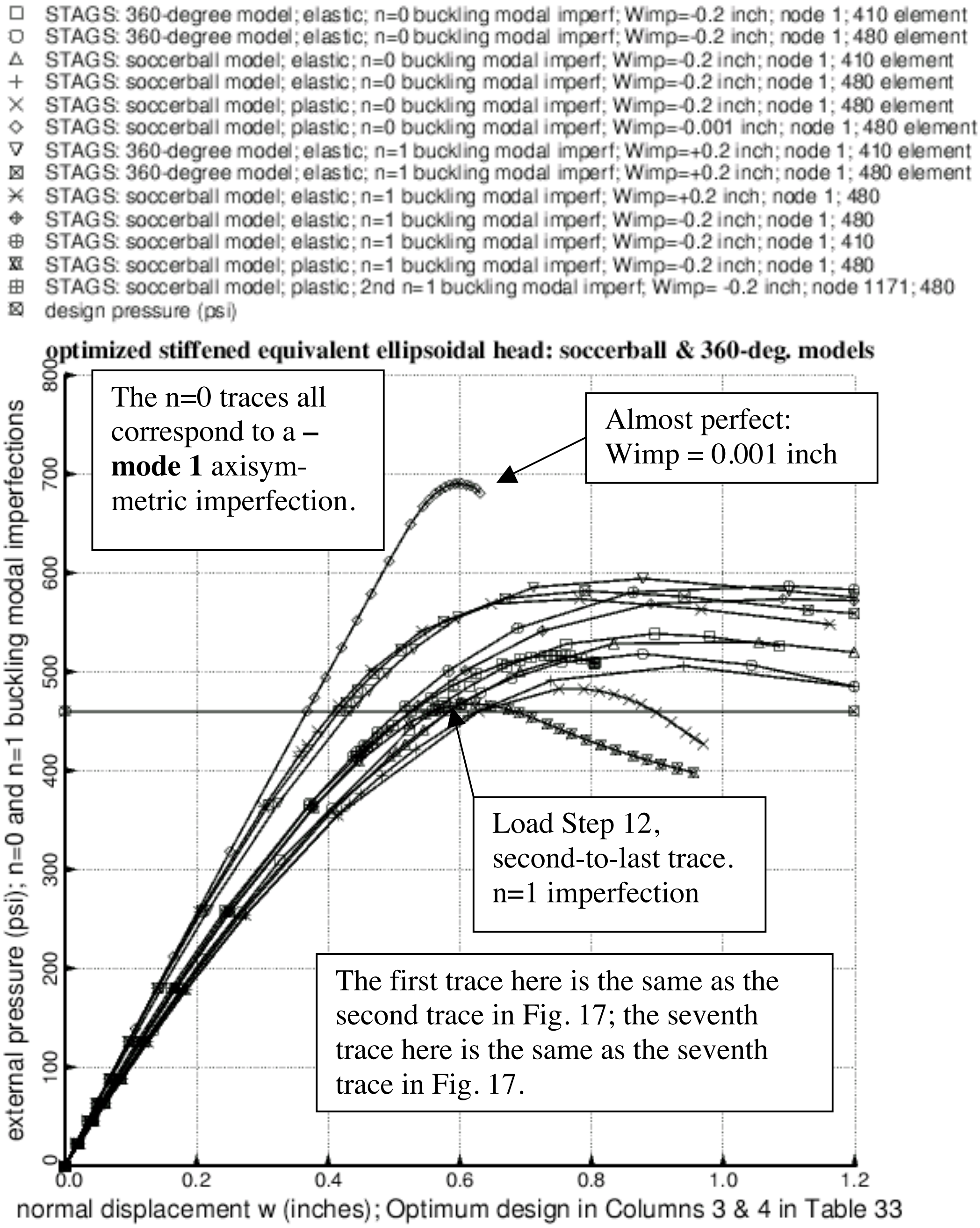

Fig. 254 Axisymmetric and non-axisymmetric collapse of optimized almost perfect (trace 6) and **imperfect isogrid-stiffened equivalent ellipsoidal shells. The optimum design, listed in columns 2 and 3 of Table 33, was obtained with linear axisymmetric (n=0) buckling modal imperfections with amplitude, Wimp = 0.2 inch.** For the 180-degree “soccerball” model the linear bifurcation buckling modal imperfection shape corresponding to axisymmetric (n=0) collapse (the first 6 traces) is shown in Fig. 257 of [26]. The linear bifurcation buckling modal imperfection shapes corresponding to non-axisymmetric (n=1) collapse (the last 7 traces) are displayed in Figs. 258 and 262. For the 360-degree models the linear bifurcation buckling modal imperfection shapes are shown for n=0 in Fig. 6 and for n=1 in Figs. 7 and 10. Compare this figure with Fig. 17.

