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## **OPTIMIZATION OF A TEE-STIFFENED PANEL UNDER AXIAL COMPRESSION, IN-PLANE SHEAR, AND NORMAL PRESSURE**

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(This is an abridged version. See the full-length paper for more: [panda2.papers/1991.teestiffened.pdf](http://panda2.papers/1991.teestiffened.pdf) )

### **ABSTRACT**

The PANDA2 computer program is used to find a minimum weight design of a flat panel with T-shaped stringers. The panel is allowed to buckle locally under the specified loading. The evolution of the design is influenced by stress and buckling constraint conditions at the midlength and at the ends of the panel. These constraint conditions are very different because bending of the panel under the pressure gives rise to very different distributions of stress over the cross section of skin and stringers which cause different buckling and postbuckling behaviors at these locations.

### **INTRODUCTION**

There is an extensive literature on the buckling and postbuckling behavior of stiffened plates and shells. This literature covers metallic panels and panels fabricated from laminated composite materials. Leissa [1] has gathered results from almost 400 sources on the buckling and postbuckling behavior of flat and cylindrical panels made of composite material with various stacking sequences and boundary conditions and subjected to various in-plane loads. The emphasis in his survey is on theoretical results, although some experimental results are included. He includes several examples in which the effect of transverse shear deformation is explored. Emphasis is given also to the effects of anisotropy on bifurcation buckling and on postbuckling behavior. Wiggenraad [2] surveys the literature on design of composite panels permitted to buckle locally under operating loads. Included in his survey are damage tolerance, fatigue, and optimization. Arnold and Parekh [3] emphasize in their survey and theoretical development the effect of in-plane shear load on the postbuckling behavior of stiffened, composite cylindrical panels. Surveys of earlier work on buckling of stiffened panels and shells appear in [4] and [5].

Among the foremost contributors of information about buckling of stiffened shells are Josef Singer and his colleagues at the Technion in Haifa, Israel. In particular, the Baruch-Singer theory [6] for averaging the properties of stiffeners over a shell surface while retaining the important eccentricity effects has been incorporated into many widely used computer programs for the stress, vibration, and buckling analysis of stiffened shells.

The literature in the field of buckling of stiffened shells can be divided into three categories, one in which test results are emphasized, a second in which structural analysis is emphasized, and a third in which optimum designs are obtained. References [7] through [17] feature test results for plates, shells, and stiffeners made of laminated composite material; [18] through [25] feature structural analysis with structural properties fixed; and

[26] through [35] feature structural analysis with optimum configurations sought in most cases via the widely used optimizers CONMIN and ADS, written by Vanderplaats and his colleagues [36 – 38]. This is just a sample of the literature on the subject. The reader is referred to the surveys given in [1] through [5] and references cited there for other sources.

## **PURPOSES OF THIS PAPER**

The purposes of this paper are to reveal the interesting buckling and postbuckling behavior of a stiffened panel under combined axial compression, in-plane shear, and normal pressure, and to demonstrate how a preliminary optimum design of such a panel can be obtained in the presence of the extremely nonlinear behavior associated with local buckling and postbuckling. Although the example presented here involves only isotropic material, the PANDA2 program was created with laminated composite wall construction in mind. The material is assumed to be elastic, although its properties can be temperature-dependent.

Details about PANDA2 and the theory on which it is based appear in [35,41,42]. Therefore only a summary of its scope is provided here.

## **REVIEW OF THE SCOPE AND PHILOSOPHY OF PANDA2**

The purpose of PANDA2 is to find the minimum weight design of a stiffened flat or curved, perfect or imperfect, panel made of laminated composite material.

