Use of GENOPT and BIGBOSOR4 to obtain optimum designs of multi-walled inflatable spherical and cylindrical vacuum chambers

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GENOPT/BIGBOSOR4 is applied to the problem of perfect elastic spherical or cylindrical "shells" the complex inflatable wall of which is a webbed sandwich. The spherical or cylindrical "shell" is stabilized by uniform pressure applied between its inner and outer walls and subjected to uniform pressure applied to its outermost wall. This paper is analogous to [1]. The distance between the inner and outer walls of the optimized spherical balloons is smaller than that for the optimized cylindrical balloons. The pre-buckling behavior of the spherical balloons is "crankier" (more nonlinear) than that of the cylindrical balloons with the result that certain special strategies have to be introduced in order to permit the generation of optimum designs via the GENOPT processor called SUPEROPT. General buckling modes of the type observed in optimized cylindrical balloons have so far not been observed in any spherical balloons, optimized or not. Local buckling modes include both axisymmetric modes and non-axisymmetric modes with many circumferential waves. Since [1] was written new versions of the "balloon" software, behavior.balloon and bosdec.balloon, have been created by means of which both cylindrical and spherical balloons and balloons with either radial webs or truss-like (slanted) webs can all be optimized and analyzed with use of the same "balloon" software. Since [1] was produced a new behavioral constraint has been added that involves a load factor corresponding to the initial loss of tension in any of the segments of the balloon wall. This new behavioral constraint is related to initial wrinkling of the balloon, which is a type of buckling that pertains to both cylindrical and spherical balloons. Optimum designs are found for balloons made of polyethylene terephthalate, which has a maximum allowable stress of 10000 psi and weight density, 0.1 lb/in^3, and for balloons made of a much stronger and lighter fictitious carbon fiber cloth, which has much higher maximum allowable tensile and compressive stresses, 75600 psi and 59600 psi, respectively, and lower weight density, 0.057 lb/in^3. The optimized weights of the balloons made of the much stronger and lighter fictitious carbon fiber cloth are 15 to 20 times lighter than those made of polyethylene terephthalate. A section is included showing optimized designs of cylindrical balloons made of the fictitious carbon fiber cloth, which is not included as a material option in [1]. Some peculiarities of the pre-buckling deformations and general buckling modes of optimized spherical and cylindrical balloons made of fictitious carbon fiber cloth are displayed. These optimized balloons, which have much thinner walls than the optimized balloons made of polyethylene terephthalate, exhibit significant spurious local "zig-zag" components of pre-buckling and bifurcation buckling modal displacements. Convergence studies with respect

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to the number of nodal points used for each segment of the balloons indicate that this spurious local "zig-zag" characteristic does not have a major influence on the prediction of the overall behavior of the balloons. Therefore, it appears that the optimized designs are valid despite the spurious local "zig-zag" characteristic, which disappears with increasing numbers of nodal points used in each segment of a balloon wall.

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SECTION 1 INTRODUCTION, PURPOSE, AND SUMMARY

1.1 Introduction and purpose

The effort that resulted in [1] and in this paper was motivated by Michael Mayo, Lockheed Martin Advanced Technology Center, Palo Alto, California, who found Ref.[2]. In [2] the stability of a multi-walled cylindrical vacuum chamber of one of the types treated in [1] is investigated: the cylindrical balloon with radial webs (Fig. 1 of [1]). The aim of the analysis in [2] is to determine the pressure between the inner and outer walls such that the cylindrical vacuum chamber subjected to a given external pressure will remain stable.

The purpose of the work reported in the present paper is to expand the generic "**balloon**" capability described in [1] to optimize (find the minimum weight of) not only cylindrical vacuum chambers but also of spherical vacuum chambers with walls of the types depicted in Fig. 1 and in Fig. 7(a) of [2]. Figures 1 and 2 of [1] show the geometries of the cross sections of 90 degrees of the circumference of a cylindrical vacuum chamber that are analogous to those shown in Fig. 1 of [2] and in Fig. 7(a) of [2], respectively. Figures 1 and 2 of the present paper show the geometries of the cross sections of 90 degrees of the meridian of a spherical vacuum chamber that are analogous to Figs. 1 and 2 of [1].

It is emphasized that the spherical vacuum chamber, frequently called "spherical balloon" in this paper, has no meridionally oriented webs or gores, only circumferentially oriented webs as displayed in Figs. 1 and 2. The outer wall of the spherical balloon has axisymmetric local bulges reminiscent of the Michelin Tire Man, not the meridionally oriented bulging gores typical of those in recreational balloons, the kind that lift tourists in baskets suspended beneath them. The BIGBOSOR4 model [4,5,6], on which this study is based, cannot handle shells or balloons with meridionally oriented bulging gores or meridionally oriented webs because structures of those types are not axisymmetric, and BIGBOSOR4 is a shell-of-revolution analyzer. The spherical vacuum chamber (balloon) reminiscent of the Michelin Tire Man is not isotropic. Hence, when the pressure, PMIDDL, is applied between the inner and outer walls, the initially spherical balloon elongates much more in the axial (vertical) direction than in the radial direction, becoming very slightly egg-shaped, as is displayed in Fig. 6, for example.

The reader should first read [1] in order more fully to understand the technology on which the present paper is based. Some of the details in [1] will not be repeated here. For example, the section in [1] about prismatic shells is not relevant to spherical vacuum chambers, which are modeled as "ordinary" shells of revolution.

To solve the problem for a given balloon wall cross section and to provide a means to optimize that wall cross section, the GENOPT system [3] is combined with the most recent version of the BOSOR4 code [4,5], a shell-of-revolution analyzer called "BIGBOSOR4" [6]. BIGBOSOR4 handles many more shell segments (up to 295 shell segments) than does the much older BOSOR4. In this paper the system created to optimize shells of revolution is called "**GENOPT/BIGBOSOR4**" [6 – 9]. A brief overview of the GENOPT program [3] is given in [1] and repeated here because the reader needs that information in order to understand how GENOPT works. Extensive details about how to use GENOPT in connection with BIGBOSOR4 are presented in [7], [9], and [13].

The GENOPT computer program [3] performs optimization with the use of a gradient-based optimizer called "ADS", created many years ago by Vanderplaats and his colleagues [10,11]. In [3] and [6 – 9] ADS is "hard wired" in a "modified-method-of-steepest-descent" (1-5-7) mode. In GENOPT a matrix of behavior constraint gradients is computed from finite differences of the behavioral constraints for the perturbed design minus the behavioral constraints for the current design in which the decision variables are perturbed one at a time by a certain percentage, usually five per cent. By "behavior" is meant buckling, stress, displacement, vibration frequency, clearance, and any other phenomena that may affect the evolution of a design during optimization cycles. In this paper the behavioral constraints are bifurcation buckling, initial loss of tension in the balloon wall, and stress.

The objective of the optimization is to minimize the weight of a spherical or a cylindrical vacuum chamber subjected to a set of specified requirements: behavioral constraints such as buckling, loss of tension, and stress.

The models used here for the optimization of the sandwich wall are BIGBOSOR4 models [4,5,6]. Therefore, the discretization is one-dimensional (strip method), which causes solution times on the computer to be much less than for the usual two-dimensionally discretized finite-element models generated in general-purpose finite-element computer programs for the analysis of structures. This property of one-dimensional discretization leads to efficient optimization. Even so, computer run times for “global” optimization via the GENOPT processor called “SUPEROPT” [12] are long, especially for configurations with many modules, such as that displayed in Fig. 24, for example. The one-dimensional discretization is in the plane of the cross section of the wall of the balloon, as displayed in Figs. 3 and 4, for examples. The variation in the direction normal to the plane of the paper (in the circumferential direction in the case of a spherical balloon and along the axis of the prismatic shell in the case of a cylindrical balloon) is trigonometric with n or N circumferential waves [4,5].

Since [1] was written there have been many changes in the definition of the problem and in the software, behavior.balloon (Table 5) and bosdec.balloon (Table 7), on which the analysis and optimization are based. Now the same generic “**balloon**” software treats both cylindrical and spherical balloons and wall configurations with either radial webs (Fig. 1) or truss-like (slanted) webs (Fig. 2). A new behavioral constraint, the load factor at which the pre-buckling tension in one or more of the segments of the model first goes to zero, has been introduced. This new “loss of tension” (wrinkling) constraint is present both for the cylindrical and for the spherical balloons. Hence, the results presented in [1] are now out of date. Since the “initial loss of pre-buckling tension” constraint has a value that is close to the bifurcation buckling constraint, the results presented in [1] are still approximately correct. The “initial loss of pre-buckling tension” constraint is usually somewhat more critical than the bifurcation buckling constraint. Hence, the latest “**balloon**” software would probably generate slightly heavier optimum designs of the cylindrical vacuum chambers than those reported in [1].

1.2 Summary

In Section 2 a brief summary of GENOPT is given. Section 3 describes how GENOPT is used to create a program system for the optimization of structures that belong to the generic class called “**balloon**” in this paper. Material properties, geometry, and decision variable candidates are introduced in Section 4. Section 5 identifies the tasks to be performed by a person called here “**the end user**”. Details of results for a particular optimized spherical balloon are explained in Section 6. Section 7 enumerates and discusses how the models created here differ from those described in [1]. Results for optimized spherical balloons made of polyethylene terephthalate are presented in Section 8. The long Section 9 and its several sub-sections give results for optimized spherical balloons made of a fictitious carbon fiber cloth, discuss real and spurious bifurcation buckling modes and real and spurious axisymmetric pre-buckling behavior, demonstrate the presence of meridional stress concentrations at junctions between segments of the complex balloon wall, discuss the convergence of predictions with respect to the number of nodal points used in one or more of these segments, identify the difficulty of finding a “global” optimum design caused by the sometimes frequent failure of Newton convergence in the attempt to find solutions of the nonlinear pre-buckling equilibrium equations during optimization cycles, and describe the sensitivity of optimized designs to changes in certain of the decision variables. Section 10, analogous in some ways to Section 9, presents results for optimized cylindrical balloons made of fictitious carbon fiber cloth, a material that is not introduced in [1].

