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Fraeijs de Veubeke: Neglected Discoverer of the "Hu-Washizu Functional" by Carlos A. Felippa

This note was motivated by a "Classical Reprint Series" appearance of Fraeijs de Veubeke's Displacement and Equilibrium Models chapter in the International Journal for Numerical Methods in Engineering [4]. In the reprint preface Olek Zienkiewicz reminds readers of several neglected aspects of de Veubeke's seminal contributions to finite element methods. This note calls attention to an ignored fundamental contribution to variational mechanics [3]. For technical details, including a step by step review of de Veubeke's derivations, interested readers are referred to a recent Technical Note in the Journal of Applied Mechanics [2].

The Canonical Functional

The three-field canonical functional of linear elastostatics, herein abbreviated to C3FLE, is identified as the "Hu-Washizu functional" in the mechanics literature. In this functional the three interior fields: displacements, stresses and strains, are independently varied. This attribution is supported by two independent publications, which, by strange coincidence, appeared simultaneouly on March 1955 [10,14]. A four-field generalization, in which surface tractions are independently varied, will be called C4FLE.

The reprinted chapter [4] is still cited as one of the early classics in the finite element literature. That article contains the first enunciation of the limitation principle, which has since served as guide in the construction of mixed models. De Veubeke's exposition of variational methods starts from the C4FLE functional, which he calls "the general variational principle." However, it does not reference Hu and Washizu as its source but an earlier technical report, written in French [3]. This appears as the third reference in that chapter.

A subsequent journal paper [5] on variational principles and the patch test is slightly more explicit. It begins: "There is a functional that generates all the equations of linear elasticity theory in the form of variational derivatives and natural boundary conditions. Its original construction [here he refers to the 1951 report] followed the method proposed by Friedrichs ..."

These references motivated me to investigate whether de Veubeke had indeed constructed that functional in the 1951 report. That would confer him priority over Hu and Washizu, although of course these two contributions were more influential in subsequent work. I was able to procure an archived copy of [3] thanks to Professors Beckers and Geradin of the University of Liege, where de Veubeke was a Professor of Aeronautical Engineering from the early 1950s until his untimely death on September 1976.

Priority Established

The technical note [2] shows that in the 1951 report de Veubeke constructs not simply the canonical three-field principle, but the four-field generalization C4FLE. This is done through the "dislocation potential" transformation method of Friedrichs [1, Ch. IV]. Consequently his priority is firmly established. The functional, however, appears as an intermediate result on the road from the Total Potential Energy (TPE) to the Total Complementary Energy (TCE) principle. The path also traverses a pair of two-field functionals, one being a generalization of the Hellinger-Reissner (HR) functional published the previous year by Reissner [12]. The full sequence presented in Part I of the 1951 report is in fact

TPE -> C4FLE -> Strain-displacement dual of HR -> HR -> TCE

The report does not call special attention to C4FLE, as well as to the strain-displacement functional that appears there for the first time. This two-field functional has escaped a name; it was later rediscovered by Oden and Reddy [11].

The bulk of [3] is actually devoted to the study of energy-based approximation methods for the analysis of monocoque wing structures, rather than to the derivation of new functionals. Its title (in English: propagation of redundants in wings with coupled longerons), technology focus and target audience (structural engineers) are likely responsible for subsequent neglect. This is reinforced by its limited dissemination and the fact that the material was apparently not submitted to an archival journal.

Variations on a Name

I posed some questions in [2] as curiosities for future science historians. For example, why are the titles of Hu and Washizu's papers almost identical? Furthermore, both de Veubeke and Washizu were visitors at MIT in 1952 and 1955, respectively, while Eric Reissner was a Professor of Mathematics there; was there a MIT connection? But these tidbits are besides the point. The priority claim is solid unless a pre-1951 publication miraculously turns up.

De Veubeke does not reference Hu or Washizu in any of the papers reprinted in the Memorial volume [8]. He does acknowledge Friedrichs, Courant and Hilbert, Prager, Reissner and Pian. On the other hand, he does not explicitly claim priority for the results discussed here. Perhaps he felt that the derivation of new functionals was not the focus of the 1951 report. And indeed it was not. The tour of five variational principles takes 8 pages out of 56. In contrast, the titles of the contributions of Hu and Washizu expressly state that to be the main objective.

Renaming a functional does have a historical precedent. The name of Hellinger was prepended to the stress-displacement functional of Reissner [12] after Hellinger's 1914 contribution was brought to attention. Gurtin [9] calls it the Hellinger-Prange-Reissner functional but this longer label has not resonated. There is justification for keeping Reissner's name in this case since he gave it as a proved theorem.

Major Contributions

This discussion, plus that of Zienkiewicz in the chapter reprint [4] should make clear that Fraeijs de Veubeke's work in applied and computational mechanics has not been well appreciated. To further that point I have compiled below what I perceive as major contributions, listed in no particular order. (Note: reference is made to the Memorial volume [8] for publications not otherwise easily accessible; unfortunately [3] was left out, perhaps because of its overall length and language translation effort.)

Two of the six canonical functionals of elastostatics, including the most general 4-field form [3]
A novel complementary energy formulation of finite elasticity [7]
The dual principles of elastodynamics [8, p. 295]
The limitation principle for mixed functionals [4]
Two-sided error bounds for FEM discretizations [8, p. 53]
Stress function models [8, p. 663]
The concept of what is now called the corotational frame for geometrically nonlinear analysis [6]
The general substructuring interface conditions [8, p. 511], which now form the basis for primal and dual domain-decomposition methods.
Equilibrium and diffusive (flux-preserving, see explanation below) models [8, p. 569]
The 6-node quadratic triangle [4], a breakthrough that opened the doors to higher order conforming elements. Midside nodes, a truly novel idea in 1963, were likely suggested by his early research in flux-preserving models.

Clarification on the next-to-last item for younger readers: "diffusive models" are not "diffuse elements". It is de Veubeke's term for mechanical elements that satisfy equilibrium weakly. This is done by enforcing only integrated flux conservation across interelement boundaries, with the objective of arriving at stiffness-like equations for use in standard FEM codes. Higher order versions of these models are plagued by spurious mechanisms, and much effort was devoted to ingenious ways to circumvent that flaw. Simple forms of these models have been recently rediscovered under the label "Discontinuous Galerkin Methods."

Conclusion

The set of contributions is indeed impressive in breadth and depth. At this point we can relate to Truesdell's epitaph on F. Reech [13, p. 300]: "it is hard to account for the oblivion in which the tradition has buried him." Oblivion is too strong a word for de Veubeke, as long as colleagues and disciples are still around. But his contributions are rarely mentioned in newer textbooks, which is a worrisome trend. Paraphrasing Truesdell's "why so?" I offer five conjectures:

His personality. An aristocrat by birth and gentleman by nature, de Veubeke never displayed greed for priority and recognition. A revealing example is the low-key referencing to his derivation of C4FLE.

A patient, dispassionate (even flat) style in his publications. Readers must search carefully for the gems; he does not highlight or hype discoveries.

Distaste for ``impressive examples'', e.g. airplanes, bridges, dams ..., that catch an engineer's roving eye. In the Memorial volume [8] I could find only a handful of application examples, none of which stands out.

Disinterest in developing early-bird, open-source FEM software as "technology transfer toolkit" that doctoral students could take along with their diplomas. (This was, by the way, a very effective scheme used at UC Berkeley in the 1960s to propagate FEM into the civil engineering community.)

His untimely death before being able to organize and tie together his wide research interests into textbooks.

I am inclined to pick the first two as primary reasons for the current neglect.

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