### **Geometry**

The kinds of stiffening handled by PANDA2 are:

1. unstiffened
2. T-shaped stiffeners
3. J-shaped stiffeners
4. Rectangular (blade) stiffeners
5. Hat-shaped stiffeners
6. Z-shaped riveted stiffeners (added after this paper was written)
7. truss-core sandwich (with no other stiffeners)
8. 2 – 4 with sub-stiffeners between stringers and/or rings (added after this paper was written)

The properties of the panel are assumed to be uniform in the axial ( $x$ ) direction and periodic (consisting of repetitive modules) in the circumferential ( $y$ ) direction. If the panel is other than of truss-core sandwich construction, it may be stiffened by uniformly spaced stringers alone, stiffened by uniformly spaced rings alone, or stiffened by both rings and stringers. All stringers must be the same. All rings must be the same. The rings can be different from the stringers. Truss-core sandwich panels cannot be further stiffened by either stringers or rings.

Note that the theory on which PANDA2 is based is valid only if the panel is either unstiffened or, if stiffeners exist in either or both coordinate directions, there are several of them within the span of the panel. One cannot

accurately determine the behavior of a panel with only one stiffener, for example. The panel, if axially stiffened, for example, has a "field" of equally spaced, identical stringers.

In PANDA2 local buckling behavior is predicted from analysis of a single module that is assumed to repeat several times over the width of the panel, as shown in Fig. 1.

A panel module consists of one stiffener plus skin of width equal to the spacing between stiffeners (Fig. 2). The single module is considered to be composed of segments, each of which has its own laminated wall construction (Fig. 3).

General instability is predicted from a model in which the stiffeners are "smeared" in the manner of Baruch and Singer [6] over the width (stringers) and length (rings) of the panel.

## **Boundary Conditions**

In the PANDA2 system the panel is assumed to be simply supported along the two edges normal to the plane of the screen (at  $y = 0$  and at  $y = \text{panel width}$ ). The panel can be either simply supported or clamped along the other two boundaries (at  $x = 0$  and  $x = L$ ), but the conditions must be the same at both of these two boundaries. The PANDA2 analysis is always performed for simple support on all four edges. However, experience has shown that for the purpose of calculating panel and general instability load factors, clamping at  $x = 0$  and at  $x=L$  can be simulated by the analysis of a shorter simply supported panel: For example, an axially compressed, flat panel clamped at  $x = 0$  and  $x = L$  has general instability loads approximately equal to those of a panel simply supported at  $x = 0$  and  $x = L/\sqrt{3.85}$ . In PANDA2, clamping at  $x = 0$  and  $x = L$  is simulated by calculation of general instability or wide column instability of a simply supported panel with a shorter length, an "effective" length that depends on the ratios of in-plane loads and on the "boundary layer length" in the axial direction. This "effective" length is calculated by PANDA2 and is provided as output.

In PANDA2 local buckling behavior and local stress concentrations near stringers are assumed to be independent of the boundary conditions along the four panel edges. This is likely to be a good assumption if there are more than two or three half waves in the local buckling pattern over the length and width of the panel.

## **Loading**

PANDA2 allows the panel to be loaded by as many as five independent sets of in-plane load combinations,  $N_x$ ,  $N_y$ ,  $N_{xy}$ ,  $M_x$ ,  $M_y$ , normal pressure  $p$ , and temperature  $T(z)$  that is nonuniform over the panel cross section but uniform in the  $(x,y)$  coordinates. Buckling loads, postbuckling behavior, and maximum stresses are calculated for each of the five load sets applied by itself. PANDA2 determines the best design that is capable of surviving all of the five load sets when each set is applied separately, as it would be during different phases of a panel's lifetime or over different areas of a large, uniform structure such as a complete cylindrical shell subjected to spatially varying loads (See [35]). Associated with each of the five independent load sets there can be two load subsets, Load Set A and Load Set B. Load Set A consists of what are termed in the PANDA2 output as "eigenvalue loads": These are loads that are to be multiplied by the critical buckling load factor (eigenvalue). Load Set B consists of loads that are not multiplied by the critical buckling load factor.