SECTION 2 ABOUT GENOPT

Section 2.1 Introduction

GENOPT [3] is a system by means of which one can convert any analysis into a user-friendly analysis and into an optimization capability. GENOPT is not limited to the field of structural mechanics. In the GENOPT "universe" there are considered to be two types of user: 1. **“the GENOPT user"**, and 2. **“the end user"**. The

GENOPT user creates the user-friendly analysis and optimization capability for a class of problems with a generic name ("**balloon**" in this paper and in [1]), and the **end user** uses that capability to find optimum designs for a member of that class with a specific name ("**try4**" and "**try7**" in this paper and "**try4**"in [1]). For the work reported here and in [1] **the GENOPT user** and **the end user** are the same person: the author.

Section 2.2 The GENOPT user

It is the duty of **the GENOPT user** to create user-friendly names, one-line definitions, and "help" paragraphs for the variables to be used in the analysis or analyses. These are listed in Tables 1 and 2 for the generic class, "**balloon**". (NOTE: Tables 1 and 2 in this paper are somewhat different from the analogous Tables 1 and 2 in [1].) **The GENOPT user** must also supply software (subroutines and/or FORTRAN statements) that perform the analysis or analyses that generate the behavioral constraints, such as buckling, stress, vibration, etc. The tasks performed by **the GENOPT user** are listed in Part 1 of Table 4 of [1]. Tables 5 – 7 in the present paper list FORTRAN coding created partly by GENOPT (listed in regular type face) and partly by **the GENOPT user** (listed in bold type face) for the generic class, "**balloon**". **The GENOPT user** must decide what behaviors will constrain the design during optimization cycles, behaviors such as general buckling, local buckling, stress, vibration, etc. While identifying each variable to be used in the generic case, **the GENOPT user** must decide which of seven roles each of these variables plays. The seven possible roles are:

1. decision variable candidate (such as a structural dimension)
2. parameter that is not a decision variable candidate (such as a material property or an index)
3. environmental variable (such as a load)
4. behavioral variable (such as a stress)
5. allowable variable (such as a maximum allowable effective stress)
6. factor of safety (such as a factor of safety for stress)
7. objective (such as weight)

Table 1 lists the variable names, one-line definitions, and roles established by **the GENOPT user** for the generic case, "**balloon**". This table is created automatically by the GENOPT processor called "GENTEXT". Table 2 lists the file called "balloon.PRO" that is created automatically by GENTEXT from the text provided by **the GENOPT user** during his long interactive GENTEXT execution. **The GENOPT user's** input provided during this long interactive GENTEXT session are preserved in the file called "balloon.INP", which is listed in Table 3. The FORTRAN statements provided by **the GENOPT user** for the generic case called "**balloon**" are listed in bold face in Tables 5 – 7. The FORTRAN coding in regular face is generated automatically by the GENTEXT processor of GENOPT.

Note that Tables 1 – 7 in this paper are different from those in [1], although they represent the same functions, respectively, as those in [1]. These tables are different from the corresponding tables in [1] because the generic "**balloon**" software has been modified since [1] was written, and there are new input indices, ISHAPE and IWEBS defined in Table 1 as follows:

```
n   (   0,   0)    2     30   ISHAPE =balloon shape index                       (NEW)
n   (   0,   0)    2     40   IWEBS  =radial (1) or truss-like (2) webs         (NEW)
```

and a new "behavior", allowable, and factor of safety defined in Table 1 as follows:

```
  y   (  20,   0)    4    200   TENLOS =load factor for tension loss            (NEW)
  y   (  20,   0)    5    205   TENLOSA=tension loss allowable (Use 1.0)        (NEW)
  y   (  20,   0)    6    210   TENLOSF=tension loss factor of safety           (NEW)
```

Also, the original name of the buckling load factor and its associated allowable and factor of safety, called GENBUK, GENBUKA, and GENBUCKF in Table 1 of [1], have been changed as follows:

```
  y   (  20,   0)    4    180   BUCKB4 =buckling load factor from BIGBOSOR4   (NEW NAME)
  y   (  20,   0)    5    185   BUCKB4A=buckling from BIGBOSOR4 allowable     (NEW NAME)
  y   (  20,   0)    6    190   BUCKB4F=buckling from BIGBOSOR4 factor of safety(NEWNAME)
```

The "**balloon**" software listed as Tables 5 – 7 in [1] will no longer work for cylindrical vacuum chambers. The **GENOPT user** must use the updated version of this "**balloon**" software, as listed in Tables 5 – 7 in the present paper and as maintained as the following files on the author's computer:

/home/progs/genopt/case/balloon/behavior.balloon
/home/progs/genopt/case/balloon/struct.balloon
/home/progs/genopt/case/balloon/bosdec.balloon

Section 2.3 The end user

It is the duty of **the end user** to provide, for a specific case, a starting design, loads, material properties allowables, and factors of safety, to choose decision variables, lower and upper bounds, equality constraints, and inequality constraints, and to choose strategy indices and whether to optimize or simply to analyze an existing design or both. Examples of these input data provided by **the end user** are listed in the many tables that are included in the run stream listed in Table 4.

Section 2.4 The GENOPT user and the end user

Please read [3] first, followed by the first part of [9], which contains many details about how to use GENOPT. Also read [1, 7, 8], which are analogous to this paper. If you have access to the GENOPT system and you want to use it, please read the file, /home/progs/genopt/doc/getting.started [13].

SECTION 3 PRODUCTION OF THE PROGRAM SYSTEM TO OPTIMIZE THE COMPLEX WALL OF SPHERICAL OR CYLINDRICAL BALLOONS

The generic case is called "**balloon**" (applicable to both cylindrical and spherical vacuum chambers).

Table 4 in this paper lists a run stream for obtaining and analyzing optimum designs for configurations in which GENOPT [3] is used in connection with BIGBOSOR4 [6]. Table 4 of [1] is divided into two parts. Part 1 pertains to tasks performed by **the GENOPT user**, and Part 2 pertains to tasks performed by **the end user**. The Table 4 in the present paper includes only the part of the run stream to be conducted by **the end user**. The reader is referred to [1] to see the part of the run stream to be conducted by **the GENOPT user**.

Tables 1 – 7 in the present paper are analogous to (but not the same as) Tables 1 – 7 in [1]. The generic **balloon** input file, balloon.INP (Tables 1 – 3) and the generic balloon software, behavior.balloon, struct.balloon, and bosdec.balloon (Tables 5, 6, and 7, respectively) have been modified so that both the cylindrical and spherical balloons are now modeled with use of the same software. Also, balloons with radial webs (Fig. 1) and balloons with truss-like (slanted) webs (Fig. 2) are now modeled with use of the same software, bosdec.balloon. There no longer exists any FORTRAN library called “bosdec.balloon2” as identified in [1]. In order that both cylindrical and spherical balloons and balloons with either radial or truss-like webs be handled with the same software, new input indices had to be introduced by **the GENOPT user** during the GENTEXT interactive session (Tables 1, 2, 3). These new input indices, ISHAPE and IWEBS, are identified above and in Table 1. In order to introduce the new behavior, TENLOS (initial loss of pre-buckling tension), and its associated allowable, TENLOSA, and its associated factor of safety, TENLOSF, there now exists a new arrangement of the “behavior” library, behavior.balloon, as listed in Table 5 of the present paper.

In [1] there exist four “behaviors”, as follows:

1. buckling (GENBUK in Table 1 of [1] could be either general or local buckling of the wall of the vacuum shell)
2. a maximum of five stress components in material number 1 (STRM1 in Table 1 of [1])
3. a maximum of five stress components in material number 2 (STRM2 in Table 1 of [1])
4. a maximum of five stress components in material number 3 (STRM3 in Table 1 of [1])

Now there exist five “behaviors”, as follows:

1. buckling (BUCKB4 in Table 1 could be either general or local buckling of the wall of the vacuum shell)
2. initial loss of tension (the new variables, TENLOS, TENLOSA, TENLOSF, in Table 1)
3. a maximum of five stress components in material number 1 (STRM1 in Table 1)
4. a maximum of five stress components in material number 2 (STRM2 in Table 1)
5. a maximum of five stress components in material number 3 (STRM3 in Table 1)

The locations of materials 1 – 3 are indicated in Fig. 4, which is taken from [1] and therefore applies to a cylindrical balloon. The locations of materials 1 – 3 in a spherical balloon are analogous to those displayed in Fig. 4. The only difference between the representations of the models of the spherical and cylindrical balloons is that for spherical balloons the origin of the abscissa is zero (Figs. 1 and 2 of this paper) whereas for cylindrical balloons the origin of the abscissa is greater than zero (Figs. 1 and 2 in [1]).

Notice that the original name for bifurcation buckling, GENBUK in [1], has been changed to BUCKB4, which means “buckling as predicted by BIGBOSOR4”. This name change was made in order to avoid the impression that the bifurcation buckling behavior signifies only general buckling.

Corresponding to the five behaviors, GENTEXT automatically creates five **skeletal** "behavioral" subroutines, SUBROUTINE BEHX1 (local or general buckling, BUCKB4), SUBROUTINE BEHX2 (initial loss of pre-buckling tension, TENLOS, in one or more of the “shell” segments, which is an indicator of the initial formation of wrinkles), SUBROUTINE BEHX3 (five components of stress, STRM1, in material no. 1), SUBROUTINE BEHX4 (five components of stress, STRM2, in material no. 2), and SUBROUTINE BEHX5 (five components of stress, STRM3, in material no. 3). The GENOPT user has to "flesh out" each of these five "behavioral" subroutines, as listed in Table 5. The GENOPT user also has to "flesh out" the subroutine that computes the objective, SUBROUTINE OBJECT in Table 5. The FORTRAN coding created by the GENOPT user is listed in bold face in Tables 5 and 6. The FORTRAN coding listed in regular face is created automatically by the GENTEXT processor of GENOPT.

In the "fleshed out" versions of SUBROUTINE BEHX1 and SUBROUTINE BEHX3 there are calls to SUBROUTINE BOSDEC. SUBROUTINE BOSDEC must entirely be written by the GENOPT user. SUBROUTINE BOSDEC ("BOSorDECk") creates a valid input file for BIGBOSOR4. A general guideline on how to go about creating SUBROUTINE BOSDEC is provided in the file associated with the GENOPT sample case called "cylinder":

/home/progs/genopt/case/cylinder/howto.bosdec.

