

Memorandum M-596

SHELL BUCKLING RESEARCH AT DELFT (1976-1988)

by

J. Arbocz

Professor of Aircraft Structures
Faculty of Aerospace Engineering
Delft University of Technology

An ERASMUS Study Group on Shell Buckling recommended that reviews be written of the research into shell buckling which has been carried out over the past decade or so at the various co-operating Institutions.

This report is the contribution from Delft to the above. The other centers involved are RWTH Aachen, ICST, London, INSA, Lyon and University of Liverpool.

SHELL BUCKLING RESEARCH AT THE AEROSPACE ENGINEERING FACULTY
OF DELFT UNIVERSITY OF TECHNOLOGY

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INTRODUCTION

The central goal of the shell research being carried out at the Aerospace Engineering Faculty of the TU-Delft is the development of an 'Improved Shell Design Criteria', which incorporates all the theoretical knowledge accumulated in the last, say, 25 years thru intensive research in the Aerospace, the Nuclear and the Offshore fields and which makes efficient use of the currently available interactive and (super) computing facilities.

To demonstrate the improvements that can be achieved the case of axially compressed isotropic, orthotropic and/or anisotropic shells has been studied in a combined experimental, analytical and numerical approach.

CURRENT DESIGN APPROACH

The dilemma of the stability analysis of axially compressed cylindrical shells is well known. Trying to explain the discrepancy between the theoretical predictions based on the linearized small deflection theory and the experimental results has occupied some of the most eminent scientists of this century.

Though certain in-plane boundary conditions can effect the buckling load considerably and for thicker shells ($R/t < 200$, say) inelastic effects must be included in the analysis, initial geometric imperfections have been accepted as the main cause of the wide experimental scatter. Despite this recognition the incorporation of the idea of imperfection sensitivity into engineering practice has not been accomplished. All the current design manuals, including the ECCS Recommendations [1], adhere to the so-called 'Lower Bound Design Philosophy' that has already been in use 50 years ago. That is, they recommend the use of an empirical knockdown factor, which is so chosen that when it is

multiplied with the classical buckling load a 'Lower Bound' to all available experimental data is obtained. In the form of a formula

$$P_a \leq \frac{\gamma}{\text{F.S.}} P_{cl} \quad (1)$$

where

P_a = allowable applied load

P_c = classical buckling load (perfect shell)

$$= \frac{2\pi Et^2}{\sqrt{3(1-\nu^2)}} \quad \text{- for isotropic shells}$$

γ = knockdown factor

$$= 1 - 0.902 \left(1 - e^{-\frac{1}{16} \sqrt{\frac{R}{t}}}\right) \quad \text{- for isotropic shells}$$

F.S. = factor of safety

For isotropic shells the knockdown factor is shown in Fig. 1.

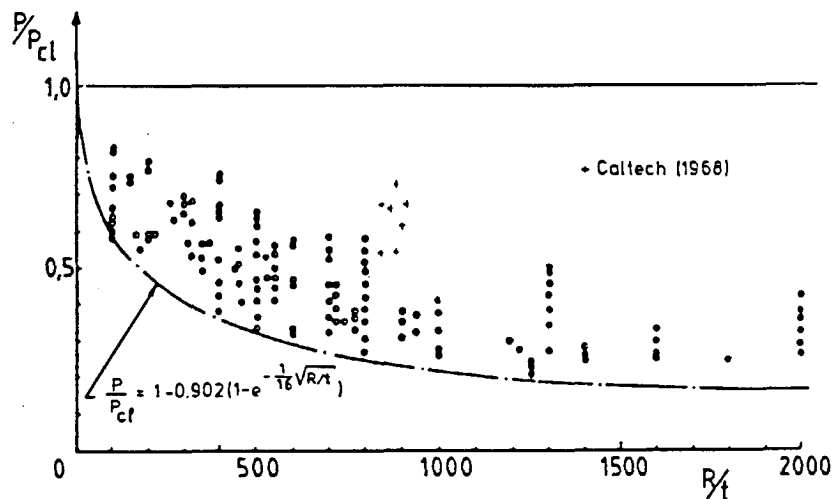


Fig. 1. Test data for isotropic cylinders under axial compression.

Much effort has been spent in the past 30 years in trying to find the cause (or the causes) for the wide experimental scatter shown in Fig. 1.