## Types of Analysis

PANDA2 performs the following analyses:

### 1. CONSTITUTIVE LAW:

- a. PANDA2 computes the integrated constitutive law [the 6x6 matrix  $C(i, j)$  that relates reference surface strains, changes in curvature, and twist to stress and moment resultants] for each segment of a panel module.
- b. It computes thermal resultants and strains from curing and from applied temperature during service for each segment of a panel module.
- c. It computes the integrated constitutive law [the 6x6 matrix  $C_s(i, j)$ ] for the panel with either and both sets of stiffeners "smeared out". ("Smearing out" the stiffeners means averaging their properties over the entire area of the panel as prescribed by Baruch and Singer [6]).
- d. It computes the tangent stiffness  $CTAN(i, j)$  of the panel skin in its locally postbuckled state, if applicable.
- e. It computes the tangent stiffness  $C_sTAN(i, j)$  of the panel with smeared stiffeners, using  $CTAN(i, j)$  for the locally postbuckled stiffness of the panel skin.

### 2. EQUILIBRIUM:

- a. PANDA2 computes bowing of the panel due to curing.
- b. It computes static response of the panel to uniform normal pressure, using either linear or nonlinear theory. Two problems are solved:
  - b(1) overall static response of the entire panel with smeared stiffeners, and
  - b(2) local static response of a single panel module with a discretized cross section.
- c. Average strain and resultant distributions in all of the panel module cross section segments are determined for:
  - c(1) the panel loaded by all loads except normal pressure. The effect of bowing of the panel due to curing, applied temperature during service, normal pressure, and edge moments is included, as well as the effect of an initial imperfection in the form of axial bowing.
  - c(2) the panel loaded by normal pressure.
- d. Stresses in material coordinates in each layer in each laminate (segment) of the panel module are calculated either for the post-locally buckled panel, or for the unbuckled panel, whichever is applicable. The effect of a local imperfection in the form of the local buckling mode is included, as well as axial bowing from either cure,

temperature change during service, pressure, edge moments, eccentrically applied axial load, initial imperfection, or any combination of these effects.

e. Tensile forces in parts of the stiffener web(s) that tend to pull the web from the panel skin are calculated, and these forces are compared to a maximum allowable "peel force" that the user has previously obtained from peel tests on sample coupons that bear some similarity to the concept for which he or she is seeking an optimum design.

### **3. BUCKLING**

a. PANDA2 computes buckling load factors from a PANDA-type of analysis (closed form, see Ref [4]) for general instability, local buckling of the panel skin, local buckling of stiffener segments, and rolling of stiffeners with and without participation of the panel skin.

b. It computes the load factor for local skin buckling from a BOSOR4-type [39] of analysis (finite strip method) in which the cross section of a single panel module is discretized, as shown in Fig. 4 of this paper.

c. It computes a load factor for wide column buckling from a BOSOR4-type of analysis of a discretized single panel module. In this analysis the reduced effective stiffness of the locally buckled panel skin is used, if applicable.

d. If the axial load varies over the width of the panel, PANDA2 computes a load factor for general instability from a BOSOR4-type of analysis of the entire panel with smeared stiffeners. The width of the panel is discretized. Again, the reduced effective stiffness of the locally buckled panel skin is used for this analysis, if applicable.

e. PANDA2 generates a refined discretized model of the entire panel width with stringer parts treated as flexible shell branches. This model can be used directly as input to BOSOR4 (now superseded by BIGBOSOR4).

#### **Philosophy embodied in PANDA2**

PANDA2 represents a more detailed treatment of certain behavior not handled by PANDA [4]. In particular, optimum designs can be obtained for imperfect panels, for panels with locally post-buckled skin, for panels with hat stiffeners, and for panels with truss-core sandwich construction. In addition, PANDA2 will handle linear or nonlinear static response to normal pressure, panels with nonuniform axial loading, panels with edge moments, and panels with thermal loading and temperature-dependent material properties. Also, PANDA2 optimizes panels for multiple sets of loads, whereas PANDA [4] optimizes for a single set of in-plane loads.