The most recent version of SUBROUTINE BOSDEC is listed in Table 7. Updated versions of the FORTRAN coding created by GENOPT and "fleshed out" by **the GENOPT user** are maintained in the following files on the author's computer:

/home/progs/genopt/case/balloon/behavior.balloon (SUBROUTINEs BEHX1, BEHX2, etc. and OBJECT)
/home/progs/genopt/case/balloon/struct.balloon (SUBROUTINE STRUCT)
/home/progs/genopt/case/balloon/bosdec.balloon (SUBROUTINE BOSDEC)

Although the stated purpose of SUBROUTINE BEHX1 is only to compute the bifurcation buckling load factor (eigenvalue) called "BUCKB4", in this "**balloon**" application the GENOPT user (the author) decided also to compute four additional behavioral constraints: the load factor, TENLOS, corresponding to the initial loss of tension and the maximum stresses in materials 1, 2 and 3, STRM1, STRM2, STRM3, then to transfer these four additional behavioral constraints to their respective appropriate subroutines, SUBROUTINE BEHX2 (TENLOS), SUBROUTINE BEHX3 (STRM1), SUBROOUTINE BEHX4 (STRM2), SUBROUTINE BEHX5 (STRM3) by means of labeled common blocks. Quantities required for computation of the objective, WEIGHT, are also generated in SUBROUTINE BEHX1 and transferred to SUBROUTINE OBJECT by a labeled common block.

Table 8 lists the most important elements associated with the updating of the generic balloon software, behavior.balloon, struct.balloon, and bosdec.balloon, since [1] was written. **Note that the versions of behavior.balloon, struct.balloon, and bosdec.balloon listed as Tables 5, 6, and 7, respectively, of [1] are no longer valid for the analysis and optimization of cylindrical balloons.**

Table 8 of this paper also lists the differences in behavior of the spherical versus that of the cylindrical balloon. Much of the information contained in Table 8 is repeated below in Section 7 for the convenience of the reader.

SECTION 4 MATERIAL, GEOMETRY, DECISION VARIABLES

Section 4.1 Material properties

The balloon material in [1] and one of the two materials in the present paper is assumed to be polyethylene terephthalate. This material is assumed to be isotropic with a Young's modulus of 435100 psi, Poisson ratio of 0.3, and density of 0.1 lb/inch^3. (The density, 0.1 lb/inch^3, is probably too high. However a change of the density would have no effect on the optimum design. Only the objective, weight/axial length for the cylindrical balloon and total weight for the spherical balloon, would be affected, and those weights are directly proportional to the density.) Note that the material, polyethylene terephthalate, although in this particular case isotropic, is handled in the same manner as if it were a ply made of elastic composite material with five different maximum allowable stress components [8]. Here the maximum allowable stress in polyethylene terephthalate is assumed to be 10000 psi for all 5 stress components: Component 1 = tension along the fibers, Component 2 = compression along the fibers, Component 3 = tension normal to the fibers, Component 4 = compression normal to the fibers, and Component 5 = in-plane shear stress.

A second material, a fictitious carbon fiber cloth, is introduced in Section 9 (Table 13). This fictitious material has much higher stress allowables than polyethylene terephthalate, with the result that optimum designs made of carbon fiber cloth weigh much less than those made of polyethylene terephthalate. See Sections 9 and 10 for details.

Section 4.2 Geometry

In [8] models for local buckling and for general buckling consist of a number of repeating modules chained together. Each module consists of several shell segments. Each shell segment is discretized in the plane of the cross section of the truss-core sandwich wall. The "**balloon**" models explored in this paper, such as those displayed in Figs. 1 – 4, are analogous to the truss-core sandwich models in [8]. A number of modules (called NMODUL in Table 1) is chained together. Here always 90 degrees of the circumference of the cylindrical "shell" or 90 degrees of the meridian of the spherical "shell" are included in the model, as displayed in Figs. 1 – 4. The word, "shell", is enclosed by quotation marks because each "shell" segment in the complex wall of the vacuum chamber behaves more like a membrane than a shell because it has a very small bending stiffness compared to its membrane stiffness. Figures 3 and 4 are taken from [1], where they are designated Fig. 4 and Fig. 5, respectively. Except for the numbers printed along the abscissa, which are incorrect for a spherical balloon, these two figures apply to the spherical balloon as well as to the cylindrical balloon.

Figure 3 shows a 90-degree sector of a vacuum chamber with **radial webs** and with only three modules. The conventions for shell segment numbering and direction of "travel" along each shell segment are identified. Figure 4 is analogous to Fig. 3 and shows the 90-degree sector of a vacuum chamber with **truss-like (slanted) webs**. In addition to the segment-numbering and direction-of-"travel" conventions, Fig. 4 also shows which parts of the structure are fabricated from which material. It is assumed in the generic "**balloon**" case that the vacuum chamber is fabricated from three different materials. The outermost and innermost membrane segments with curvature are made of Material No. 1; the outer and inner straight membrane segments are made of Material No. 2; the webs are made of Material No. 3. **In the studies reported in [1] and in this paper all three materials have the same properties. The entire balloon is effectively made of one material even though three different materials are specified.**

As is indicated in Figs. 1 and 2, symmetry boundary conditions are applied at circumferential coordinates 0 and 90 degrees of the cylindrical balloon or at meridional coordinates 0 and 90 degrees of the spherical balloon.

Section 4.3 Decision variable candidates

Figure 1 identifies the eight decision variable candidates listed in Table 1:

1. HEIGHT is the radial distance from the inner to outer walls of the vacuum chamber measured as shown in Fig. 1.
2. RINNER is the radius of curvature of each segment of the curved innermost wall.
3. ROUTER is the radius of curvature of each segment of the curved outermost wall.
4. TINNER is the thickness of each segment of the curved innermost wall.
5. TOUTER is the thickness of each segment of the curved outermost wall.
6. TFINNR is the thickness of each segment of the inner wall that consists entirely of straight segments.
7. TFOUTR is the thickness of each segment of the outer wall that consists entirely of straight segments.
8. TFWEBS is the thickness of each web.

The end user selects in the DECIDE processor (Table 4) which of the eight decision variable candidates are to be actual decision variables for optimization.

SECTION 5 THE END USER PERFORMS HIS INITIAL TASKS

As in [1], the end user again chooses "**try4**" as the specific case name for the balloons made of polyethylene terephthalate. "**try4**" is an end-user-chosen specific member of the generic class called "**balloon**". In [1] "**try4**" represents a cylindrical balloon. In the present paper "**try4**" represents a spherical balloon. For balloons made of the fictitious carbon fiber cloth the end user chooses "**try7**" for the specific case name for both the spherical and cylindrical balloons.

Table 4 lists the many activities of the **end user** that relate to the specific case, **try4.** The reader is referred to [1] for a description of these activities. The description in [1] of the value used for POUTER (5.0 psi rather than 15.0 psi) and of the value used for the factor of safety for buckling (3.0 rather than 1.0) applies in this paper also. Since the newly introduced behavior, TENLOS (initial loss of pre-buckling tension), actually represents a type of buckling (wrinkling), the same factor of safety used for buckling, BUCKB4F, should also be assigned to the factor of safety, TENLOSF, associated with the initial loss of pre-buckling tension, TENLOS.

In this paper the input data for BEGIN, DECIDE, MAINSETUP, etc. are listed within Table 4. Therefore, in this paper there are no separate tables listing input data for the GENOPT processors called BEGIN, DECIDE, MAINSETUP, etc. that are analogous to Tables 8, 9, 10, etc. in [1].

In Table 4 are listed many re-starts of SUPEROPT. At the time Table 4 was created SUPEROPT often bombed because of failure of convergence of the Newton iterations for the solution of the nonlinear pre-buckling equations. Since then, a strategy has been established by means of which SUPEROPT continues on despite occasional failures of convergence during the approximately 470 design cycles specified for a complete execution of SUPEROPT. The effect of this new strategy is demonstrated near the beginning of Table 4 and discussed in Item 7d of Table 8 and in Sections 7 and 9 and shown in Figs. 51b and 52b.

SECTION 6 DISCUSSION OF THE RESULTS LISTED IN THE try4.OPM FILE (Table 9)

This section, analogous to Section 8 of [1], contains brief descriptions of some of the results listed in the try4.OPM file (Table 9) for the design obtained after optimization of a spherical balloon made of polyethylene terephthalate. This output file pertains to the optimized design of a spherical vacuum chamber with 8 modules and with truss-like (slanted) webs (Figs. 5, 6, and 10a-k). The items discussed here are identified with the strings, “**Item 1**”, “**Item 2”**, “**Item 3”**, …, “**Item19**”, in Table 9. These items are listed in bold face in Table 9.

NOTE: After Table 9 was generated, and after the text in this section was written, certain modifications were made to the “balloon” software, behavior.balloon (Table 5). These modifications would probably now lead to a somewhat different optimum design for the spherical balloon with 8 modules and truss-like webs than that listed in Table 9. The purpose of this section is to describe the meaning of the output data listed in the *.OPM file. The actual values of the data that would be changed by the modifications are not crucial.

Item 1: The try4.OPM file listed in Table 9 is the result of the analysis of a fixed design (ITYPE = 2). The fixed design in this example is the optimum design obtained after completion of the entire optimization process carried out with use of the new strategy mentioned in the last paragraph of the previous section.

Item 2: The optimized values of the decision variable candidates, HEIGHT, RINNER, ROUTER, TINNER, TOUTER, TFINNR, TFOUTR, and TFWEBS are listed. In this particular case all of the decision variable candidates (Role 1 variables in Table 1) are chosen by the end user to be decision variables.

