Gottfried Wilhelm Leibniz (1646 – 1716)

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Leibniz occupies a prominent place in the history of mathematics and the history of philosophy. He developed the infinitesimal calculus independently of Isaac Newton, and Leibniz's mathematical notation has been widely used ever since it was published. He became one of the most prolific inventors in the field of mechanical calculators. While working on adding automatic multiplication and division to Pascal's calculator, he was the first to describe a pinwheel calculator in 1685 and invented the Leibniz wheel, used in the arithmometer, the first mass-produced mechanical calculator. He also refined the binary number system, which is at the foundation of virtually all digital computers. In philosophy, Leibniz is mostly noted for his optimism, e.g., his conclusion that our Universe is, in a restricted sense, the best possible one that God could have created. Leibniz, along with René Descartes and Baruch Spinoza, was one of the three great 17th century advocates of rationalism. The work of Leibniz anticipated modern logic and analytic philosophy, but his philosophy also looks back to the scholastic tradition, in which conclusions are produced by applying reason to first principles or prior definitions rather than to empirical evidence. Leibniz made major contributions to physics and technology, and anticipated notions that surfaced much later in biology, medicine, geology, probability theory, psychology, linguistics, and information science. He wrote works on politics, law, ethics, theology, history, philosophy, and philology. Leibniz's contributions to this vast array of subjects were scattered in various learned journals, in tens of thousands of letters, and in unpublished manuscripts. As of 2011, there is no complete gathering of the writings of Leibniz.

Gottfried Leibniz was born on July 1, 1646 in Leipzig, Saxony (at the end of the Thirty

Years' War), to Friedrich Leibniz and Catharina Schmuck. Friedrich noted in his family journal: "On Sunday 21 June [NS: 1 July] 1646, my son Gottfried Wilhelm is born into the world after six in the evening, æ to seven [ein Viertel uff sieben], Aquarius rising." His father, who was of Sorbian ancestry, died when Leibniz was six years old, and from that point on he was raised by his mother. Her teachings influenced Leibniz's philosophical thoughts in his later life.

Leibniz's father had been a Professor of Moral Philosophy at the University of Leipzig and Leibniz inherited his father's personal library. He was given free access to this from the age of seven. While Leibniz's schoolwork focused on a small canon of authorities, his father's library enabled him to study a wide variety of advanced philosophical and theological works - ones that he would not have otherwise been able to read until his college years. Access to his father's library, largely written in Latin, also led to his proficiency in the Latin language. Leibniz was proficient in Latin by the age of 12, and he composed three hundred hexameters of Latin verse in a single morning for a special event at school at the age of 13.

He enrolled in his father's former university at age 15, and he completed his bachelor's degree in philosophy in December 1662. He defended his Disputatio Metaphysica de Principio Individui, which addressed the Principle of individuation, on June 9, 1663. Leibniz earned his master's degree in philosophy on February 7, 1664. He published and defended a dissertation Specimen Quaestionum Philosophicarum ex Jure collectarum, arguing for both a theoretical and a pedagogical relationship between philosophy and law, in December 1664. After one year of legal studies, he was awarded his bachelor's degree in Law on September 28, 1665.

In 1666, (at age 20), Leibniz published his first book, On the Art of Combinations, the first part of which was also his habilitation thesis in philosophy. His next goal was to earn his license and doctorate in Law, which normally required three years of study then. In 1666, the University of Leipzig turned down Leibniz's doctoral application and refused to grant him a doctorate in law, most likely due to his relative youth (he was 21 years old at the time). Leibniz subsequently left Leipzig.

Leibniz then enrolled in the University of Altdorf, and almost immediately he submitted a thesis, which he had probably been working on earlier in Leipzig. The title of his thesis was Disputatio Inauguralis De Casibus Perplexis In Jure. Leibniz earned his license to practice law and his Doctorate in Law in November 1666. He next declined the offer of an academic appointment at Altdorf, saying that “my thoughts were turned in an entirely different direction.”

As an adult, Leibniz often introduced himself as "Gottfried von Leibniz". Also many posthumously-published editions of his writings presented his name on the title page as "Freiherr G. W. von Leibniz." However, no document has ever been found from any contemporary government that stated his appointment to any form of nobility.