Optimization is carried out based on several independently treated structural models of the panel. These might be classified into three model types, as follows:

1. Model type 1: Included are PANDA-type models [4] for general, local, and panel buckling, bifurcation buckling of stiffener parts, and rolling of stiffeners with and without participation of the panel skin. With PANDA-type models, buckling load factors are calculated from closed-form equations rather than from discretized models. The formulas are given in [4]. (See Table 1 and Figs. 1-4 of [4]).

2. Model type 2: Buckling load factors and post-local buckling behavior are calculated for what is termed in PANDA2 a "panel module." A module includes the cross section of a stiffener plus the panel skin of width equal to the spacing between stiffeners. In this model the panel module cross section is divided into segments, each of which is discretized and analyzed via the finite difference energy method [39] (finite strip method). Variation of deflection in the axial direction is assumed to be harmonic [ $\sin(nx)$  or  $\cos(nx)$ ]. This one-dimensional discretization is similar to that used in the BOSOR programs for the analysis of shells of revolution [39]. In fact, many of the subroutines for buckling and vibration analysis are taken from BOSOR4 and modified slightly in order to handle prismatic structures instead of shells of revolution.

The single module model gives a good approximation to the local skin buckling mode if there are more than three equally spaced stringers in the panel. What goes on locally between interior stringers in a panel, stringers that are rotating about their axes only, not bending, is only weakly affected by the boundary conditions at panel edges that may be several bays away.

Both local and wide-column instability can be handled with the same discretized structural model. For all except truss-core sandwich panels, symmetry conditions are applied at the left and right edges of the single module model, that is, symmetry conditions are applied midway between stringers. Edge conditions for the single module of the truss-core sandwich panel are discussed later.

The wide column buckling model in PANDA2 is applied to an axial length of panel between adjacent rings, or if there are no rings, to the entire axial length of the panel,  $L$  or for clamped panels the modified length  $L/\sqrt{3.85}$ . The wide-column buckling load predicted from the single panel module is always lower and, if the panel is not of the truss-core sandwich type and is stringer-stiffened, usually reasonably close to the general instability load of the entire width of the panel between rings because the axial bending stiffness of a stringer-stiffened panel is usually much, much greater than the transverse bending stiffness of the portion of the panel between adjacent rings. Hence, the strain energy in the buckled panel, and therefore the buckling behavior, is only weakly dependent on bending of the panel transverse to the stringers. Therefore, the boundary conditions along the edges of the panel parallel to the stringers are not important. On the other hand, local bending of the skin and local deformation of the stringer parts in the wide column buckling mode may significantly affect the wide column buckling load. These effects are not included in the closed-form PANDA-type model of general instability, but they are included in the single panel module model of wide column buckling.

The wide-column buckling model should not be used for prediction of general buckling of truss-core sandwich panels. It is too conservative because, unlike T, J, Blade, and Hat-stiffened panels, sandwich panels have bending stiffnesses in the  $x$  and  $y$  directions which are of the same magnitude. Therefore, one cannot ignore the longitudinal boundaries in the calculation of general instability load factors of truss-core sandwich panels.

3. Model type 3: Also included in the PANDA2 collection of models is a discretized model of the entire width of the panel, treated in this case with stiffeners smeared out. This model is introduced only if the axial load varies across the width of the panel or if there exists normal pressure.

The purpose of PANDA2 is to yield optimum PRELIMINARY designs of rather sophisticated panels that may experience very complex and very nonlinear behavior. The goal is to do this without having to use large, general-purpose programs with their elaborate data base management systems. This goal is achieved through the use of several separate relatively simple models, each designed to capture a specific phenomenon, rather

than through the use of a single multi-dimensionally discretized finite element model with a large number of degrees of freedom.