Item 3: As mentioned in the previous section and as discussed in [1], the pressure applied to the outermost wall, POUTER = 5.0 psi, is associated with the factors of safety, BUCKB4F = 3.0 and TENLOSF = 3.0. We actually wish to obtain the optimum design of a balloon with outer pressure, POUTER = 15.0 psi. However, it may happen that there occurs an unrecoverable error caused by failure of convergence of the Newton iterations for the solution of the nonlinear pre-buckling equations that govern equilibrium of the balloon under the total load, PINNER, PMIDDL, and POUTER, since POUTER is applied in a single load step and the solution with POUTER = 0.0 psi is used as a starting solution for the Newton iteration process. The system with POUTER = 5.0 psi and factors of safety, BUCKB4F = TENLOSF = 3.0 is almost equivalent to the system with POUTER = 15.0 psi and factors of safety, BUCKB4F = TENLOSF = 1.0 psi with respect to the following two margins listed under Item 15 of Table 9:

```
MARGIN CURRENT
 NO.      VALUE            DEFINITION
  1     4.368E-02  (BUCKB4(1 )/BUCKB4A(1 )) / BUCKB4F(1 )-1; F.S.=  3.00
  2    -8.217E-04  (TENLOS(1 )/TENLOSA(1 )) / TENLOSF(1 )-1; F.S.=  3.00
```

Item 4: The file, try4.LOADB, is a valid input file for BIGBOSOR4 for an execution of BIGBOSOR4 with INDIC = 0 (nonlinear axisymmetric analysis) with the application of PINNER, PMIDDL, and DELTAT (if any DELTAT) only, that is, with application of only the loads in Load Set B. (Load B = “fixed”, non-eigenvalue loads.) The outer pressure, POUTER, is equal to zero. The file, try4.LOADB, can be used for an execution of BIGBOSOR4 outside the GENOPT context in order to obtain plots of the pre-buckled states of the balloon under PINNER, PMIDDL, and DELTAT (if any DELTAT). Figures 5 and 6 are obtained via an execution of BIGBOSOR4 outside the GENOPT context with use of the file, try4.LOADB = try4.ALL, as input. Figure 5 displays the pre-buckled state of the optimized spherical balloon with 8 modules and truss-like webs under PINNER = 0 psi, PMIDDL = 0.002 x 60 psi, DELTAT = 0.0 degrees, and Figure 6 displays the pre-buckled state of that balloon under PINNER = 0 psi, PMIDDL = 1.002 x 60 psi, DELTAT = 0.0 degrees. It is obvious from Figs. 5 and 6 that the axisymmetric behavior is highly nonlinear. This highly nonlinear behavior is discussed in Item 6 below.

Item 5: The weight of the 90-degree portion of the spherical balloon that is included in the BIGBOSOR4 model is computed by BIGBOSOR4 as the variable TOTMAS. TOTMAS is generally the mass of the BIGBOSOR4 model of a shell structure. In this application TOTMAS is the weight rather than the mass because the density, DENSTY (Table 1), provided by the end user is the weight density rather than the mass density of the material. The total weight of the balloon is twice TOTMAS because in this application only 90 degrees of the meridian of the balloon is included in the BIGBOSOR4 model, with symmetry conditions applied at the equator.

Item 6: The nonlinear pre-buckling solution for the equilibrium of the balloon under the "fixed" (non-eigenvalue) loads, PINNER = 0 psi, PMIDDL = 60 psi, and DELTAT = 0 degrees (DELTAT = 0 only for the spherical balloon) is obtained by BIGBOSOR4 in 11 load steps with increasing multipliers, P (pressure multiplier) and TEMP (temperature multiplier). In the first load step the pressure multiplier, P = 0.002. In the second load step the pressure multiplier, P = 0.102. In each successive load step DP = 0.1 is added to the pressure multiplier, P. Therefore, at the 11th and last load step the pressure multiplier, P = 1.002. The same holds for the temperature multiplier, TEMP, if any. (With spherical balloons there never is any thermal loading: DELTAT = 0.0.) At any load step the load on the shell is equal to P x PINNER, P x PMIDDL, and TEMP x DELTAT. The outer pressure, POUTER = 0.0 psi during this entire nonlinear pre-buckling analysis. Listed in the try4.OPM file (Table 9) are the load step number, the number of Newton iterations required for convergence of the nonlinear equilibrium equations at each load step, and the value of the maximum displacement in the entire BIGBOSOR4 model at each load step. Typically (but not always) the second load step requires the most Newton iterations for convergence. Figure 7 shows a plot of the maximum displacement at the second load step in an optimized spherical balloon with 15 modules and radial webs (Fig. 1) versus Newton iteration number. Figures 8 and 9 are plots of the normal displacement at the apex of the optimized spherical balloon with 15 modules and radial webs (Fig. 1) as a function of the pressure multiplier, P. It is seen that in the range of small pressure multiplier, $0 < P < 0.05$, the behavior of the "shell" is extremely nonlinear (Fig. 8), and for larger values of P the behavior is nearly linear (Fig. 9). This behavior is typical of all the spherical balloons regardless of the number of modules. The pattern of Newton iterations required for each load step reflects this "initially-very-nonlinear-followed-by-almost-linear" behavior. At pressure multiplier, P, greater than about 0.1 the developing tension in the various segments of the spherical balloon stabilize these segments and render the subsequent behavior almost linear with increasing pressure multiplier, P. Since this subsequent behavior is almost linear, few Newton iterations are required for convergence at load steps 4 - 11.

Item 7: The file, try4.BEHX1, is a valid input file for BIGBOSOR4. It can be used for the execution of BIGBOSOR4 in a context independent of GENOPT. The main purpose of execution of BIGBOSOR4 independently of GENOPT is to obtain plots of the type shown, for a cylindrical balloon, in Fig. 6a and Figs. 11 – 14 of [1], for examples. Unfortunately, in the case of the spherical balloon the pre-buckling behavior appears to be far more nonlinear, as described in Item 6, than the pre-buckling behavior of a cylindrical balloon. Therefore, while the file, try4.BEHX1, for a spherical balloon still contains valid input for BIGBOSOR4, an execution of BIGBOSOR4 with that file as input bombs in the case of a spherical balloon because of failure of convergence of the Newton iterations for solution of the nonlinear pre-buckling equations for the state of the spherical balloon under only the "fixed" (non-eigenvalue) loads, PINNER = 0 psi, PMIDDL = 60 psi, and DELTAT = 0 degrees (Load Set B). As explained in the try4.OPM file (under Item 7 in Table 9), in the version of BIGBOSOR4 that is executed outside the GENOPT context, Load Set B cannot be divided into the 11 (or more) sub-steps as described in Item 6. Therefore, BIGBOSOR4 attempts to determine the nonlinear solution for Load Set B in a single load step. In the case of the cylindrical balloons this simple strategy usually works. However, for the spherical balloons the simple strategy usually fails because the pre-buckling behavior of the spherical balloon is more "cranky" (nonlinear) than that of the cylindrical balloon. Item 4 in Table 8 describes how to obtain buckling modes from BIGBOSOR4 for spherical balloons.

Item 8: The initial loss of meridional tension and the initial loss of circumferential tension in each of the six segments of various modules of the spherical balloon with truss-like (slanted) webs are listed. (Figure 4 shows the segment numbering convention within each module of a balloon with truss-like webs.) The initial loss of **meridional** tension occurs in Segment 3 (the inner membrane with thickness, TFINNR) of Module No. 3, that is, in Segment No. 15 = (2 modules) x (6 segments per module) + (3 segments) of the model of the spherical balloon with 8 modules over 90 degrees of meridian, at a load factor, 3.3225. This initial loss of **meridional** tension would cause local buckling (wrinkling) that is axisymmetric (zero circumferential waves in the buckling mode). The initial loss of **circumferential** tension occurs in Segment 3 of Module No. 8, that is, Segment No. 45 = 7 x 6 + 3 of the model of the spherical balloon with 8 modules over 90 degrees of meridian, at a load factor, 2.9975. This initial loss of **circumferential** tension would cause local buckling (wrinkling) that is non-axisymmetric (many circumferential waves). The locations of the initial loss of **meridional** and **circumferential** tension are identified in Item 10, to be discussed below. The computation of the initial loss of **meridional** and **circumferential** tension is described in Item 2 of Table 8, and the relationship of the initial loss of tension to buckling as predicted from the BIGBOSOR4 stability (eigenvalue) analysis is discussed in Item 8 of Table 8.

Item 9: Bifurcation buckling eigenvalues are obtained by BIGBOSOR4 with the BIGBOSOR4 analysis type index, INDIC = 1. The eigenvalue problem is described at the beginning of Table 10. If the analysis type, ITYPE = 2, in the try4.OPT file (input for the GENOPT processor called MAINSETUP), GENOPT computes buckling load factors from BIGBOSOR4 over a range of circumferential wave numbers if the balloon is spherical. Listed in the try4.OPM file (Table 9) are the bifurcation buckling load factors and numbers of full circumferential waves in the format: 3.1317E+00(64) = eigenvalue(number of circumferential waves). The end user can obtain plots of selected buckling modes as described in Table 10 and in Item 4 of Table 8. Figures 10a – 10k show the buckling modes from BIGBOSOR4 corresponding to the numbers of circumferential waves, N = 0, 8, 16, 24, 32, 40, 48, 56, 64, 72, and 80. The eigenvalues corresponding to most of the values of N, especially for 32 < N < 88, are very close to one another.

NOTE: In optimization runs ONLY the bifurcation buckling load factor (eigenvalue) corresponding to axisymmetric buckling (N = 0 circumferential waves) is computed by BIGBOSOR4 for the establishment of the buckling constraint, BUCKB4. This is because, for the much stronger fictitious carbon fiber cloth, the bifurcation buckling load factors corresponding to non-axisymmetric buckling modes of optimized balloons are usually very small and correspond to spurious buckling modes. See Table 14 and the discussion in Section 9.2. Non-axisymmetric buckling is "covered" by the initial-loss-of-tension constraint, TENLOS. Therefore, optimized designs will not be unconservative because of the neglect during optimization cycles of non-axisymmetric bifurcation buckling as predicted by BIGBOSOR4.