Thanks to the contributions of many scientists the concensus reached is that the experimental buckling loads are mainly affected by 3 factors, namely

1. Boundary conditions,
2. Initial Geometric Imperfections,
3. Inelastic Effects.

THE EFFECTS OF BOUNDARY CONDITIONS AND NONLINEAR PREBUCKLING DEFORMATIONS

The effect of experimental boundary conditions has been studied extensively in the past by Hoff and Soong², Almroth³, Weller et al.⁴, Arbocz and Sechler^{5,6} and others. The effect can be separated into two major items that will be discussed separately. These are the effect of end fixity on the buckling deformation (eigenfunction) and its associated buckling load (eigenvalue) and the effect of nonlinear prebuckling deformation caused by the end constraint of the shell.

As can be seen from the results shown in Table 1 for the stringer stiffened shell AS-2 (see Table 2 for its geometric and material properties) the buckling load with membrane prebuckling depends strongly on the boundary conditions specified. Stiffening the boundary conditions raises the buckling load by about 12% for C-3, by about 34% for SS-4 and by about 39% for the C-4 boundary condition. On the other hand, the inclusion of the nonlinear prebuckling deformations (with the shell loaded through the shell midsurface) has an insignificant effect. The integers in the parenthesis indicate the number of full waves of the buckling pattern in the circumferential direction.

TABLE 1

Buckling loads of the perfect stringer stiffened shell AS-2^[7]

	SS-3	SS-4	C-3	C-4
Membrane prebuckling (N/cm)	229.8(10)	300.7(14)	256.9(10)	320.8(14)
Nonlinear prebuckling (N/cm)	224.0(10)	280.0(14)	256.0(14)	316.8(14)

It must further be mentioned that in most practical applications the shell edges are supported elastically by rings. Cohen⁸ has shown in a 1966 paper that there is a critical size of the end-ring below which the ring strain energy controls the buckling. In this case the large deformation of the end rings leads to an inextensional buckling mode with 2 full-waves in the circumferential direction at a relatively low buckling load. With the SRA computer code one can compute the critical buckling loads for the stringer stiffened shell AS-2 with elastic end-rings of varying sizes. As can be seen from Fig. 2 also for shell AS-2 there is a critical size of the end rings which separates the region where edge buckling prevails from the region where general overall buckling is critical. The computations were carried out for symmetrically placed end rings of square cross-section where $A_1 = Ct^2$. If one introduces the following rigidity ratio

$$\frac{(EI) \text{ RING}}{D \frac{L}{2} \text{ SHELL}} \quad (2)$$

where for stringer stiffened shells

$$D = \frac{Et^3}{12(1-\nu^2)} + \frac{E(I_{11} + A_1 e_1^2)}{d_1} = C_{44} \quad (3)$$

then the transition point corresponds approximately to a critical rigidity ratio of 100, the same that was found by Cohen⁸ for isotropic shells.

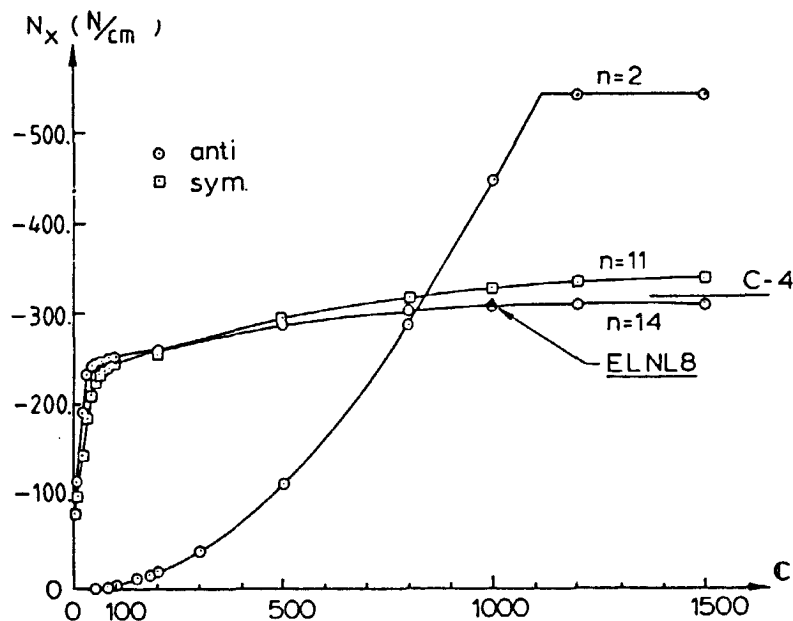


Figure 2. Critical loads for shell AS-2 with elastic end rings.