For example, PANDA-type models (Model type 1) are used in PANDA2 to obtain quick, preliminary designs which one can then use as starting designs in optimization analyses based on the more elaborate discretized panel module model. (2011 NOTE: Since this paper was written computers have become so fast that this two-step process is never required or even advised. The PANDA2 user should always use the more elaborate discretized panel module model!) Also, PANDA-type models are used to obtain buckling load factors in cases for which the discretized panel module model is not applicable, to obtain knockdown factors for the effect of in-plane shear loading, to obtain preliminary estimates of how much growth in any initial panel bowing to expect under compressive in-plane loads, and to check if it is likely that a curved panel with uniform external pressure will collapse under the pressure acting by itself. (2011 NOTE: This is still done in PANDA2.)

Models of type 2 (single discretized module) and type 3 (discretization of entire width with smeared stiffeners) are used in tandem to obtain from nonlinear theory the complex behavior of a stiffened plate or shell loaded by normal pressure. Model type 3 is the only one that is valid if the axial load varies across the width of the panel.

In the panels designed by PANDA2 the skin between stringers and the stringer parts will deform if they are locally imperfect, and even if they are perfect they may buckle well before failure of the panel. The maximum stress components and therefore stress constraints in the optimization analysis are computed including local prebuckling deformation and local post buckling growth and modification of the local skin buckling mode as predicted by a modified form of a theory formulated by Koiter in 1946 [40, 35]. Model type 2 (single discretized module) is the only model in PANDA2 valid for these analyses.

After the optimum design is obtained, the user can, if no in-plane shear load is applied, check the accuracy of the general instability load predicted from the single-module model by running a multi-module model with BOSOR4 [39] (2011 NOTE: now BIGBOSOR4, which supersedes BOSOR4). The input data file for this multi-module model is generated automatically by the PANDA2 system.

### **Architecture of the PANDA2 system**

As with PANDA [4], the program PANDA2 [35] consists of several independently executable processors which share a common data base. In the processor BEGIN the user supplies a starting design (perhaps a design produced by PANDA). In DECIDE the user chooses decision variables for the optimization analysis and their upper and lower bounds, linking variables and their factors of proportionality, and "escape" variables (explained in [35]). In MAINSETUP the user chooses up to five sets of combined in-plane loads and normal pressure; factors of safety for general instability, panel (between rings) instability, local instability, and material failure; strategy parameters such as number and range of axial half-waves in the local buckling mode; and number of design iterations in the optimization problem. [2011 NOTE: PANDA2 now includes the effects of general, inter-ring, and local imperfections in the shapes of the general, inter-ring, and local buckling modes. In MAINSETUP the PANDA2 user now also supplies amplitudes for these buckling modal imperfection shapes. Furthermore, in MAINSETUP the PANDA2 user now also supplies a "conservativeness" index, ICONSV = -1 (least conservative approximations) or 0 (medium conservative approximations) or 1 (most conservative approximations, the preferred value). Still further, in MAINSETUP the PANDA2 user now chooses whether or not to employ an alternative buckling formulation (double trigonometric series solutions for local, inter-ring, and general buckling).]

The command PANDAOPT initiates a “batch” run of the PANDA2 mainprocessor, which consists of two main branches: in one branch the structural analyses (stress, buckling and post-buckling) are performed and in the other new designs are produced by the optimizer ADS [37, 38].