Item 10: Here are listed the locations (Segment number, j, Nodal point number, i) corresponding to the initial loss of **meridional** tension and **circumferential** tension in the six segments in various modules of the spherical balloon with 8 modules and with truss-like webs. The load factors, EIGEN1(i,j) and EIGEN2(i,j), and the corresponding values of **meridional** and **circumferential** stress resultants, N1FIX, N1VAR, N2FIX, N2VAR, are also listed. In this list "Load Step 1" means application of only the "fixed" (non-eigenvalue) loads, PINNER, PMIDDL, and DELTAT. "Load Step 2" means application of the total loads, PINNER, PMIDDL, DELTAT, and POUTER. The “eigenvalue”, EIGEN1(i,j) corresponding to the initial loss of **meridional** tension is computed from the equation,

meridional tension = 0 = N1FIX(i,j) + EIGEN1(i,j) x [N1VAR(i,j) – N1FIX(i,j)]

The “eigenvalue”, EIGEN2(i,j) corresponding to the initial loss of **circumferentia**l tension is computed from the equation,

circumferential tension = 0 = N2FIX(i,j) + EIGEN2(i,j) x [N2VAR(i,j) – N2FIX(i,j)]

These “eigenvalues”, EIGEN1(i,j) and EIGEN2(i,j), are closely related to the local buckling eigenvalues computed by BIGBOSOR4 as described in Item 9 above (for real buckling modes, not spurious buckling modes). The variable, TENLOS, is the minimum “eigenvalue” obtained from the above two equations. In this case the load factors from the BIGBOSOR4 stability analysis and from the “initial loss of tension” analysis are as follows:

Design Constraint	Load factor	Definition of the load factor
1	3.131052	buckling load factor from BIGBOSOR4: BUCKB4(1)
2	2.997535	load factor for tension loss: TENLOS(1)

The load factor corresponding to initial loss of tension, TENLOS, is usually somewhat less than the bifurcation buckling load factor from BIGBOSOR4 because the BIGBOSOR4 stability analysis includes the finite bending stiffness of the “shell” segment. The two design constraints just listed correspond to the first two margins listed under Item 15, which is described below. Buckling margins are related to the buckling load factor by the equation: Buckling Margin = (Buckling load factor)/(Factor of safety) – 1.0.

Item 11: The maximum stresses are computed from membrane theory. The maximum stress in each segment of various modules is equal to the maximum stress resultant divided by the wall thickness of that segment. Listed in the try4.OPM file (Table 9) are the location of the maximum stress (Segment, j, Nodal point, i) and the meridional and circumferential stresses from the “fixed” (non-eigenvalue) loads, PINNER, PMIDDL, and DELTAT (Load Set B = Load Step 1) and from the total loads, PINNER, PMIDDL, DELTAT, and POUTER (Load Step 2).

NOTE: The stresses listed under Item 14 and the stress margins listed under Item 15 are computed from the maximum membrane stresses corresponding to the TOTAL loads even though these stresses are smaller than those corresponding to the application only of the “fixed” loads. This implies that with the actual structure POUTER = 5.0 psi would be applied before the cavity between the inner and outer walls of the balloon is inflated.

ANOTHER NOTE: If POUTER = 15.0 psi were applied before the cavity between the inner and outer walls of the balloon were inflated, the maximum stresses experienced by the balloon would be lower than those listed in Item 11. Therefore, the use of POUTER = 5.0 psi is conservative with respect to the stress constraints.

Item 12: The results listed in Table 9 under this item are computed in SUBROUTINE BEHX1 from a BIGBOSOR4 model corresponding to analysis type, INDIC = 1. (See Table 10.) With INDIC = 1 the loads in Load Step B ('fixed", non-eigenvalue loads, PINNER, PMIDDL, and DELTAT) are applied in a single load step, not applied in a series of 11 load steps as described in Item 6 above. Newton iterations converge after only one iteration (ITER = 1, FMAX = 5.5215E+00) because the solution vector from the pre-buckling equilibrium analysis described in Item 6 has been saved and is now used as the starting solution for the balloon loaded only by the "fixed" (non-eigenvalue) loads, PINNER, PMIDDL, DELTAT, and the starting solution is the same as the converged solution because the loading is the same. The additional load, POUTER, is then applied in one load step. In this particular case only two Newton iterations are required (ITER = 2, FMAX = 4.2222E+00) because the balloon wall as loaded by the "fixed" (non-eigenvalue) loads, PINNER, PMIDDL, DELTAT, has previously been stabilized by the pressure, PMIDDL = 60 psi. In all the cases run to date that involve the material, polyethylene terephthalate, similar convergence characteristics are observed in this INDIC = 1 phase of the GENOPT/BIGBOSOR4 analysis. Note that the same favorable convergence characteristics do not exist when BIGBOSOR4 is executed outside the GENOPT context. See Item 7 above for a discussion. BIGBOSOR4 acts differently inside the GENOPT context than outside the GENOPT context because of certain modifications to the GENOPT/BIGBOSOR4 software, /home/progs/bosdec/sources/addbosor4.src. These and other special modifications of addbosor4.src are listed in Table 11. The file, addbosor4.src, is the BIGBOSOR4 computer program as modified for use in the GENOPT context.

Item 13: In SUBROUTINE BEHX3, only if the analysis type index, ITYPE = 2, the maximum stresses for each material type are computed by BIGBOSOR4, including bending stresses. There are huge bending stress concentrations at the junctions between shell segments. Therefore, these very large and very localized maximum stresses are much larger than the maximum stresses computed from membrane theory as described in Item 11 above. However, these very large extreme-fiber maximum stresses are not used for the computation of the stresses listed under Item 14 or for the computation of the stress margins listed under Item 15.

The large bending stress concentrations exhibit rather peculiar convergence characteristics with increasing nodal point density in the neighborhoods of shell junctions, probably because the "shell" structure acts like a balloon, that is, a structure in which the wall of each segment has essentially zero bending stiffness. Figures 12 – 17 demonstrate in the case of a spherical balloon with 8 modules over 90 degrees of meridian and with **radial webs**. Figure 12 shows the overall axisymmetric deformation of the spherical balloon under the "fixed" (non-eigenvalue) loads, PINNER = 0 psi, PMIDDL = 60 psi, DELTAT = 0 degrees.

This particular study of the convergence of the maximum stresses with respect to nodal point density involves only "shell" segment no. 31. Figure 13 is a "zoomed" plot of the undeformed and deformed spherical balloon in the neighborhood of "shell" segment no. 31. With careful inspection one can detect a slight "zig-zag" pattern of the displacement from nodal point to nodal point in the neighborhood of the junction of Segment 31 with its neighboring segments. Figure 14 is a plot of the extreme fiber meridional stress along segment no. 31 corresponding to the nodal point distribution displayed in Fig. 13. The plot starts at the beginning of segment 31 where segment 31 is joined to several other segments. Results from only the first part of segment 31 are included in the plot. Note the huge and local bending stress concentration near the beginning of segment 31. Figure 15 is a "zoomed" plot of the undeformed and deformed spherical balloon in the neighborhood of segment no. 31 that has a much more dense distribution of nodal points only in segment 31 than that displayed in Fig. 13. In this model an especially dense nodal point distribution is used near the beginning of segment 31. Figure 16 is analogous to Fig. 14. Note, however, that the amplitude of the bending stress concentration is greatly reduced and that the bending stress concentration is much more localized than that displayed in Fig. 14.

Figure 17 includes the extreme fiber meridional stress data from both Figs. 14 and 16. Figure 17 demonstrates the dramatic reduction of the amplitude of the bending stress concentration and the greatly reduced arc length over which this bending stress concentration decays in the model with the dense nodal point distribution near the beginning of segment 31 (Fig. 15). An analogous study of meridional stress concentration is reported in Section 9 for a spherical balloon made of the fictitious carbon fiber cloth. Figures 41 – 43 referred to in Section 9 are analogous to Figs. 14, 16, and 17 referred to here.

These results lead to two conclusions:

1. The use of membrane stresses to compute stress margins is probably justified for preliminary design.

2. The stress concentrations in the immediate neighborhoods of the junctions between shell segments can probably be reduced to tolerable levels by local reinforcement of the seams between segments of the balloon.

As a result of this study one might ask, "Why not use very high nodal point density near the ends of all of the segments in the model of the balloon?" Doing this would require too much computer time for optimization and the storage requirements for BIGBOSOR4 would be exceeded, especially for models with large numbers of modules over 90 degrees of the meridian of a spherical balloon and with large numbers of modules over 90 degrees of the circumference of a cylindrical balloon.

Figure 11 shows the pre-buckled equilibrium state of a spherical balloon with only four modules subjected to the loads in Load Set B: PINNER = 0 psi, PMIDDL = 60 psi, DELTAT = 0 degrees. Here the "zig-zag" pattern of the displacement from nodal point to nodal point in the neighborhoods of several of the "shell" segment junctions is much more pronounced than the barely noticeable "zig-zag" pattern near the beginning of segment 31 shown in Fig. 13.

Item 14: Listed here are the membrane stresses used to compute the stress margins for material types 1 (STRM1), Material 2 (STRM2), and Material 3 (STRM3). There are five stress components for each material:

Stress component no. 1 = tension along the fibers [STRMi(1,1)]
Stress component no. 2 = compression along the fibers [STRMi(1,2)]
Stress component no. 3 = tension transverse to the fibers [STRMi(1,3)]
Stress component no. 4 = compression transverse to the fibers [STRMi(1,4)]
Stress component no. 5 = in-plane shear stress [STRMi(1,5)]

Stress components with very, very small values (0.1000E-09) indicate that the balloon structure exhibits no stress components of those types. The row index, i, in the two dimensional arrays, STRM1(i,j), STRM2(i,j), STRM3(i,j), is the load set number, ILOADX. In all the cases in this project there is only one load set. This one load set has two parts: the "fixed" (non-eigenvalue) loads, PINNER, PMIDDL, DELTAT, and the total loads, PINNER, PMIDDL, DELTAT, and POUTER.

Item 15: The margins corresponding to the results listed in Items 9, 10, and 14 are given here. Since this is an optimized "shell" there are several critical margins. Note that several of the margins are negative. Negative margins are permitted by GENOPT as follows:
1. A design is accepted by GENOPT as "FEASIBLE" if the most critical margin is greater than minus 0.01.