Leibniz's first position was as a salaried alchemist in Nuremberg, even though he knew nothing about the subject. He soon met Johann Christian von Boyneburg (1622 _1672), the dismissed chief minister of the Elector of Mainz, Johann Philipp von Schönborn. Von Boineburg hired Leibniz as an assistant, and shortly thereafter reconciled with the Elector and introduced Leibniz to him. Leibniz then dedicated an essay on law to the Elector in the hope of obtaining employment. The stratagem worked; the Elector asked Leibniz to assist with the redrafting of the legal code for his Electorate. In 1669, Leibniz was appointed Assessor in the Court of Appeal. Although von Boineburg died late in 1672, Leibniz remained under the employment of his widow until she dismissed him in 1674.

Von Boineburg did much to promote Leibniz's reputation, and the latter's memoranda and letters began to attract favorable notice. Leibniz's service to the Elector soon followed a diplomatic role. He published an essay, under the pseudonym of a fictitious Polish nobleman, arguing (unsuccessfully) for the German candidate for the Polish crown. The main European geopolitical reality during Leibniz's adult life was the ambition of Louis XIV of France, backed by French military and economic might. Meanwhile, the Thirty Years' War had left German-speaking Europe exhausted, fragmented, and economically backward. Leibniz proposed to protect German-speaking Europe by distracting Louis as follows. France would be invited to take Egypt as a stepping stone towards an eventual conquest of the Dutch East Indies. In return, France would agree to leave Germany and the Netherlands undisturbed. This plan obtained the Elector's cautious support. In 1672, the French government invited Leibniz to Paris for discussion, but the plan was soon overtaken by the outbreak of the Franco-Dutch War and became irrelevant. Napoleon's failed invasion of Egypt in 1798 can be seen as an unwitting implementation of Leibniz's plan.

Thus Leibniz began several years in Paris. Soon after arriving, he met Dutch physicist and mathematician Christiaan Huygens and realised that his own knowledge of mathematics and physics was patchy. With Huygens as mentor, he began a program of self-study that soon pushed him to making major contributions to both subjects, including inventing his version of the differential and integral calculus. He met Nicolas Malebranche and Antoine Arnauld, the leading French philosophers of the day, and studied the writings of Descartes and Pascal, unpublished as well as published. He befriended a German mathematician, Ehrenfried Walther von Tschirnhaus; they corresponded for the rest of their lives. In 1675 he was admitted as a foreign honorary member of the French Academy of Sciences, which he continued to follow mostly by correspondence.

When it became clear that France would not implement its part of Leibniz's Egyptian plan, the Elector sent his nephew, escorted by Leibniz, on a related mission to the English government in London, early in 1673. There Leibniz came into acquaintance of Henry Oldenburg and John Collins. After demonstrating a calculating machine he had been designing and building since 1670 to the Royal Society, the first such machine that could execute all four basic arithmetical operations, the Society made him an external member. The mission ended abruptly when news reached it of the Elector's death, whereupon Leibniz promptly returned to Paris and not, as had been planned, to Mainz.

The sudden deaths of Leibniz's two patrons in the same winter meant that Leibniz had to find a new basis for his career. In this regard, a 1669 invitation from the Duke of Brunswick to visit Hanover proved fateful. Leibniz declined the invitation, but began corresponding with the Duke in 1671. In 1673, the Duke offered him the post of Counsellor which Leibniz very reluctantly accepted two years later, only after it became clear that no employment in Paris, whose intellectual stimulation he relished, or with the Habsburg imperial court was forthcoming.

Leibniz managed to delay his arrival in Hanover until the end of 1676 after making one more short journey to London, where he was later accused by Newton of being shown some of Newton's unpublished work on the calculus. This fact was deemed evidence supporting the accusation, made decades later, that he had stolen the calculus from Newton. On the journey from London to Hanover, Leibniz stopped in The Hague where he met Leeuwenhoek, the discoverer of microorganisms. He also spent several days in intense discussion with Spinoza, who had just completed his masterwork, the Ethics. Leibniz respected Spinoza's powerful intellect, but was dismayed by his conclusions that contradicted both Christian and Jewish orthodoxy.

In 1677, he was promoted, at his request, to Privy Counselor of Justice, a post he held for the rest of his life. Leibniz served three consecutive rulers of the House of Brunswick as historian, political adviser, and most consequentially, as librarian of the ducal library. He thenceforth employed his pen on all the various political, historical, and theological matters involving the House of Brunswick; the resulting documents form a valuable part of the historical record for the period.