TABLE 2

Geometric and material properties of shell AS-2

t	=	1.96596 x 10 ⁻²	cm
L	=	13.97	cm
R	=	10.16	cm
d ₁	=	8.03402 x 10 ⁻¹	cm
e ₁	=	3.36804 x 10 ⁻²	cm
A ₁	=	7.98708 x 10 ⁻³	cm ²
I ₁₁	=	1.50384 x 10 ⁻⁶	cm ⁴
I _{t1}	=	4.94483 x 10 ⁻⁶	cm ⁴
E	=	6.89472 x 10 ⁺⁶	N/cm ²
v	=	0.3	

THE IMPERFECTION SENSITIVITY THEORY

Mainly due to the pioneering work of Koiter¹⁰ and Budiansky and Hutchinson¹¹, for thin shells that buckle elastically, initial geometric imperfections have been accepted as the main cause of the wide scatter of experimental results seen in Fig. 1. Because of the complexity of the problem in the early investigations the initial imperfection representations were restricted to the simplest possible form. For an axially compressed isotropic shell Koiter¹² in 1963 assumed an initial imperfection in the form of the classical axisymmetric buckling mode

$$\bar{w} = t\bar{\xi}_1 \cos i_{cl} \frac{\pi x}{L} \quad (4)$$

where

$$i_{cl} = \frac{L}{\pi} \sqrt{\frac{2c}{Rt}} \quad (5)$$

and calculated the critical load at which bifurcation from the axisymmetric prebuckling state into an asymmetric buckling mode

$$\hat{W} = C_{k\ell} \sin \frac{k\pi x}{L} \cos \frac{\ell y}{R} \quad (6)$$

will occur. He found that the minimum buckling load occurs when $k = \frac{1}{2} i_{c\ell}$ and that the value of ℓ , the number of full waves in the circumferential direction depends on the shell geometry and the amplitude of the initial imperfection $\bar{\xi}_1$.

It is often stated that for a given imperfection amplitude from all the possible imperfection shapes the one affine to the critical buckling mode produces the lowest buckling load. Since for the stringer stiffened shell AS-2 the critical buckling mode is asymmetric, therefore an initial imperfection shape

$$\bar{W} = t \bar{\xi}_2 \sin \frac{\pi x}{L} \cos \frac{10y}{R} \quad (7)$$

must be very damaging.

If the amplitude of the initial imperfection $\bar{\xi}_2$ is known, then from Koiter's formula¹⁰

$$(1-\rho_s)^{3/2} = \frac{3}{2} \sqrt{-3b} |\bar{\xi}_2| \rho_s \quad (8)$$

one can calculate the collapse load $\rho_s = \lambda_s / \lambda_{c_{k\ell}}$ of the shell AS-2. If (say) $\bar{\xi}_2 = 0.4$ then $\rho_s = 0.74$.

As soon as measured initial imperfection surveys were published (see Fig. 3 for shell AS-2) it became doubtful whether the effect of the initial imperfections occurring in practice could indeed be represented by a single trigonometric function. Thus in 1974 Arbocz and Babcock¹³ presented the so-called Multimode Analysis, where the measured initial imperfections were represented by the following double Fourier series

$$\begin{aligned} \bar{W} = t \sum \bar{W}_{i0} \cos i \frac{\pi x}{L} \\ + t \sum \sum \sin k \frac{\pi x}{L} (\bar{W}_{k\ell} \cos \frac{\ell y}{R} + \bar{W}'_{k\ell} \sin \frac{\ell y}{R}) \end{aligned} \quad (9)$$

With the code MIUTAM^[13] extensive correlation studies have been carried out at Caltech^[14] and at Technion^[15]. It was found that in most cases the

theoretical predictions based on the measured initial imperfections were within about 10% of the experimental collapse loads.