2. A design is accepted by GENOPT as “ALMOST FEASIBLE” if the most critical margin is greater than minus 0.05.
In this particular case the optimized design is “ALMOST FEASIBLE”.

Item 16: The definition for WEIGHT is given as “**`weight/length of the balloon`**”. However, note that in the case of the spherical balloon the quantity, WEIGHT, is actually the total weight of the spherical balloon, that is, 2.0 x TOTMAS, in which TOTMAS is computed by BIGBOSOR4. See Item 7i in Table 8.

Item 17: In the case of the spherical balloon the input variable, LENGTH, plays no role. Nevertheless, the end user must supply a positive value for LENGTH during the BEGIN interactive session. See Item 7j in Table 8.

Item 18: The user-provided value of the external pressure, POUTER = 5.0 psi, has previously been discussed. We are actually interested in POUTER = 15 psi. However, the use of POUTER = 15 psi might possibly lead to failure of convergence of the Newton iterations for nonlinear pre-buckling equilibrium undertaken in the second part of Item 12: the computation of pre-buckling equilibrium under the total loads, PINNER, PMIDDL, DELTAT, and POUTER, a computation which is attempted with the use of a single load step and in which the equilibrium state obtained with POUTER = 0 psi is used as a starting solution vector. The user-specified value of POUTER is associated with factors of safety for buckling, BUCKB4F, and for the initial loss of meridional or circumferential tension, TENLOSF. Associated with POUTER = 5.0 psi we use factors of safety, BUCKB4F = TENLOSF = 3.0, as shown in the next item.

Item 19: The factors of safety, BUCKB4F and TENLOSF, must be inversely proportional to the user-specified value of POUTER. With POUTER = 5.0 psi we use BUCKB4F = TENLOSF = 3.0. If POUTER were equal to 15.0 psi we would use BUCKB4F = TENLOSF = 1.0.

SECTION 7 SOME RESULTS LISTED IN TABLE 8 AND REPEATED HERE FOR EMPHASIS AND FOR THE CONVENIENCE OF THE READER

Table 8 contains a summary of the improvements in the generic “**balloon**” software, behavior.balloon and bosdec.balloon, since [1] was written and describes some of the differences in behavior of the cylindrical and spherical balloons. The following items included in Table 8 are repeated here for emphasis and for the convenience of the reader:

Section 7.1 From Item 1 of Table 8:

```
*********** IMPORTANT NOTE *************************
The versions of the generic "balloon" software,
behavior.balloon, struct.balloon, and bosdec.balloon,
listed as Tables 5 - 7, respectively, of [1] are
now out of date. The up-to-date versions of
behavior.balloon, struct.balloon, and bosdec.balloon
are listed as Tables 5 - 7 in the more recent paper,
"Use of GENOPT and BIGBOSOR4 to obtain optimum designs
of multi-walled inflatable spherical and cylindrical
vacuum chambers, February, 2011. bosdec.balloon2 no longer
```

exists. Also, the file, balloon.INP, listed in Table 3 of [1] is out of date. The up-to-date version of balloon.INP is listed as **Table 3** in the "spherical balloon" paper just cited.
****

************** EMPHASIZE IMPORTANT NOTE *****************
Tables 5, 6, and 7 of [1] are now out-of-date and no longer applicable to the generic "balloon" case. Use only the versions of the balloon sofware, behavior.balloon, struct.balloon, and bosdec.balloon that are stored in the following files:

/home/progs/genopt/case/balloon/behavior.balloon
/home/progs/genopt/case/balloon/struct.balloon
/home/progs/genopt/case/balloon/bosdec.balloon

The up-to-date file, balloon.INP, is stored in the file:

/home/progs/genopt/case/balloon/balloon.INP

These versions of the generic "balloon" software and the GENTEXT input file, balloon.INP, are always kept up to date.
****

Section 7.2 From Item 2 of Table 8:

ITEM 2. There are new variables introduced by the GENOPT user into the balloon.INP file via the GENOPT utility called "INSERT". Table 1 in [1] should be replaced with the following table:

TABLE 1 GLOSSARY OF VARIABLES USED IN "balloon"

ARRAY ?	NUMBER OF (ROWS,COLS)	ROLE	PROMPT NUMBER (balloon.PRO)	NAME	DEFINITION OF VARIABLE	NEW INPUT SINCE [1] WAS WRITTEN
n	(0, 0)	2	10	LENGTH	= length of the cylindrical shell	
n	(0, 0)	2	15	RADIUS	= inner radius of the cylindrical balloon	
n	(0, 0)	2	20	NMODUL	= number of modules over 90 degrees	
n	(0, 0)	2	30	ISHAPE	= balloon shape index	**(NEW)**
n	(0, 0)	2	40	IWEBS	= radial (1) or truss-like (2) webs	**(NEW)**
n	(0, 0)	2	50	IEMOD1	= material number in EMOD1(IEMOD1)	
y	(10, 0)	2	55	EMOD1	= elastic modulus, meridional direction	
y	(10, 0)	2	60	EMOD2	= elastic modulus, circumferential direction	
y	(10, 0)	2	65	G12	= in-plane shear modulus	
y	(10, 0)	2	70	G13	= out-of-plane (s,z) shear modulus	
y	(10, 0)	2	75	G23	= out-of-plane (y,z) shear modulus	
y	(10, 0)	2	80	NU	= Poisson ratio	
y	(10, 0)	2	85	ALPHA1	= meridional coef. thermal expansion	
y	(10, 0)	2	90	ALPHA2	= circumf.coef.thermal expansion	
y	(10, 0)	2	95	TEMPER	= delta-T from fabrication temperature	

```
  y   (  10,    0)    2    100   DENSTY  = weight density of material
  n   (   0,    0)    1    110   HEIGHT  = height from inner to outer membranes
  n   (   0,    0)    1    115   RINNER  = radius of curvature of inner membrane
  n   (   0,    0)    1    120   ROUTER  = radius of curvature of outer membrane
  n   (   0,    0)    1    125   TINNER  = thickness of the inner curved membrane
  n   (   0,    0)    1    130   TOUTER  = thickness of the outer curved membrane
  n   (   0,    0)    1    135   TFINNR  = thickness of inner truss-core segment
  n   (   0,    0)    1    140   TFOUTR  = thickness of the outer truss segment
  n   (   0,    0)    1    145   TFWEBS  = thickness of each truss-core web
  n   (   0,    0)    2    155   NCASES  = Number of load cases (number of environments)
  y   (  20,    0)    3    160   PINNER  = pressure inside the inner membrane
  y   (  20,    0)    3    165   PMIDDL  = pressure between inner and outer membranes
  y   (  20,    0)    3    170   POUTER  = pressure outside the outer membrane
  y   (  20,    0)    4    180   BUCKB4  = buckling load factor from BIGBOSOR4     (NEW NAME)
  y   (  20,    0)    5    185   BUCKB4A = allowable for buckling load factor      (NEW NAME)
  y   (  20,    0)    6    190   BUCKB4F = buckling from BIGBOSOR4 factor of safety(NEW NAME)
  y   (  20,    0)    4    200   TENLOS  = load factor for tension loss               (NEW)
  y   (  20,    0)    5    205   TENLOSA = tension loss allowable (Use 1.0)           (NEW)
  y   (  20,    0)    6    210   TENLOSF = tension loss factor of safety              (NEW)
  n   (   0,    0)    2    215   JSTRM1  = stress component number in STRM1(NCASES,JSTRM1)
  y   (  20,    5)    4    220   STRM1   = stress component in material 1
  y   (  20,    5)    5    225   STRM1A  = allowable stress in material 1
  y   (  20,    5)    6    230   STRM1F  = factor of safety for stress in material 1
  y   (  20,    5)    4    235   STRM2   = stress component in material 2
  y   (  20,    5)    5    240   STRM2A  = allowable for stress in material 2
  y   (  20,    5)    6    245   STRM2F  = factor of safety for stress in material 2
  y   (  20,    5)    4    250   STRM3   = stress component in material 3
  y   (  20,    5)    5    255   STRM3A  = allowable for stress in material 3
  y   (  20,    5)    6    260   STRM3F  = factor of safety for stress in material 3
  n   (   0,    0)    7    270   WEIGHT  = weight/length of the balloon
================================================================================
```

Note that the name of the buckling behavior has been changed from GENBUK [1], which erroneously implies general buckling, to BUCKB4, which denotes "buckling load factor obtained from BIGBOSOR4". In keeping with this name change are the changes in the names of the allowable and factor of safety from GENBUKA and GENBUKF in [1] to BUCKB4A and BUCKB4F, respectively.

The new input datum, ISHAPE, controls the shape of the balloon, either cylindrical (ISHAPE = 1) or spherical (ISHAPE = 2). The new input datum, IWEBS, controls whether the webs are radial as in Fig. 1 of [1] (IWEBS = 1) or truss-like (slanted) as in Fig. 2 of [1] (IWEBS = 2). (Also see **Figs. 1 and 2** of the present paper on spherical balloons.)

The new input data, TENLOS, TENLOSA, TENLOSF, represent the introduction by the GENOPT user of a new behavioral constraint (TENLOS) and its associated allowable (TENLOSA) and factor of safety (TENLOSF). The segments of a balloon act as if they have no bending stiffness. As soon as they lose tension they wrinkle ("buckle"). TENLOS is the load factor to be multiplied by POUTER which corresponds to the initial loss of tension in one of the "shell" segments of the cross-section of the multi-walled balloon. The "wrinkling load factor", TENLOS, is computed from the equations:

```
TENLOS(loss of MERIDIONAL tension) =
         N1FIX(node,seg.)/[N1FIX(node,seg.) -N1VAR(node,seg.)]

TENLOS(loss of CIRCUMFERENTIAL tension) =
         N2FIX(node,seg.)/[N2FIX(node,seg.) -N2VAR(node,seg.)]
```

in which N1FIX(node,seg.) is the meridional stress resultant from the "fixed" (non-eigenvalue) loads, PINNER, PMIDDL, and DELTAT, at nodal point "node" and segment number "seg.", and N1VAR(node,seg.) is the meridional stress resultant from the total loads, PINNER, PMIDDL, DELTAT, and POUTER. N2FIX and N2VAR (circumferential stress resultants) are analogous respectively to N1FIX and N1VAR. For the cylindrical balloon the behavioral constraint, TENLOS, is the minimum value of N1FIX(node,seg.)/[N1FIX(node,seg.) -N1VAR(node,seg.)] over all the nodes of all the segments in the model, For the spherical balloon TENLOS is the minimum value of both N1FIX(node,seg.)/[N1FIX(node,seg.) -N1VAR(node,seg.)] and N2FIX(node,seg.)/[N2FIX(node,seg.) -N2VAR(node,seg.)]. The circumferential "wrinkling" from N2FIX(node,seg.)/[N2FIX(node,seg.) -N2VAR(node,seg.)] is assumed, in the "initial loss of tension" analysis, not to occur in the case of the cylindrical balloon because it is assumed that under the external pressure, POUTER, the cylindrical balloon is free to expand in the axial direction (normal to the plane of the paper in Fig.1 of [1]). This assumption does not apply in the case of the spherical balloon.