Among the few people in north Germany to accept Leibniz were the Electress Sophia of Hanover (1630 _1714), her daughter Sophia Charlotte of Hanover (1668 _1705), the Queen of Prussia and his avowed disciple, and Caroline of Ansbach, the consort of her grandson, the future George II. To each of these women he was correspondent, adviser, and friend. In turn, they all approved of Leibniz more than did their spouses and the future king George I of Great Britain.

The population of Hanover was only about 10,000, and its provinciality eventually grated on Leibniz. Nevertheless, to be a major courtier to the House of Brunswick was quite an honor, especially in light of the meteoric rise in the prestige of that House during Leibniz's association with it. In 1692, the Duke of Brunswick became a hereditary Elector of the Holy Roman Empire. The British Act of Settlement 1701 designated the Electress Sophia and her descent as the royal family of England, once both King William III and his sister-in-law and successor, Queen Anne, were dead. Leibniz played a role in the initiatives and negotiations leading up to that Act, but not always an effective one. For example, something he published anonymously in England, thinking to promote the Brunswick cause, was formally censured by the British Parliament.

The Brunswicks tolerated the enormous effort Leibniz devoted to intellectual pursuits unrelated to his duties as a courtier, pursuits such as perfecting the calculus, writing about other mathematics, logic, physics, and philosophy, and keeping up a vast correspondence. He began working on the calculus in 1674; the earliest evidence of its use in his surviving notebooks is 1675. By 1677 he had a coherent system in hand, but did not publish it until 1684. Leibniz's most important mathematical papers were published between 1682 and 1692, usually in a journal which he and Otto Mencke founded in 1682, the Acta Eruditorum. That journal played a key role in advancing his mathematical and scientific reputation, which in turn enhanced his eminence in diplomacy, history, theology, and philosophy.

The Elector Ernest Augustus commissioned Leibniz to write a history of the House of Brunswick, going back to the time of Charlemagne or earlier, hoping that the resulting book would advance his dynastic ambitions. From 1687 to 1690, Leibniz traveled extensively in Germany, Austria, and Italy, seeking and finding archival materials bearing on this project. Decades went by but no history appeared; the next Elector became quite annoyed at Leibniz's apparent dilatoriness. Leibniz never finished the project, in part because of his huge output on many other fronts, but also because he insisted on writing a meticulously researched and erudite book based on archival sources, when his patrons would have been quite happy with a short popular book, one perhaps little more than a genealogy with commentary, to be completed in three years or less. They never knew that he had in fact carried out a fair part of his assigned task: when the material Leibniz had written and collected for his history of the House of Brunswick was finally published in the 19th century, it filled three volumes.

In 1708, John Keill, writing in the journal of the Royal Society and with Newton's presumed blessing, accused Leibniz of having plagiarized Newton's calculus. Thus began the calculus priority dispute which darkened the remainder of Leibniz's life. A formal investigation by the Royal Society (in which Newton was an unacknowledged participant), undertaken in response to Leibniz's demand for a retraction, upheld Keill's charge. Historians of mathematics writing since 1900 or so have tended to acquit Leibniz, pointing to important differences between Leibniz's and Newton's versions of the calculus.

In 1711, while traveling in northern Europe, the Russian Tsar Peter the Great stopped in Hanover and met Leibniz, who then took some interest in Russian matters for the rest of his life. In 1712, Leibniz began a two-year residence in Vienna, where he was appointed Imperial Court Councillor to the Habsburgs. On the death of Queen Anne in 1714, Elector George Louis became King George I of Great Britain, under the terms of the 1701 Act of Settlement. Even though Leibniz had done much to bring about this happy event, it was not to be his hour of glory. Despite the intercession of the Princess of Wales, Caroline of Ansbach, George I forbade Leibniz to join him in London until he completed at least one volume of the history of the Brunswick family his father had commissioned nearly 30 years earlier. Moreover, for George I to include Leibniz in his London court would have been deemed insulting to Newton, who was seen as having won the calculus priority dispute and whose standing in British official circles could not have been higher. Finally, his dear friend and defender, the Dowager Electress Sophia, died in 1714.

Leibniz died in Hanover in 1716: at the time, he was so out of favor that neither George I (who happened to be near Hanover at the time) nor any fellow courtier other than his personal secretary attended the funeral. Even though Leibniz was a life member of the Royal Society and the Berlin Academy of Sciences, neither organization saw fit to honor his passing. His grave went unmarked for more than 50 years. Leibniz was eulogized by Fontenelle, before the Academie des Sciences in Paris, which had admitted him as a foreign member in 1700. The eulogy was composed at the behest of the Duchess of Orleans, a niece of the Electress Sophia.