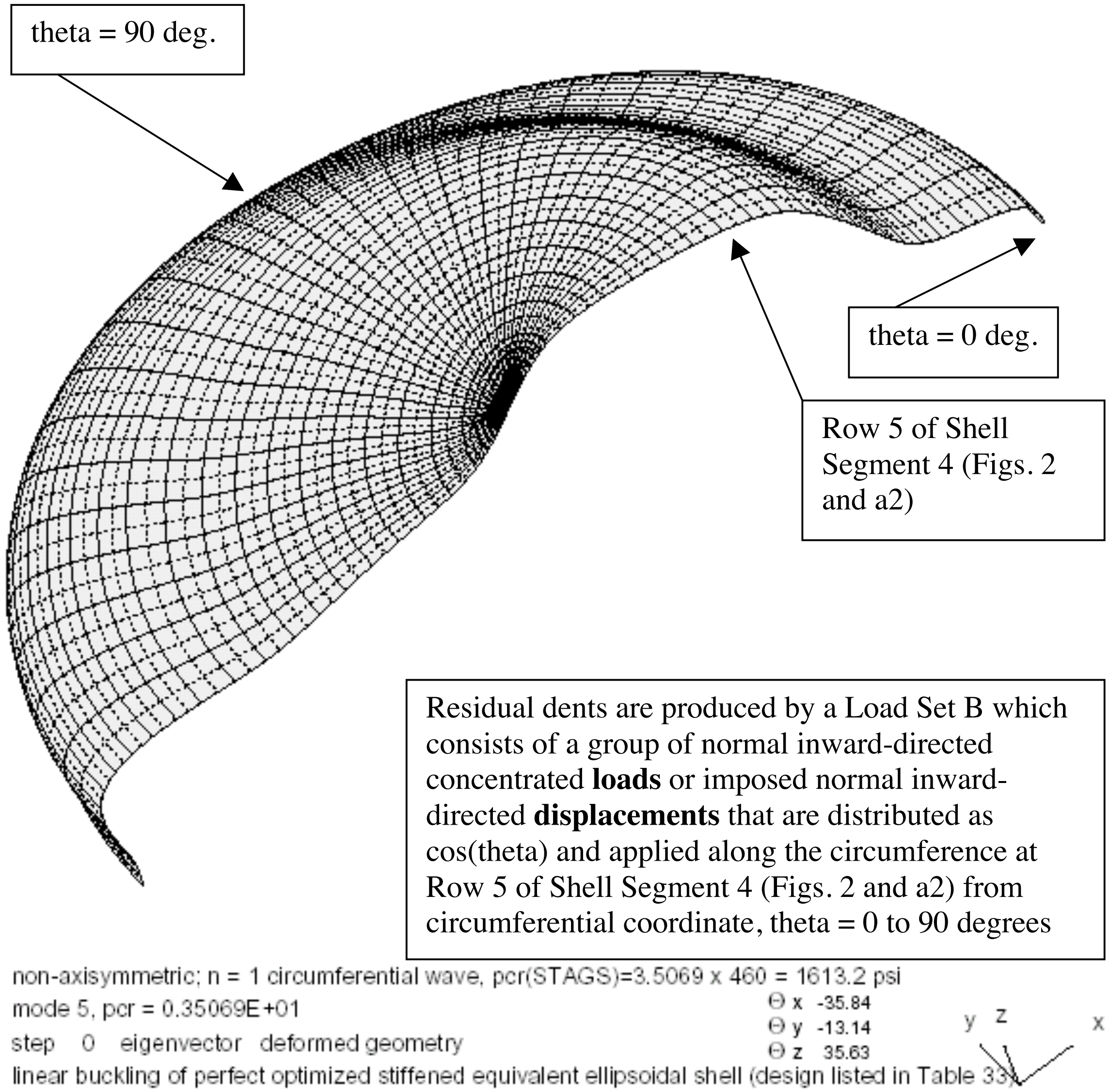

Fig. 262 **STAGS "soccerball" model of the optimized imperfect isogrid-stiffened equivalent ellipsoidal shell. The optimum design, listed in columns 2 and 3 of Table 33, was obtained with plus and minus axisymmetric (n=0) mode 1 and mode 2 linear buckling modal imperfection shapes with amplitude, Wimp = 0.2 inch.** This is the non-axisymmetric (2nd n=1 circumferential wave) linear buckling modal imperfection shape used as the n = 1 imperfection corresponding to the last trace in Fig. 254. Compare with the 360-degree STAGS model displayed in Fig. 10. The difference in the eigenvalue, 3.5069 here vs 3.5518 in Fig. 10, is caused primarily by the difference in the finite element used in the STAGS model: STAGS Element 480 here vs STAGS Element 410 in Fig. 10. Indicated in this figure is the location where normal inward-directed concentrated loads or displacements are imposed in a "cos(theta)" distribution in order to produce a dent that **locally** resembles the negative of this linear buckling mode shape. (from AIAA 50th Structures, Structural Dynamics, and Materials Conference, Palm Springs, California, 2009, AIAA Paper 2009-2702)