



Professor James G. A. Croll & Rafael

From: S. Yamada, J. G. A. Croll, and N. Yamamoto, "Nonlinear Buckling of Compressed FRP Cylindrical Shells and Their Imperfection Sensitivity," Journal of Applied Mechanics, vol. 75, no. 4, p. 041005, 2008

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 Department of Civil Engineering
 University College London

Qualifications:
 Fellow of the Royal Academy of Engineering
 Fellow of the Institution of Civil Engineers
 Fellow of the Institution of Structural Engineers

Fellow of the Institute for mathematics and its Applications

Fellow of the Royal Academy of Arts

Chartered Engineer

Bachelor of Engineering (1st class honours), University of Canterbury, Christchurch, New Zealand

Doctor of Philosophy, University of Canterbury, Christchurch, New Zealand.

Experience includes:

40 years as lecturer (1967-83) Professor and Head of Department of Civil Engng at UCL (1983-present), more than 200 books and research publications; specialist consultancies services to around 35 UK and other International companies; court experience at public enquiries; forensic work for structural collapses; visiting professorships in 9 countries; considerable professional committee work - most recent as Chair of the Joint Board of Moderators responsible for the accreditation of civil and structural engineering degrees in the UK and abroad.

Structural engineering interests have covered most materials but especially those constructed from: concrete, steel, composite steel/concrete, timber, advanced composites including fibre reinforced polymer.

Consultancy, teaching and research has covered: concrete roof shells, concrete cooling tower shells, steel and concrete water tanks, oil storage tanks, silos, pressure vessels, steel and concrete offshore platforms, bouyancy tanks, submarine hulls, ship hulls, onshore and subsea pipelines, lightweight roof systems, cable net roofs, spaceframe roofs, trusses, arches, box girder bridges, cable stayed bridges, suspension bridges, buildings.

Research interests also include: stability and buckling of structures, vibrations and dynamic response of structures, behaviour of concrete and asphalt pavements, thermal buckling of structures, ice mechanics, soil structure interaction.

Selected Publications:

Elements of structural stability by J. G. A. Croll and A. C. Walker, Wiley, 1972, 223 pages

J.G.A. Croll (Department of Civil and Municipal Engineering, University College, London UK), "Continuum perturbation method in the branching analysis of conservative systems", International Journal of Mechanical Sciences, Vol. 13, No. 7, July 1971, pp. 605-613, doi:10.1016/0020-7403(71)90031-2

ABSTRACT: Techniques for the branching analysis of structural systems possessing a potential function are classified according to the order in which the conditions of stationarity, perturbation and discretization are imposed. It is shown that the recently developed algorithm for branching analysis of discrete systems, in which contraction of the algebraic perturbation equations plays a significant role, has an even closer analogy with the related continuum analysis than hitherto supposed. The proposed algorithm for the continuum branching analysis is outlined in general terms, and its use is illustrated by reference to the "elastica". More important than the analytical treatment of the resulting differential perturbation equations, however, is that the method proposed offers a viable alternative means of deriving practical numerical solution analogues. Broadly, numerical analogues derived on the present basis possess all the advantages and disadvantages of the generalized finite difference technique over the generalized finite element technique.

Batista, R. C. and Croll, J. G. A., 'A design approach for axially compressed unstiffened cylinders' in *Stability problems in engineering structures and components*, 1979 (Institute of Physics Conference, Cardiff, 1978). Edited by T. H. Richards and P. Stanley (Applied Science Publishers, London).

J.G.A. Croll, "Towards simple estimates of shell buckling loads", *Der Stahlbau*, 1975

De Souza VCM. Croll JGA. An energy analysis of the free vibrations of isotropic spherical shells. *J. Sound Vibr.* 1980;73;379–404.

Ellinas, C. P., Batista, R. C., and Croll, J. G. A., 'Overall buckling of stringer stiffened cylinders', *Proc. Instn civ. Engrs*, Part 2, 1981, 71, 479–512.

Ellinas, C. P. and Croll, J. G. A., 'Overall buckling of ring stiffened cylinders', *Proc. Instn civ. Engrs*, Part 2, 1981, 71, 637–661.