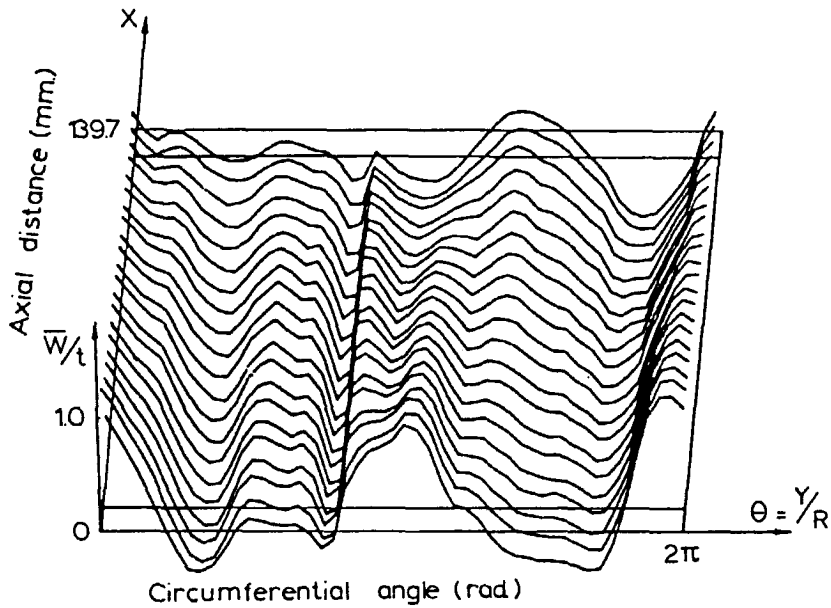


Figure 3. Measured initial shape of the stringer-stiffened shell AS-2^[14]

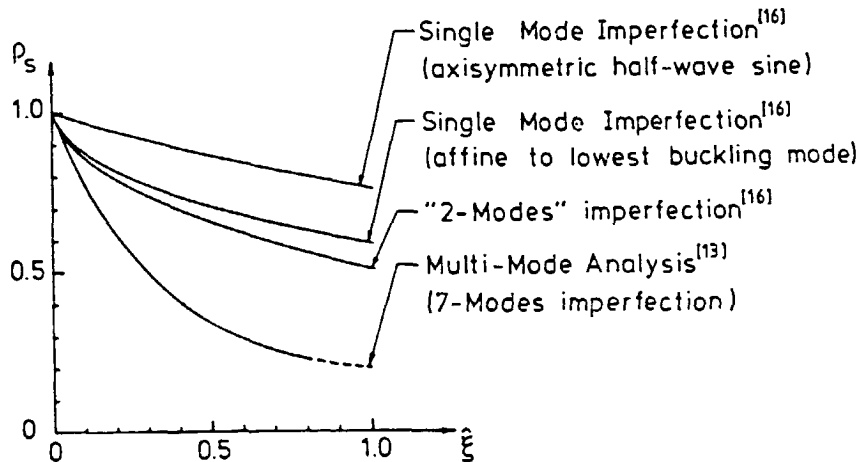


Figure 4. Imperfection sensitivity for different models (shell AS-2)

Figure 4 summarizes for shell AS-2 the buckling load predictions based on the different imperfection models. Looking at this figure it becomes clear that for reliable prediction one must know both the shape and the size of the initial imperfections. This brings up the critical question every shell designer must face, once it has been established that the buckling load of the proposed structure is imperfection sensitive:

'Is it cheaper to use a large knockdown factor and a large factor of safety to account for the uncertainties involved, or should one apply the Imperfection Sensitivity Theory in order to arrive at an optimal design?'

INITIAL IMPERFECTION DATA BANKS

It is true that for many cases, especially in applications where the total weight of the structure is of no major concern, the Lower Bound Design Method provides safe and reliable buckling load prediction. However, it penalizes innovative shell design because of the poor experimental results obtained with shells produced and tested under completely different circumstances, sometimes half-a-century ago.

If, however, the total weight is of critical importance, then a more sophisticated design approach is called for. That is, the designer must estimate how much the expected imperfections will decrease the buckling load of the chosen configuration. It is obvious, that the main difficulty in using the Imperfection Sensitivity Theory in practical design problems with weight sensitive applications is related to the fact, that it requires some advanced knowledge of the geometric imperfections that will be present once the structure under consideration has been built, an information that is rarely available.

The fact that any further improvements in our buckling load prediction capability is dependent on the availability of imperfection data has been recognized and has lead to the establishment of an International Imperfection Data Bank with the first two branches in Delft [17] and at the Technion [18] in Haifa.

It is encouraging to see that practically all the current experimental programs include initial imperfection surveys on buckling sensitive structures

[19]. The initial analysis of the imperfection data assembled so far has shown clearly that it is possible to associate characteristic initial imperfection distributions with the different manufacturing processes [20]. This fact makes it possible to combine the available imperfection data with a statistical analysis of both the imperfections and corresponding critical loads into a (say) Statistical Imperfection Sensitivity Analysis.