Leibniz never married. He complained on occasion about money, but the fair sum he left to his sole heir, his sister's stepson, proved that the Brunswicks had, by and large, paid him well. In his diplomatic endeavors, he at times verged on the unscrupulous, as was all too often the case with professional diplomats of his day. On several occasions, Leibniz backdated and altered personal manuscripts, actions which put him in a bad light during the calculus controversy. On the other hand, he was charming, well-mannered, and not without humor and imagination. He had many friends and admirers all over Europe.

Leibniz's philosophical thinking appears fragmented, because his philosophical writings consist mainly of a multitude of short pieces: journal articles, manuscripts published long after his death, and many letters to many correspondents. He wrote only two philosophical treatises, of which only the Théodicée of 1710 was published in his lifetime.

Leibniz dated his beginning as a philosopher to his Discourse on Metaphysics, which he composed in 1686 as a commentary on a running dispute between Nicolas Malebranche and Antoine Arnauld. This led to an extensive and valuable correspondence with Arnauld; it and the Discourse were not published until the 19th century. In 1695, Leibniz made his public entrÈe into European philosophy with a journal article titled "New System of the Nature and Communication of Substances". Between 1695 and 1705, he composed his New Essays on Human Understanding, a lengthy commentary on John Locke's 1690 An Essay Concerning Human Understanding, but upon learning of Locke's 1704 death, lost the desire to publish it, so that the New Essays were not published until 1765. The Monadologie, composed in 1714 and published posthumously, consists of 90 aphorisms.

Leibniz met Spinoza in 1676, read some of his unpublished writings, and has since been suspected of appropriating some of Spinoza's ideas. While Leibniz admired Spinoza's powerful intellect, he was also forthrightly dismayed by Spinoza's conclusions, especially when these were inconsistent with Christian orthodoxy.

Unlike Descartes and Spinoza, Leibniz had a thorough university education in philosophy. He was influenced by his Leipzig professor Jakob Thomasius, who also supervised his BA thesis in philosophy. Leibniz also eagerly read Francisco Suárez, a Spanish Jesuit respected even in Lutheran universities. Leibniz was deeply interested in the new methods and conclusions of Descartes, Huygens, Newton, and Boyle, but viewed their work through a lens heavily tinted by scholastic notions. Yet it remains the case that

Leibniz's methods and concerns often anticipate the logic, and analytic and linguistic philosophy of the 20th century.

Leibniz variously invoked one or another of seven fundamental philosophical Principles:

1. Identity/contradiction. If a proposition is true, then its negation is false and vice versa.
2. Identity of indiscernibles. Two things are identical if and only if they share the same and only the same properties. Frequently invoked in modern logic and philosophy. The "identity of indiscernibles" is often referred to as Leibniz's Law. It has attracted the most controversy and criticism, especially from corpuscular philosophy and quantum mechanics.
3. Sufficient reason. "There must be a sufficient reason [often known only to God] for anything to exist, for any event to occur, for any truth to obtain."
4. Pre-established harmony. "[T]he appropriate nature of each substance brings it about that what happens to one corresponds to what happens to all the others, without, however, their acting upon one another directly." (Discourse on Metaphysics, XIV) A dropped glass shatters because it "knows" it has hit the ground, and not because the impact with the ground "compels" the glass to split.
5. Law of Continuity. Natura non saltum facit.
6. Optimism. "God assuredly always chooses the best."
7. Plenitude. "Leibniz believed that the best of all possible worlds would actualize every genuine possibility, and argued in ThÈodicÈe that this best of all possible worlds will contain all possibilities, with our finite experience of eternity giving no reason to dispute nature's perfection."

Leibniz would on occasion give a speech for a specific principle, but more often took them for granted.

Leibniz believed that much of human reasoning could be reduced to calculations of a sort, and that such calculations could resolve many differences of opinion:

“The only way to rectify our reasonings is to make them as tangible as those of the Mathematicians, so that we can find our error at a glance, and when there are disputes among persons, we can simply say: Let us calculate [calculemus], without further ado, to see who is right.”

Leibniz's calculus ratiocinator, which resembles symbolic logic, can be viewed as a way of making such calculations feasible. Leibniz wrote memoranda that can now be read as groping attempts to get symbolic logic -and thus his calculus- off the ground. But Gerhard and Couturat did not publish these writings until modern formal logic had emerged in Frege's Begriffsschrift and in writings by Charles Sanders Peirce and his students in the 1880s, and hence well after Boole and De Morgan began that logic in 1847.