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"Geometric discontinuities in thin shell finite element formulations", *Computers & Structures*, Vol. 14, Nos. 1-2, 1981, pp. 37-41, doi:10.1016/0045-7949(81)90081-X

ABSTRACT: The implementation of equilibrium and compatibility conditions along discontinuities in the geometry of thin shells is considered for finite element displacement formulations in which the displacements and their first derivatives are specified as generalised coordinates. A method is proposed in which the transformations of generalised coordinates affect only the element stiffness matrix. Applied to the solution of a rotationally symmetric shell with discontinuities in the geometric curvature the method is shown to eliminate the numerical disturbances that can otherwise occur in the solution.

J. G. A. Croll (1) and R. C. Batista (2)

(1) Department of Civil and Municipal Engineering, University College, London, England

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"Explicit lower bounds for the buckling of axially loaded cylinders", *International Journal of Mechanical Sciences*, Vol. 23, No. 6, 1981, pp. 331-343, doi:10.1016/0020-7403(81)90063-1

ABSTRACT: Previously postulated, lower bound estimates of the buckling loads for axially loaded unstiffened cylinders are simplified by means of a Donnell-type approximation to provide compact, explicit, analytical expressions which could prove particularly suited to design. For a wide range of practical geometric parameters these simplified expressions are shown to provide close approximations of the exact lower bound loads and associated modes. In addition they allow the independent and significant influence of length to radius, radius to thickness, and Poisson's ratio to be isolated, and suggest a convenient means of summarising buckling loads in terms of a single composite geometric parameter.

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(1) Dept. of Civil Engineering, COPPE-Engenharia Civil Universidade Federal do Rio de Janeiro, Rio de Janeiro, Brazil

(2) Dept. of Civ. Engrg., Univ. College London, London, England

"Simple Buckling Theory for Pressurized Cylinders", *ASCE Journal of the Engineering Mechanics Division*, Vol. 108, No. 5, September/October 1982, pp. 927-944

ABSTRACT: Energy distribution within the critical modes of a classical energy eigenvalue analysis for circular cylindrical seashells under external pressure shows that a significant contribution to the stability of the shell is derived from membrane stiffness. It is argued from physical reasoning that the unstable post-critical behavior is the result of the loss of this membrane stiffness. To account for the combined effect of mode coupling and imperfections in eroding this membrane stiffness, a simplified theoretical analysis is described in which appropriate terms in the membrane potential energy are neglected. It is shown, by comparison with the writers' experiments, and with a collection of data found in the literature, that this reduced stiffness method provides lower bounds which, for shells with moderate imperfections, are reliable lower bounds of the experimental buckling pressures in the modes observed to trigger buckling. This lower boundedness combined with the conceptual and analytic simplicity of the reduced stiffness method make it a particularly attractive basis for the design.

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“Pressure buckling of end supported shells of revolution”, *Engineering Structures*, Vol. 4, NO. 4, October 1982, pp. 222-232, doi:10.1016/0141-0296(82)90027-X

ABSTRACT: A reduced stiffness theoretical analysis of imperfection sensitive elastic buckling for end supported cooling tower-type shells, provides lower bounds of experimentally recorded buckling pressures. Classical analysis of these shells with the same empirically realistic boundary support conditions, in contrast, provides unreliable upper bounds of test buckling pressures. By attempting to attribute this imperfection sensitivity to notionally weakened boundary conditions, past interpretations may have provided non-conservative advice for the estimation of design buckling pressures. The reduced stiffness method provides a simple, alternative basis for designing against imperfection sensitive buckling.