IMPROVED (STOCHASTIC) SHELL DESIGN PROCEDURE

The improvements in the currently recommended shell design procedures are primarily sought in a more selective approach by the definition of the 'knockdown' factor λ . Thus, for instance, if a company takes great care in producing its shells very accurately and if it can show experimentally that the boundary conditions are defined in such a way that no additional imperfections (especially at the shell edges) are introduced, then the use of an improved (higher) 'knockdown' factor λ derived by a stochastic approach should be allowed. The proposed new Improved Shell Design Procedure can be presented by the following formula:

$$P_a \leq \frac{\lambda_a}{F.S.} P_c$$

where

- P_a = allowable buckling load
- P_c = buckling load of 'perfect' structure computed via shell codes
- λ_a = reliability based improved (higher) 'knockdown' factor
- F.S. = factor of safety

The steps involved in the definition of such a reliability based improved (higher) 'knockdown' factor λ_a can be summarized as follows:

1. Compute the Fourier coefficients of the initial imperfection surveys of a relatively small sample (say 4) nominally identical shells.
2. Calculate the mean vector and the variance-covariance matrix of the Fourier coefficients of the experimental sample.

3. Compute the reliability function $R(\lambda)$ by a first Order Second Moment Analysis [21] of the buckling of shells with the random imperfections of steps 1 and 2.
4. Determine the improved (higher) 'knockdown' factor λ_a for a given reliability from the plot $R(\lambda)$ vs λ (see Fig. 5).

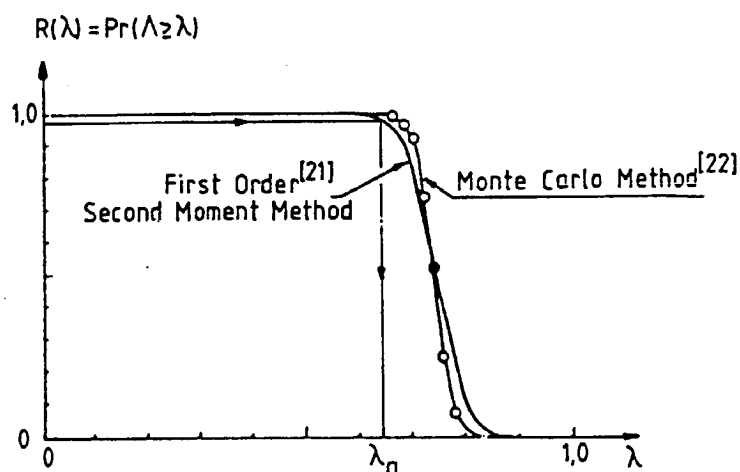


Fig. 5. Reliability function $R(\lambda)$ for a given R/t ratio.

Notice that by replacing the Monte Carlo Method [22] by the First Order Second Moment Analysis the number of deterministic buckling load calculations needed to derive the reliability function $R(\lambda)$ is greatly reduced (from, say, 1098 to 15).

Also, as can be seen from Fig. 5, the reliability function $R(\lambda)$ obtained by the First Order Second Moment Analysis is somewhat conservative in the region of high reliability when compared to the results obtained via the Monte Carlo Method.

If the R/t values of the shells in the small experimental sample vary slightly (see Caltech shells [23] in Fig. 1) then it is sufficient to derive just a single reliability function $R(\lambda)$ for a group of shells produced by the same fabrication process. One uses then the mean values for the geometric parameters involved like radius R , wall-thickness t , length L , Young's modulus

E and Poisson's ratio ν . However, if the geometric parameters of the shells in question vary widely it is necessary to calculate several reliability functions for a given fabrication process in order to obtain an Improved Lower Bound (see Fig. 6).