Leibniz thought symbols were important for human understanding. He attached so much importance to the invention of good notations that he attributed all his discoveries in mathematics to this. His notation for the infinitesimal calculus is an example of his skill in this regard. C.S. Peirce, a 19th-century pioneer of semiotics, shared Leibniz's passion for symbols and notation, and his belief that these are essential to a well-running logic and mathematics.

But Leibniz took his speculations much further. Defining a character as any written sign, he then defined a "real" character as one that represents an idea directly and not simply as the word embodying the idea. Some real characters, such as the notation of logic, serve only to facilitate reasoning. Many characters well known in his day, including Egyptian hieroglyphics, Chinese characters, and the symbols of astronomy and chemistry, he deemed not real. Instead, he proposed the creation of a characteristica universalis or "universal characteristic", built on an alphabet of human thought in which each fundamental concept would be represented by a unique "real" character:

It is obvious that if we could find characters or signs suited for expressing all our thoughts as clearly and as exactly as arithmetic expresses numbers or geometry expresses lines, we could do in all matters insofar as they are subject to reasoning all that we can do in arithmetic and geometry. For all investigations which depend on reasoning would be carried out by transposing these characters and by a species of calculus.

Complex thoughts would be represented by combining characters for simpler thoughts. Leibniz saw that the uniqueness of prime factorization suggests a central role for prime numbers in the universal characteristic, a striking anticipation of Gödel numbering. Granted, there is no intuitive or mnemonic way to number any set of elementary concepts using the prime numbers. Leibniz's idea of reasoning through a universal language of symbols and calculations however remarkably foreshadows great 20th century developments in formal systems, such as Turing completeness, where computation was used to define equivalent universal languages (see Turing degree).

Because Leibniz was a mathematical novice when he first wrote about the characteristic, at first he did not conceive it as an algebra but rather as a universal language or script. Only in 1676 did he conceive of a kind of "algebra of thought", modeled on and including conventional algebra and its notation. The resulting characteristic included a logical calculus, some combinatorics, algebra, his analysis situs (geometry of situation), a universal concept language, and more.

What Leibniz actually intended by his characteristica universalis and calculus ratiocinator, and the extent to which modern formal logic does justice to the calculus, may never be established.

Leibniz is the most important logician between Aristotle and 1847, when George Boole and Augustus De Morgan each published books that began modern formal logic. Leibniz enunciated the principal properties of what we now call conjunction, disjunction, negation, identity, set inclusion, and the empty set. The principles of Leibniz's logic and, arguably, of his whole philosophy, reduce to two:

1. All our ideas are compounded from a very small number of simple ideas, which form the alphabet of human thought.
2. Complex ideas proceed from these simple ideas by a uniform and symmetrical combination, analogous to arithmetical multiplication.

The formal logic that emerged early in the 20th century also requires, at minimum, unary negation and quantified variables ranging over some universe of discourse.

Leibniz published nothing on formal logic in his lifetime; most of what he wrote on the subject consists of working drafts. In his book History of Western Philosophy, Bertrand Russell went so far as to claim that Leibniz had developed logic in his unpublished writings to a level which was reached only 200 years later.

Although the mathematical notion of function was implicit in trigonometric and logarithmic tables, which existed in his day, Leibniz was the first, in 1692 and 1694, to employ it explicitly, to denote any of several geometric concepts derived from a curve, such as abscissa, ordinate, tangent, chord, and the perpendicular. In the 18th century, "function" lost these geometrical associations.

Leibniz was the first to see that the coefficients of a system of linear equations could be arranged into an array, now called a matrix, which can be manipulated to find the solution of the system, if any. This method was later called Gaussian elimination. Leibniz's discoveries of Boolean algebra and of symbolic logic, also relevant to mathematics, are discussed in the preceding section. The best overview of Leibniz's writings on the calculus may be found in Bos (1974).

Leibniz is credited, along with Sir Isaac Newton, with the invention of infinitesimal calculus (that comprises differential and integral calculus). According to Leibniz's notebooks, a critical breakthrough occurred on November 11, 1675, when he employed integral calculus for the first time to find the area under the graph of a function $y = \int f(x)$. He introduced several notations used to this day, for instance the integral sign $\int$ representing an elongated S, from the Latin word summa and the d used for differentials, from the Latin word differentia. This cleverly suggestive notation for the calculus is probably his most enduring mathematical legacy. Leibniz did not publish anything about his calculus until 1684. The product rule of differential calculus is still called "Leibniz's law". In addition, the theorem that tells how and when to differentiate under the integral sign is called the Leibniz integral rule.