C P Ellinas, J G A Croll (Department of Civil and Municipal Engineering, University College London), “Experimental and theoretical correlations for elastic buckling of axially compressed ring stiffened cylinders”, *The Journal of Strain Analysis for Engineering Design*, Vol. 18, No. 2, 1983, pp. 81-93, doi: 10.1243/03093247V182081

ABSTRACT: Re-examination of some 85 past experiments on the elastic buckling of axially loaded ring stiffened cylinders shows the existence of two distinctive behavioural regimes. Lightly stiffened cylinders, like isotropic cylinders, buckle into non-axisymmetric modes having long axial wavelengths at loads that are sensitive to the precise magnitudes of small initial imperfections. Heavily stiffened cylinders are characterised by snap buckling into axisymmetric modes which, for elastic behaviour at least, show only limited sensitivity to initial imperfections. Each of these characteristics is shown to be predicted by the recently developed ‘reduced stiffness analysis’ method, which, despite the relatively perfect nature of test specimens, predicts reliable lower bounds to the experimental scatter. Taken together with the previously demonstrated empirical validity of the reduced stiffness analysis for the prediction of buckling modes and lower bounds to buckling loads for both isotropic and stringer stiffened cylinders, the present comparisons provide further support to the recommendation that this method be considered as an alternative basis for future design.

J.G.A. Croll and C.P. Ellinas (University College London, Gower Street, London WC1E 5BT, England), “Reduced stiffness axial load buckling of cylinders”, *International Journal of Solids and Structures*, Vol. 19, No.5, 1983, pp. 461-477, doi:10.1016/0020-7683(83)90056-2

ABSTRACT: A reduced stiffness method is presented for the estimation of lower bounds to the imperfection sensitive general buckling of axially loaded, orthotropically stiffened, elastic cylinders. It predicts many previously inexplicable empirical observations and provides lower bounds to the scatter of available test buckling loads, thus becoming useful as a design tool.

J. G. A. Croll (Department of Civil and Municipal Engineering, University College, Gower Street, London WC1E 6BT, UK), "Analysis of buckle propagation in marine pipelines", *Journal of Constructional Steel Research*, Vol. 5, No. 2, 1985, pp. 103-122, doi:10.1016/0143-974X(85)90009-4

ABSTRACT: This paper shows how the analysis of minimum, quasi-static, buckle propagation pressures for subsea pipelines may be exactly formulated in terms of the characteristics of ring collapse. A simple mechanistic approach to ring collapse is described which enables the rational incorporation of the effects of material strain hardening. Theoretical predictions resulting from this analysis are shown to reproduce past empirically observed propagation pressures; they also successfully predict the variations in the forms of ring collapse modes resulting from differing strain-hardening properties. Upon suitable non-dimensionalisation this analysis shows how predicted propagation pressures may be represented in terms of just one composite material and geometric 'propagation parameter'. This eliminates the need for recourse to empirically fitted design curves and allows resolution of certain anomalies experienced in past analyses. It emphasises the need for more complete information regarding material strain-hardening properties if test results are to be properly interpreted. Most importantly, the present analysis offers the potential for future design of pipelines being at once more rationally and parametrically complete, and yet compact and simple to apply.

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(1) J. P. Kenny & Partners Ltd., † High Holborn, London, U.K.

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"Elastic-plastic buckling design of cylindrical shells subject to combined axial compression and pressure loading", *International Journal of Solids and Structures*, Vol. 22, No. 9, 1986, pp. 1007-1017, doi:10.1016/0020-7683(86)90033-8

ABSTRACT: A recently developed procedure for predicting the elastoplastic axisymmetric collapse of cylinders subjected to combinations of axial compression and pressure loading is described. This allows the modelling of radial pressure induced deformations, boundary effects and initial geometric imperfections in terms of an equivalent imperfection in a "column type" bifurcation analysis. Together with the incorporation of a more rational means of specifying initial geometric tolerances, it is used to develop compact design-orientated procedures for predicting safe design loads for this form of elastoplastic collapse of cylinders.

Yamada, S., Croll, J.G.A., Buckling behavior of pressure loaded cylindrical panels (1989) *J Eng Mech*, 115 (2), pp. 327-344

P. B. Gonçalves and J. G. A. Croll, "Axisymmetric buckling of pressure-loaded spherical caps," *Journal of Structural Engineering*, vol. 118, no. 4, pp. 970-985, 1992.

S. Yamada and J.G.A. Croll, "Buckling and postbuckling characteristics of pressure-loaded cylinders", *ASME Journal of Applied Mechanics*, vol. 60, pp. 290-299 (1993).