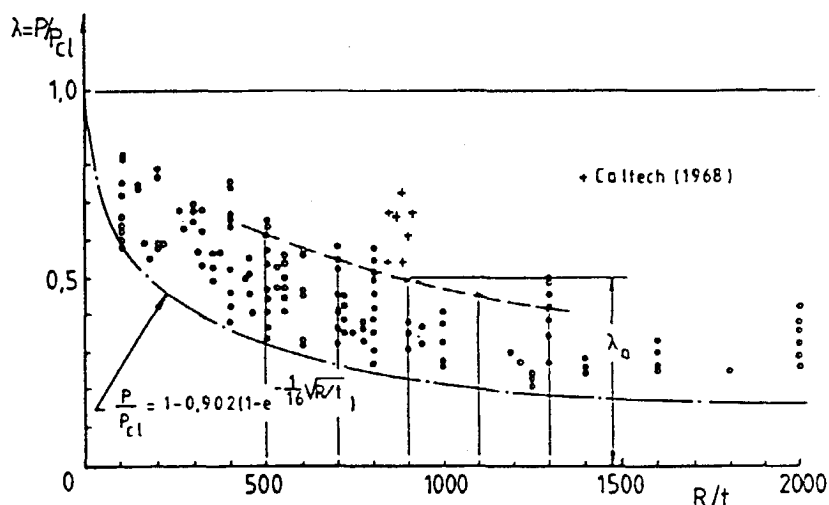


Fig. 6. Definitions of the Improved Lower Bound Design Curve.

Using the First Order Second Moment Analysis to derive reliability functions one is combining the Lower Bound Design Philosophy with the notion of Goodness Classes. Thus shells manufactured by a process, which produces inherently a less damaging initial imperfection distribution, will not be penalized because of the low experimental results obtained with shells produced by another process, which generates a more damaging characteristic initial imperfection distribution.

INTERACTIVE SHELL DESIGN CODE 'DISDECO'

The key to the success of any Stochastic Stability Analysis lies in the reliability and accuracy of the underlying deterministic buckling analysis used. As has been pointed out by Arbocz and Babcock²⁴ the success of the deterministic buckling load analysis depends very heavily on the appropriate choice of the nonlinear model used, which in turn requires considerable knowledge by the user as to the physical behaviour of imperfect shell structures. This knowledge can be acquired by first using the series of imperfection sensitivity analysis of increasing complexity that have been described in the literature [24,25].

In order to facilitate the introduction of the proposed Improved Shell Design Procedure the development of DISDECO^[7], the Delft Interactive Shell Design Code has been initiated. The purpose of this project is to make the accumulated theoretical, numerical and practical knowledge of the last 25 years readily accessible to users interested in the design of buckling sensitive thin-walled shell structures. With this open-ended, hierarchical, modular, interactive computer code the user can access from his workstation successively programs of different complexity. Also, at every step of his analysis the user can call upon extensive HELP files containing useful information about the potential design solutions.

The steps involved in calculation the reliability based improved (higher) knockdown factor λ_a are done with the module called STOCH. This program needs as input besides the geometric properties of the shell under consideration also the Fourier coefficients of the measured initial imperfections of a relatively small sample (say 4) of nominally identical shells and information about the nonlinear model to be used for the deterministic buckling load calculations.

CONCLUSIONS

For a successful implementation of the proposed Improved Shell Design Procedure the companies involved must be prepared to do the initial investments in carrying out complete imperfection surveys on a (small) sample of shells that are representative of their production-line. With the modern

measuring and data acquisition systems one can carry out a complete surface map of very large shells at a negligible small fraction of their production cost. What is more expensive is the data reduction and the analysis that must be carried out in order to get reliability functions. The help and encouragement of Supervising Governmental Agencies and the Engineering Societies would be very beneficial for this endeavor.

The Solid Mechanics Group of the Aerospace Engineering Faculty of the Delft University of Technology is prepared to set up cooperative programs with interested companies in order to advise them how they can carry out the necessary imperfection surveys in an optimal manner, and to perform the necessary data reduction and the analysis involved in getting the reliability functions at minimal costs.

It is the author's opinion that, as the amount of data characteristic initial imperfection distributions classified according to fabrication processes increases, we shall succeed with the help of the increased computational speed of the next generation of computers to make the Improved Shell Design Procedure available to more and more shell designers. This, hopefully, will result in the desired dissemination of the vast amount of theoretical knowledge accumulated over the past 75 years about shell buckling behaviour. Thus finally, the academic world will be able to point to the successful solution of one of the most perplexing problems in Mechanics.

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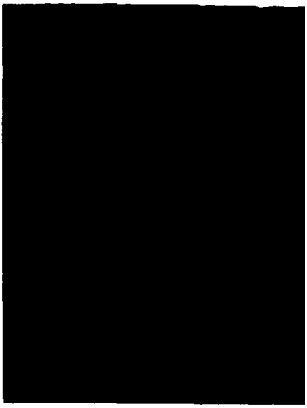
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