Since wrinkling from the initial loss of tension in one or more of the segments of the complex wall of the balloon is a kind of buckling, it is expected that the design margin corresponding to bifurcation buckling from a BIGBOSOR4 model (BUCKB4) will not be too different from the design margin corresponding to initial loss of tension (TENLOS). For an optimized spherical balloon with 8 modules over 90 degrees we have, for example, the following two "buckling" margins:

```
MARGIN CURRENT
NO.     VALUE            DEFINITION
 1    6.170E-02 (BUCKB4(1)/BUCKB4A(1))/BUCKB4F(1)-1; F.S.= 3.00
 2   -7.231E-03 (TENLOS(1)/TENLOSA(1))/TENLOSF(1)-1; F.S.= 3.00
```

Computer optimizations carried out so far show that the "initial loss of tension (TENLOS)" margin is usually somewhat more critical than the "bifurcation buckling from BIGBOSOR4 (BUCKB4)" margin.

Section 7.3 From Item 3 of Table 8:

```
ITEM 3. The "behavior.balloon" library has been significantly
modified. Because of the new behavioral constraint, TENLOS,
there are now five subroutines, BEHXi, i = 1, 2, 3, 4, 5,
instead of the four listed in Table 5 of [1]. The five
subroutines, BEHXi, compute the following behavioral constraints:

SUBROUTINE BEHX1 = buckling as predicted by BIGBOSOR4 (BUCKB4)
SUBROUTINE BEHX2 = wrinkling due to initial loss of tension (TENLOS)
SUBROUTINE BEHX3 = 5 stress components in material type 1 (STRM1)
SUBROUTINE BEHX4 = 5 stress components in material type 2 (STRM2)
SUBROUTINE BEHX5 = 5 stress components in material type 3 (STRM3)

Typical margins produced by these five subroutines for an optimized
design of a spherical balloon with 8 modules are as follows:

MARGINS CORRESPONDING TO CURRENT DESIGN (F.S.= FACTOR OF SAFETY)
MARGIN CURRENT
NO.     VALUE            DEFINITION
 1    6.170E-02 (BUCKB4(1)/BUCKB4A(1))/BUCKB4F(1)-1; F.S.= 3.00
 2   -7.231E-03 (TENLOS(1)/TENLOSA(1))/TENLOSF(1)-1; F.S.= 3.00
 3   -2.840E-04 (STRM1A(1,1)/STRM1(1,1))/STRM1F(1,1)-1; F.S.= 1.00
 4    1.609E-01 (STRM1A(1,3)/STRM1(1,3))/STRM1F(1,3)-1; F.S.= 1.00
 5   -5.128E-03 (STRM2A(1,1)/STRM2(1,1))/STRM2F(1,1)-1; F.S.= 1.00
 6    3.480E-01 (STRM2A(1,3)/STRM2(1,3))/STRM2F(1,3)-1; F.S.= 1.00
 7    7.867E-03 (STRM3A(1,1)/STRM3(1,1))/STRM3F(1,1)-1; F.S.= 1.00
 8    3.024E-01 (STRM3A(1,3)/STRM3(1,3))/STRM3F(1,3)-1; F.S.= 1.00

There are more margins than the five subroutines, BEHXi, because
there are up to five stress constraints corresponding to
each of the three material types. In this example only two of the
five stress components for each material produce margins:
STRMi(1,1) (tensile stress along the fibers), i = 1 or 2 or 3, and
STRMi(1,3) (tensile stress normal to the fibers), i = 1 or 2 or 3,
in which "i" denotes "material type".
```

Section 7.4 From Item 4 of Table 8:

```
ITEM 4. SUBROUTINE BEHX1 of the file, behavior.balloon, now
has the following additional statement:

      IF (ITYPEX.EQ.2) CALL B4POST

The BIGBOSOR4 processor, B4POST, creates the file, *.PLT2,
that contains the buckling mode that corresponds to the
variable called BUCKB4. One can obtain a plot of this
```

buckling mode (which may be either local or general buckling) by porting the file, *.PLT2, to a directory from which one wants to execute BIGBOSOR4 in a context independent of GENOPT and then typing the command, "BOSORPLOT", there. Here is the run stream:

```
   COMMAND                     MEANING OF COMMAND

cd /home/progs/work6    (go to a working space, "work6")
bigbosor4log    (activate the BIGBOSOR4 set of commands)
cp /home/progs/genoptcase/try4.PLT2 .   (get BIGBOSOR4 plot file)

bosorplot       (choose what to plot. Use "x" in response to
                 prompt if you want plot on your screen. Use "p"
                 in response to prompt if you want plot to be
                 in the file called "metafile.ps".)

gv metafile.ps  (get a plot on your screen via the "ghost view"
                 utility, if "ghost view" is available on your
                 workstation. Figs. 10a – 10k are edited versions
                 of what appears on your screen.)
```

NOTE: One does not run BIGBSOSOR4, that is, one does not type the command, "BIGBOSORALL", as described in the file, balloon.runstream (Table 4 in the paper [1] cited at the beginning of this file). The "bosorplot" execution requires as input only the file, *.PLT2, in which "*" denotes the name that one has established for the specific case.

In the case of the cylindrical balloon the valid BIGBOSOR4 input data are contained in the *.BEHX1 file. Therefore, instead of the command listed above,

```
cp /home/progs/genoptcase/try4.PLT2 .   (get BIGBOSOR4 plot file)
```

there exists, in Table 4 of [1] the following command:

```
cp /home/progs/genoptcase/try4.BEHX1 try4.ALL   (get BIGBOSOR4 input file)
```

One can type the last command in the case of the spherical balloon also. However, the BIGBOSOR4 execution subsequently launched via the command, BIGBOSORALL, will bomb because of failure of convergence of the Newton iterations for the nonlinear solution of the pre-buckling equilibrium equations. It appears that (without modifying the BIGBOSOR4 computer program) one can obtain buckling modes of the spherical balloon only via the two commands,

```
cp /home/progs/genoptcase/try4.PLT2 .   (get BIGBOSOR4 plot file)
```

```
bosorplot
```

Section 7.5 From Item 5 of Table 8:

```
ITEM 5. SUBROUTINE BEHX1 of the behavior.balloon library now creates the
additional file, *.LOADB, in which "*" signifies the specific name
that you have assigned. The file, *.LOADB, contains a valid input
file for BIGBOSOR4 for a case in which the BIGBOSOR4 analysis type,
INDIC = 0, and for which only the "fixed" loads are applied, that
is, only PINNER, PMIDDL, and DELTAT are applied. In order to get
plots of the deformations at various load steps, you run BIGBOSOR4
in a context independent of GENOPT in the same manner as directed
in the file, balloon.runstream (Table 4). You do the following:

   COMMAND                  MEANING OF COMMAND

cd /home/progs/work6    (go to a working space, "work6")
bigbosor4log    (activate the BIGBOSOR4 set of commands)
cp /home/progs/genoptcase/try4.LOADB try4.ALL   (get BIGBOSOR4 input file)
bigbosorall     (execute BIGBOSOR4: input file = try4.ALL .
                 NOTE: valid input files for bigbosor4 always have the
                 suffix, ".ALL")

Next, you want to plot the pre-buckling deformations for one or more
load steps. For each load step for which you want a plot, do the
following:

bosorplot       (choose what to plot. Use "x" in response to
                 prompt if you want plot on your screen. Use "p"
                 in response to prompt if you want plot to be
                 in the file called "metafile.ps".)

gv metafile.ps  (get a plot on your screen via the "ghost view"
                 utility, if "ghost view" is available on your
                 workstation. Figs. 5 and 6 are edited versions
                 of what appears on your screen.)

When you are finished, type the following command:

cleanup         (Clean up the files generated by BIGBOSOR4)
```

Section 7.6 From Item 7a of Table 8:

```
ITEM 7a. In the cylindrical vacuum chamber the pre-buckled state
of every module is the same. Hence, one can use the pre-buckled
state of the segments of the first module to represent that
```

of every module. In the spherical vacuum chamber the pre-buckled stress resultants vary over the 90 degrees of meridian included in the model. The stress resultants vary not only from module to module, but also within each "shell" segment of each module.

Section 7.7 From Item 7b of Table 8:

ITEM 7b. The pre-buckling behavior of the cylindrical vacuum chamber is uniform over the entire 90 degrees of circumference included in the model, as displayed in Figs. 9 and 10 of [1]. In contrast, the pre-buckling behavior of the spherical vacuum chamber is not uniform over the 90 degrees of meridian included in the model. The spherical vacuum chamber is not isotropic because there are webs oriented only in the circumferential direction, not in the meridional direction. There cannot be webs in the meridional direction in the GENOPT/BIGBOSOR4 model because then the shell would not be axisymmetric. BIGBOSOR4 can handle only axisymmetric or prismatic shell structures. **Figure 6** of the present paper on spherical balloons demonstrates the non-isotropic nature of the pre-buckling axisymmetric deformations of the spherical balloon. Under the loads in Load Set B (non-eigenvalue) loads, PINNER = 0 psi, PMIDDL = 60 psi, the originally spherical shell becomes slightly egg shaped (elongates in the axial direction more than in the radial direction).