Leibniz's approach to the calculus fell well short of later standards of rigor (the same can be said of Newton's). We now see a Leibniz proof as being in truth mostly a heuristic argument mainly grounded in geometric intuition. Leibniz also freely invoked mathematical entities he called infinitesimals, manipulating them in ways suggesting that they had paradoxical algebraic properties. George Berkeley, in a tract called The Analyst and also in De Motu, criticized these.

From 1711 until his death, Leibniz was engaged in a dispute with John Keill, Newton and others, over whether Leibniz had invented the calculus independently of Newton. This subject is treated at length in the article Leibniz-Newton controversy.

Infinitesimals were officially banned from mathematics by the followers of Karl Weierstrass, but survived in science and engineering, and even in rigorous mathematics, via the fundamental computational device known as the differential. Beginning in 1960, Abraham Robinson worked out a rigorous foundation for Leibniz's infinitesimals, using model theory, in the context of a field of hyperreal numbers. The resulting non-standard analysis can be seen as a belated vindication of Leibniz's mathematical reasoning. Robinson's transfer principle is a mathematical implementation of Leibniz's heuristic law of continuity.

Leibniz may have been the first computer scientist and information theorist. Early in life, he documented the binary numeral system (base 2), then revisited that system throughout his career. He anticipated Lagrangian interpolation and algorithmic information theory. His calculus ratiocinator anticipated aspects of the universal Turing machine. In 1934, Norbert Wiener claimed to have found in Leibniz's writings a mention of the concept of feedback, central to Wiener's later cybernetic theory.

In 1671, Leibniz began to invent a machine that could execute all four arithmetical operations, gradually improving it over a number of years. This "Stepped Reckoner" attracted fair attention and was the basis of his election to the Royal Society in 1673. A number of such machines were made during his years in Hanover, by a craftsman working under Leibniz's supervision. It was not an unambiguous success because it did not fully mechanize the operation of carrying. Couturat reported finding an unpublished note by Leibniz, dated 1674, describing a machine capable of performing some algebraic operations.

Leibniz was groping towards hardware and software concepts worked out much later by Charles Babbage and Ada Lovelace. In 1679, while mulling over his binary arithmetic, Leibniz imagined a machine in which binary numbers were represented by marbles, governed by a rudimentary sort of punched cards. Modern electronic digital computers replace Leibniz's marbles moving by gravity with shift registers, voltage gradients, and pulses of electrons, but otherwise they run roughly as Leibniz envisioned in 1679.

Selected works (The year given is usually that in which the work was completed, not of its eventual publication.):

1666. De Arte Combinatoria (On the Art of Combination); partially translated in Loemker ß1 and Parkinson (1966).
1671. Hypothesis Physica Nova (New Physical Hypothesis); Loemker ß8.I (partial).
1673 Confessio philosophi (A Philosopher's Creed); an English translation is available.
1684. Nova methodus pro maximis et minimis (New method for maximums and minimums); translated in Struik, D. J., 1969. A Source Book in Mathematics, 1200 _1800. Harvard University Press: 271 _81.
1686. Discours de métaphysique; Martin and Brown (1988), Ariew and Garber 35,

Loemker ß35, Wiener III.3, Woolhouse and Francks 1. An online translation by Jonathan Bennett is available.

1703. Explication de l'Arithmétique Binaire (Explanation of Binary Arithmetic); Gerhardt, Mathematical Writings VII.223. An online translation by Lloyd Strickland is available.

1710. Théodicée; Farrer, A.M., and Huggard, E.M., trans., 1985 (1952). Wiener III.11 (part). An online translation is available at Project Gutenberg.

1714. Monadologie; translated by Nicholas Rescher, 1991. The Monadology: An Edition for Students. University of Pittsburg Press. Ariew and Garber 213, Loemker ß67, Wiener III.13, Woolhouse and Francks 19. Online translations: Jonathan Bennett's translation; Latta's translation; French, Latin and Spanish edition, with facsimile of Leibniz's manuscript.

1765. Nouveaux essais sur l'entendement humain; completed in 1704. Remnant, Peter, and Bennett, Jonathan, trans., 1996. New Essays on Human Understanding. Cambridge University Press. Wiener III.6 (part). An online translation by Jonathan Bennett is available.