James G. A. Croll (Department of Civil and Environmental Engineering, University College, London, UK, WC1E 6BT), "Towards a rationally based elastic-plastic shell buckling design methodology", *Thin-Walled Structures*, Vol. 23, Nos. 1-4, 1995, pp. 67-84, Special Issue: Buckling Strength of Imperfection-sensitive

Shells, doi:10.1016/0263-8231(95)00005-X

ABSTRACT: The 'reduced stiffness method' for the analysis of shell buckling was developed to overcome a trend towards increasingly sophisticated analysis that has become divorced from its basically simple underlying physics. This paper outlines the developments of the reduced stiffness method from its origins in the late 1960s, through its experimental confirmation, generalisation and elaboration over the past 20 years, to its more recent consolidation using carefully controlled non-linear numerical experiments. It is suggested that the method has now reached a stage where it could profitably be adopted as a basis for an improved shell buckling design methodology.

J.G.A. Croll, "Shell buckling: A return to basic mechanics", in *Applied Mechanics in the Americas*, vol.1, American Academy of Mechanics, pp. 410-417 (1995).

Croll, JGA and Wisuthseriwong, P (1998) A fresh look at the buckling of concrete shell roofs. In: (Proceedings) Structural engineering world congress. : San Fransisco.
(no abstract given)

A. A. Popov, J. M. T. Thompson and J. G. A. Croll, "Bifurcation Analyses in the Parametrically Excited Vibrations of Cylindrical Panels", *Nonlinear Dynamics*, Vol. 17, No. 3, 1998, pp. 205-225,
doi: 10.1023/A:1008396603655

ABSTRACT: We consider parametrically excited vibrations of shallow cylindrical panels. The governing system of two coupled nonlinear partial differential equations is discretized by using the Bubnov–Galerkin method. The computations are simplified significantly by the application of computer algebra, and as a result low dimensional models of shell vibrations are readily obtained. After applying numerical continuation techniques and ideas from dynamical systems theory, complete bifurcation diagrams are constructed. Our principal aim is to investigate the interaction between different modes of shell vibrations under parametric excitation. Results for system models with four of the lowest modes are reported. We essentially investigate periodic solutions, their stability and bifurcations within the range of excitation frequency that corresponds to the parametric resonances at the lowest mode of vibration.

J. G. A. Croll and G. D. Gavrilenko, "Substantiation of the method of reduced stiffness", *Strength of Materials*, Vol. 30, No. 5, 1998, pp. 481-496, doi: 10.1007/BF02522630

ABSTRACT: We suggest the theoretical fundamentals of the method of reduced stiffness for the determination of the lower bounds of sensitivity of longitudinally compressed shells whose shapes are close to cylindrical to imperfections of the shape under the conditions elastic buckling. The theoretical and experimental data demonstrating the validity of the method of reduced stiffness are also presented

S. Yamada and J. G. A. Croll. Contributions to understanding the behavior of axially compressed cylinders. *ASME J. Appl. Mech.*, 66:299–309, 1999.

J. G. A. Croll and G. D. Gavrilenko, "Reduced-stiffness method in the theory of smooth shells and the classical analysis of stability (review)", *Strength of Materials*, Vol. 31, No. 2, 1999, pp. 138-154,
doi: 10.1007/BF02511103

ABSTRACT: We present a promising method for the investigation of the load-carrying capacity of imperfect shells based on simple analytic approaches. This method is called the reduced-stiffness method. In many cases, it enables one to obtain analytic relations for the estimation of the lower bounds of buckling loads for actual shells. We present the exact lower bounds of the buckling loads for compressed smooth cylindrical shells. For

comparison, we also used the classical approach to the analysis of the critical loads for shells.

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“Reduced-stiffness method in the theory of buckling of stiffened shells”, *Strength of Materials*, Vol. 32, No. 2, 2000, pp. 168-177, doi: 10.1007/BF02511677

ABSTRACT: The reduced-stiffness method is used to establish the lower bounds of buckling loads for stringer- or ring-stiffened cylindrical shells. The numerical results obtained by using this method are compared with the experimental data. We also consider the problem of applicability of the reduced-stiffness method in design practice as well as the prospects for its development and generalization.