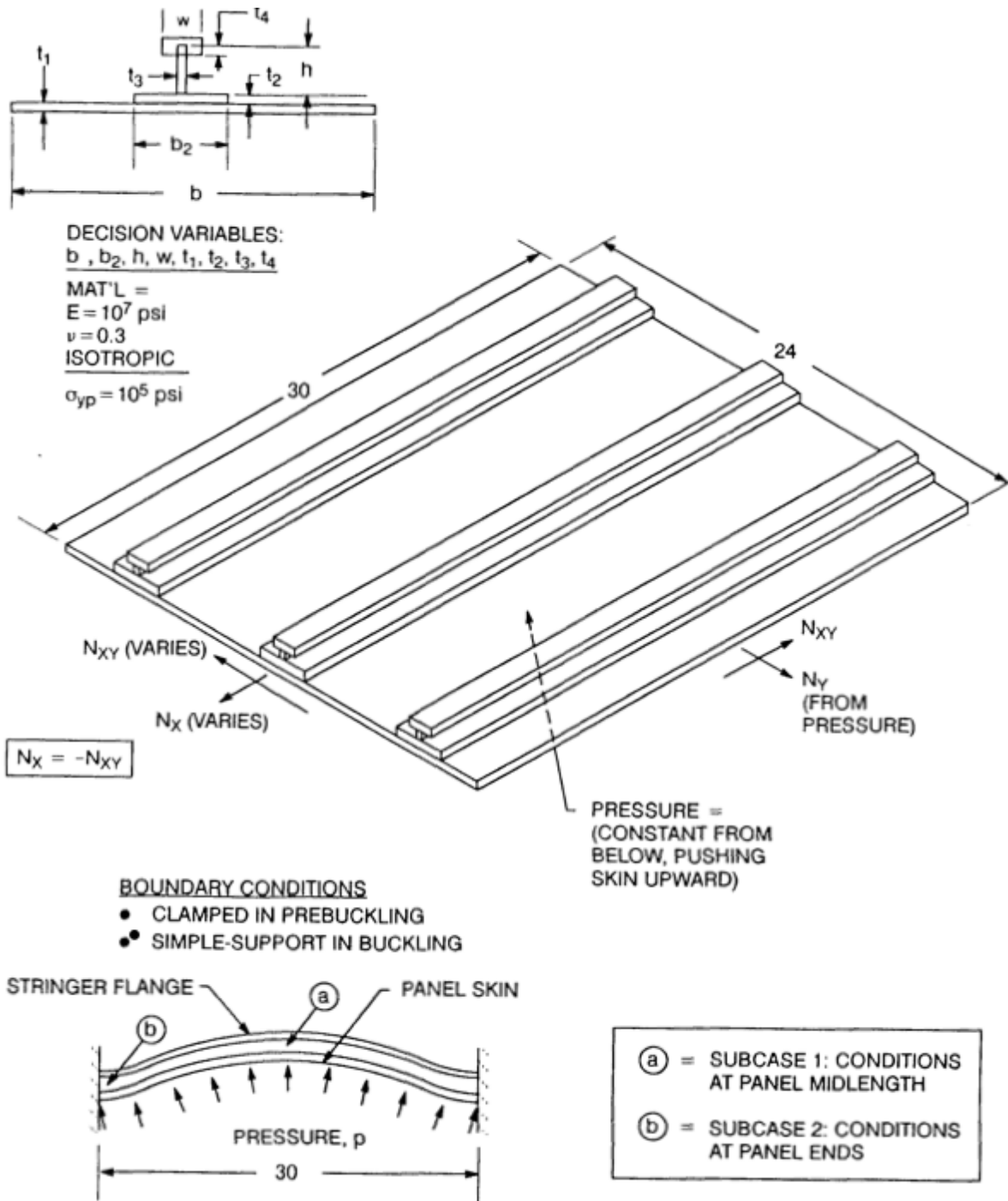


Fig. 4 Panel geometry, material properties, and loading. (Presented at AIAA 32nd Structures, Structural Dynamics and Materials Conference, AIAA Paper 91- 1207-CP)