Reduced-Stiffness Method in the Theory of Shells

G. D. Gavrilenko and J. G. A. Croll

International Applied Mechanics, 2004, Volume 40, Number 7, Pages 715-743

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“Reduced stiffness buckling of sandwich cylindrical shells under uniform external pressure”, *Thin-Walled Structures*, Vol. 43, No. 8, August 2005, pp. 1188-1201, doi:10.1016/j.tws.2005.03.006

ABSTRACT: A reduced stiffness lower bound method for the buckling of laterally pressure loaded sandwich cylindrical shell is proposed. Also, an attempt is made to assess the validity of the proposed reduced stiffness lower bound with FEM numerical examples. In addition, the proposed method is compared with classical and Plantema's approaches of the buckling of the laterally pressure loaded sandwich cylindrical shell. Comparison of the proposed reduced stiffness lower bound with that obtained from non-linear FEM analysis verifies that it indeed provides a safe lower bound to the buckling of laterally pressure loaded sandwich cylindrical shells. The attractive feature of the proposed reduced stiffness method is that it can be readily used in designing laterally pressure loaded sandwich cylindrical shells without being concerned about geometrical imperfections.

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“Lower bound buckling strength of axially loaded sandwich cylindrical shell under lateral pressure”, *Thin-Walled Structures*, Vol. 44, No. 7, July 2006, pp. 800-807, doi:10.1016/j.tws.2006.04.013

ABSTRACT: Effects of the lateral pressure on the FEM and reduced stiffness lower bound buckling strength of axially loaded sandwich cylindrical shell are examined. Further, a reduced stiffness lower bound buckling strength for the axially loaded sandwich cylindrical shell under lateral pressure is proposed. The effect of the lateral pressure on the FEM and reduced stiffness lower bounds are corresponding; it causes them to reduce slightly. However, reduced stiffness buckling mode shape remains the same. In addition, the proposed reduced stiffness lower bound buckling strength is shown to provide effective and valid for cores having different shear

stiffness. It provides comparatively close lower bounds to short axially loaded sandwich cylindrical shells under lateral pressure. Further, it provides a safe lower bound that does not depend on precise geometrical imperfection spectra and lateral pressure and it is simple and easy to employ.

J. G. A. Croll, "Stability in shells," *Nonlinear Dynamics*, vol. 43, no. 1-2, pp. 17–28, 2006.

Sosa, E.M., Godoy, L.A. and Croll, J.G.A. (2006). "Computation of lower-bound buckling loads using general-purpose finite element codes", *Computers and Structures*, Vol. 84, No. 29–30, pp. 1934–1945.

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"A reduced stiffness approach for the buckling of open cylindrical tanks under wind loads", *Thin-Walled Structures*, Vol. 45, No. 9, September 2007, pp. 727-736, doi:10.1016/j.tws.2007.07.001

ABSTRACT: This paper reports on the implementation of a lower bound approach to the buckling analysis of cylindrical shells for tanks subjected to wind loads. The formulation is based on a reduced energy model of the shell adapted to a special purpose, semi-analytical, finite element program in which it is possible to separately compute the membrane and bending energy contributions. First, the energy components are investigated, in order to identify stabilizing and destabilizing contributions. Second, an eigenvalue analysis is carried out using a reduced value of the stiffness, in which membrane components are eliminated on the basis that they are assumed to be eroded as a result of mode coupling catalyzed by imperfections. The methodology is employed for thin-walled, above ground, tanks under wind pressures. It is shown that the resulting critical loads constitute lower bounds to those obtained using a nonlinear analysis of the shell, including imperfections, and also to those obtained from experiments.

H. Wang, and J. G. A Croll, Buckling design optimisation of fibre reinforced polymer shells using lower bound post-buckling capacities. *Int. Conf. on Experimental Mechanics, ICEM13*, July 1-6 2007, Alexandroupolis, Greece

Hongtao Wang and James G.A. Croll (Department of Civil and Environmental Engineering University College London, WC1 E 6BT), "Optimisation of shell buckling using lower bound capacities", *Thin-Walled Structures*, Vol. 46, Nos. 7-9, July-September 2008, pp. 1011-1020, Special issue to mark the Retiral of Professor Jim Rhodes, Founding Editor, doi:10.1016/j.tws.2008.01.035

ABSTRACT: In the context of aerospace and marine applications there are considerable incentives for designers to adopt thin shells, whose performances are enhanced by appropriately chosen rib stiffeners or using high-performance composite materials. Imperfection-sensitive buckling in these circumstances is controlled by extremely high numbers of independent material and geometric parameters. As a basis for design, traditional reliance upon scatter of test results is suggested to be untenable and the increasing tendency to replace this approach by use of nonlinear finite elements is argued to bring with it all sorts of other quite considerable practical problems. This paper describes how the long established and very simple "reduced stiffness method" (RSM) is able to provide an alternative design strategy. It shows how a very straightforward extension of classical critical load analysis allows the definition of lower bounds to the potential imperfection sensitivity in each mode and consequently the delineation of the mode and load likely to provide the controlling influence on

design. Reliability of its predictions is briefly demonstrated through comparisons with extensive test programmes and confirmation through carefully controlled nonlinear numerical studies. Use of the RSM is shown to offer scope for identifying material and geometric parameters that result in improved and even “optimum” buckling loads. Case studies from past and a current programme of research looking at the buckling of composite shells are used to illustrate this design potential.

Hongtao Wang and James G A Croll (Department of Civil, Environmental, and Geomatic Engineering, University College London, London, WC1E 6BT, UK E-mail: hongtao.wang@ucl.ac.uk), “Finite Element Validation of a Lower-Bound Design Method for Optimising Buckling Capacities of FRP Shells”, (cocomat.de, publisher and date not given in the pdf file; most recent reference is 2007)

ABSTRACT: The buckling loads of thin FRP laminated shells are sensitive to initial geometric imperfections. A large number of geometric and material variables prohibit the traditional lower-bound experimental design methodology for isotropic shells from being applied to composite shells. As an alternative, a so-called “reduced stiffness method” (RSM) has been applied to the lower-bound buckling analysis of FRP laminated shells. The RSM analysis has shown that the classical critical mode which gives the lowest reduced stiffness critical load is often different from the mode giving the minimum classical critical load. This paper presents a series of finite element numerical experiments to test the validity of the RSM. By examining the effects of initial imperfections in the form of the recognised classical critical modes, it is shown that the RSM provide a reliable lower-bound method for buckling design of FRP shells.

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“Reduced Stiffness Buckling Analysis of Aboveground Storage Tanks with Thickness Changes”, *Advances in Structural Engineering* Vol. 14 No. 3 2011, pp. 475-487

ABSTRACT: The Reduced Stiffness Analysis (RSA) to compute lower bounds to buckling loads of shells has been employed by a number of researchers as a simple way to evaluate the buckling capacity of shells that display unstable behavior and imperfection-sensitivity. It allows the use of simple eigenvalue analysis, without having to perform incremental nonlinear analysis, and is based on the physical behavior of the shell which recognizes that a significant contribution to the stability of a shell under lateral pressure is provided by its membrane stiffness. Unstable post-critical behavior is associated with the loss of this stabilizing membrane contribution. Past use of the approach has been mainly restricted to cases of uniform shell thickness and uniform pressures in the circumferential direction, in which case analytical solutions are possible. Recent applications by the authors and other researchers have shown ways to compute the lower bounds using finite element analysis, for which a modified eigenvalue analysis is constructed by neglecting the membrane contributions to the matrix containing the initial stresses. This paper illustrates the application of the methodology to cases of pressure loaded shells with thickness changes in the meridional direction. A semi-analytical finite element code has been employed for the buckling analysis when uniform pressures act on aboveground steel tanks. The tanks are representative of those constructed for the oil industry, with diameter to thickness ratios of the order of 3000, and height to diameter ratios lower than one.