○ = WEIGHT OF THE ENTIRE PANEL

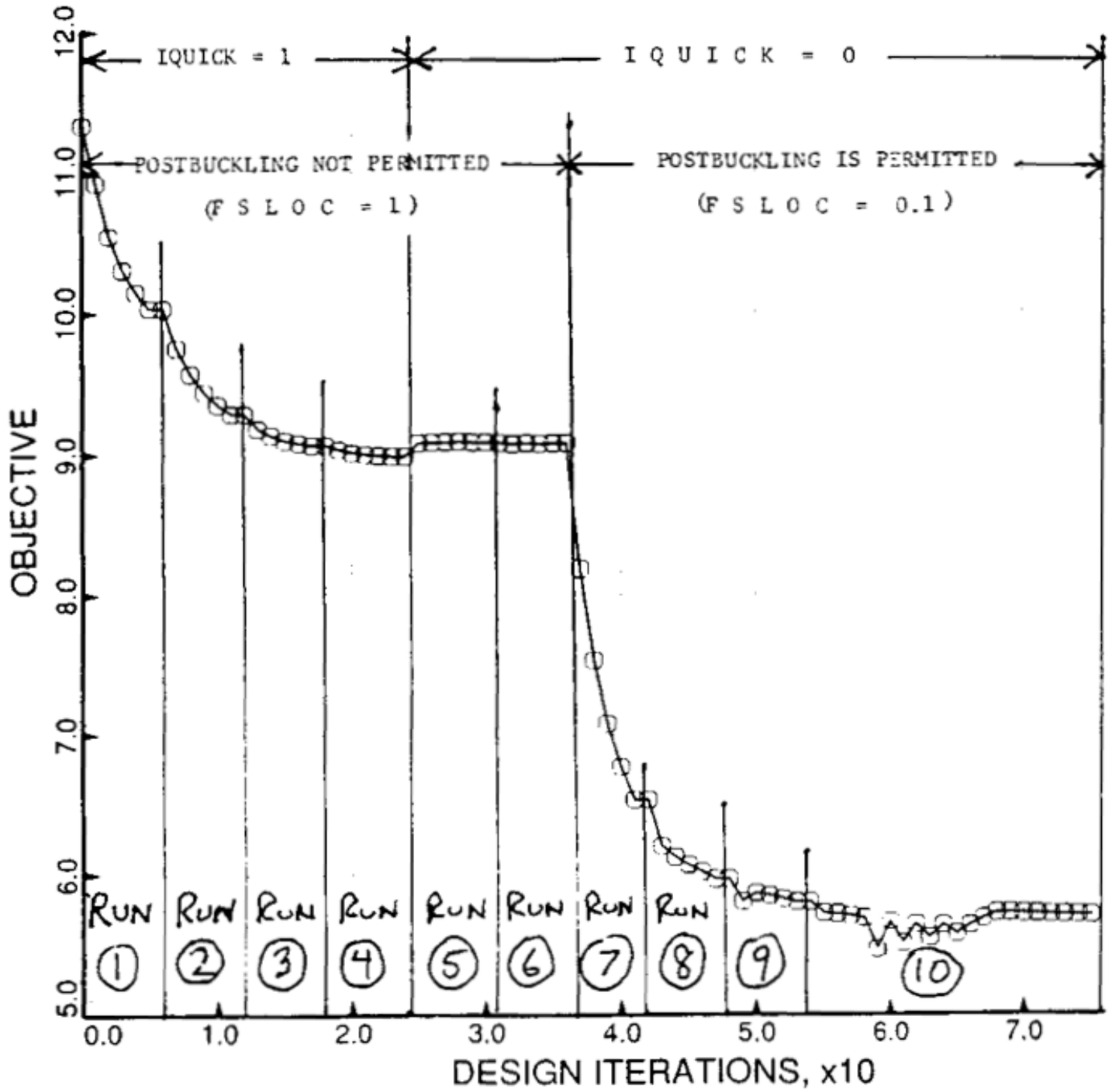


Fig. 5 Panel weight versus design iterations. Spacing between the stringers,  $b$ , held at 8 inches. (Presented at AIAA 32nd Structures, Structural Dynamics and Materials Conference, AIAA Paper 91-1207-CP)