Section 7.8 From Item 7c of Table 8:

ITEM 7c. The cylindrical vacuum chamber is modeled as a "true prismatic shell", whereas the spherical vacuum chamber is modeled as an ordinary shell of revolution. There is a difference in the governing equations, as described in Refs.[8] and [9] cited in the paper identified as [1] near the beginning of this file.

Section 7.9 From Item 7d of Table 8:

ITEM 7d. From the computer runs executed so far it appears that the nonlinear pre-buckling equilibrium states are often more difficult to obtain for the spherical vacuum chamber than for the cylindrical vacuum chamber. In the case of the spherical balloon there are more occurrences of failure of convergence of the Newton iterations required for generating the nonlinear pre-buckling equilibrium states. Failure of Newton convergence previously caused early termination of SUPEROPT optimization runs. The FORTRAN coding in SUBROUTINE BEHX1 was modified in order to prevent early termination of SUPEROPT runs caused by failure of

Newton convergence. A typical modification in SUBROUTINE BEHX1 follows:

```
      ILETW = INDEX(WRDCOL,'INITIAL LOADS TOO HIGH FOR THIS STRUCT')
      IF (ILETW.NE.0) THEN
C BEG NOV 2010
         EIGMIN = 10.E+16
         IF (ITYPEX.EQ.2)
     1   WRITE(IFILE,'(/,A)') ' ********* ABORT **********'
         IF (ITYPEX.NE.2)
     1   WRITE(IFILE,'(/,A)') ' ********* WARNING **********'
         WRITE(IFILE,'(A)') ' THIS IS THE INDIC=1 BUCKLING ANALYSIS'
         WRITE(IFILE,'(A)') ' INITIAL LOADS TOO HIGH FOR THIS STRUCT'
         IF (ITYPEX.EQ.2)
     1   WRITE(IFILE,'(A,I2)') ' Run is now aborting: IMODX=',IMODX
         WRITE(IFILE,'(A,/,A,/,A,/,A,/,A,/,A,/,A)')
     1 ' Look near the end of the *.OPP file for the "FEASIBLE" or for',
     1 ' the "ALMOST FEASIBLE" design. Choose whichever of those you',
     1 ' prefer, and use CHANGE to save that design. Then, if you want',
     1 ' to continue with SUPEROPT, execute SUPEROPT again. Bushnell',
     1 ' has found that this execution of SUPEROPT may run for many',
     1 ' iterations before bombing again, or it may run to completion',
     1 ' (a total of about 470 design iterations).'
         IF (ITYPEX.NE.2)
     1   WRITE(IFILE,'(A,I2)') ' IABORT is now set to 1: IMODX=',IMODX
         IF (ITYPEX.NE.2) WRITE(IFILE,'(/,A,/,A,/,A,/,A)')
     1 ' WEIGHT and TOTMAS and EIGMIN are being set equal to large',
     1 ' numbers. If you want a plot of the objective edit the *.PL5',
     1 ' file by removing all entries with very large numbers, then',
     1 ' execute DIPLOT.'
         WRITE(IFILE,'(A)') ' ********************************'
         IABORT = 1
C BEG DEC 2010
         CALL MOVER(0.,0,STRS1V,1,6)
         CALL MOVER(0.,0,STRS2V,1,6)
         EIGMIN = 10.E16
C END DEC 2010
         WEIGHT = 10.E20
         TOTMAS = 10.E20
         IF (ITYPEX.EQ.2) CALL ERREX
C BEG DEC 2010
         GO TO 1000
C END DEC 2010
C END NOV 2010
       ENDIF
```

In the FORTRAN fragment just listed the index, ITYPEX, governs the type of analysis:

ITYPEX = 1 means optimization
ITYPEX = 2 means analysis of a fixed design (no optimization)
ITYPEX = 3 means design sensitivity analysis.

Early termination of an execution (CALL ERREX) only occurs if ITYPEX = 2
The variables, WEIGHT and TOTMAS, are set equal to very high numbers
so that the current objective will be much higher than the previously
obtained objective values that correspond to FEASIBLE and to ALMOST
FEASIBLE designs. The variable, EIGMIN, is set equal to a very high
number so that the optimizer, ADS, will not change the design during
the current execution of OPTIMIZE in the SUPEROPT script.

Now with optimization runs (ITYPEX = 1), instead of "CALL ERREX"
to terminate execution in the
event of failure of convergence of the Newton iterations for
solution of the pre-buckling equilibrium equations, SUBROUTINE
BEHX1 sets WEIGHT and TOTMAS to a very large number and keeps
on executing. In a particular specific case (a spherical balloon
with 12 modules and radial webs) the following typical entries
now exist in the *.OPP file:

```
 ----------------------------------------------------------OPTIMIZE
   215         5.8165E+03          FEASIBLE            3
   216         5.6096E+03     MILDLY UNFEASIB          4
   217         5.3223E+03     MILDLY UNFEASIB          4
   218         5.7909E+03     ALMOST FEASIBLE          4
   219         5.7172E+03     ALMOST FEASIBLE          4
   220         5.7791E+03     ALMOST FEASIBLE          4
 --------------------------------------------------------AUTOCHANGE
 ----------------------------------------------------------OPTIMIZE
   221         1.0000E+21          FEASIBLE            0
 ----------------------------------------------------------OPTIMIZE
   222         1.0000E+21          FEASIBLE            0
 ----------------------------------------------------------OPTIMIZE
   223         1.0000E+21          FEASIBLE            0
 ----------------------------------------------------------OPTIMIZE
   224         1.0000E+21          FEASIBLE            0
 ----------------------------------------------------------OPTIMIZE
   225         1.0000E+21          FEASIBLE            0
 --------------------------------------------------------AUTOCHANGE
 ----------------------------------------------------------OPTIMIZE
   226         9.8328E+03      NOT FEASIBLE            3
   227         1.0274E+04      NOT FEASIBLE            3
   228         1.0879E+04     MORE UNFEASIBLE          2
   229         1.1307E+04     MORE UNFEASIBLE          2
   230         1.1601E+04     MILDLY UNFEASIB          2
   231         1.1822E+04     ALMOST FEASIBLE          2
```

```
------------------------------------------------------OPTIMIZE
```

The large numbers, 1.0000E+21, for the objective signify that failure of convergence of the Newton iterations for the solution of the nonlinear pre-buckling equilibrium equations has occurred in design iterations 221 - 225. These large numbers affect the *.PL5 file, which is produced by the GENOPT processor called "CHOOSEPLOT" and which contains a "plot" of the objective versus design iterations. If the user wants meaningful plots of objective versus design iterations, he or she will have to eliminate any "large-number" entries in the *.PL5 file. The part of the unedited *.PL5 file that corresponds to the above list involving design iterations 215 - 231 follows:

```
 0.21500E+03 0.56096E+04
 0.21600E+03 0.53223E+04
 0.21700E+03 0.57909E+04
 0.21800E+03 0.57172E+04
 0.21900E+03 0.57791E+04
 0.22000E+03 0.10000E+22 <--- remove this entry before plotting
 0.22100E+03 0.10000E+22 <--- remove this entry before plotting
 0.22200E+03 0.10000E+22 <--- remove this entry before plotting
 0.22300E+03 0.10000E+22 <--- remove this entry before plotting
 0.22400E+03 0.10000E+22 <--- remove this entry before plotting
 0.22500E+03 0.98328E+04
 0.22600E+03 0.10274E+05
 0.22700E+03 0.10879E+05
 0.22800E+03 0.11307E+05
 0.22900E+03 0.11601E+05
 0.23000E+03 0.11822E+05
 0.23100E+03 0.11822E+05
```

For another case with 15 modules and truss-like (slanted) webs, **Figure 21** shows the plot of objective versus design iterations before an editing of the *.PL5 file to remove all the "high-number" entries. **Figure 22** shows the same plot after removal of all the "high-number" entries in the *.PL5 file.

In these cases the plots are obtained via the following commands:

diplot (the input file is try4.PL5)
gv try4.5.ps (obtain plot of objective versus design iterations)

Section 7.10 From Item 7f of Table 8:

ITEM 7f. In the case of the cylindrical balloons the buckling wave number used to compute buckling over the very long length, 6000 inches [1], is N = 1, which signifies one half wave over the axial length, 6000 inches. In the case of the spherical balloons the buckling wave number used to compute "critical" buckling in optimization or in design sensitivity runs (ITYPE = 1 or 3 in the *.OPT file) is N = 0, which signifies axisymmetric buckling. In runs involving the analysis of a "fixed" design (ITYPE = 2 in the *.OPT file) bifurcation buckling load factors from BIGBOSOR4 are sought over a wide range of N in order to search for a minimum. This occurs only in the case of a spherical balloon (ISHAPE = 2 in the *.BEG file). The output in the *.OPM file from this search for a minimum (critical) buckling load factor for an optimized spherical balloon with 8 modules and radial webs follows:

```
 BUCKLING LOAD FACTORS AND MODES FROM BIGBOSOR4 (BEHX1)

        3.6727E+00(     0)  <--axisymmetric buckling (N=0 circ.waves,
        3.4612E+00(    16)     Figure 18)
        3.2674E+00(    32)
        3.1965E+00(    48)
        3.1851E+00(    64)  <--critical buckling (N=64 circ.waves,
        3.2020E+00(    80)     Figure 19)
        3.2379E+00(    96)
        3.2909E+00(   112)
        3.3580E+00(   128)
        3.4376E+00(   144)
        3.5291E+00(   160)
 Critical buckling load factor, BUCKB4=  3.1851E+00
 Critical number of circumferential full-waves, NWVCRT=   64
```

The same *.OPM file lists the following values for the minimum load factors corresponding to initial loss of meridional and circumferential tension in each segment (ISEG) of any module:

```
 LOSS OF    MERIDIONAL   TENSION: (EIG1MN(1,ISEG),ISEG=1,5)=
  4.1691E+00  1.0315E+01  3.6024E+00  3.7359E+01  1.2758E+01

 LOSS OF CIRCUMFERENTIAL TENSION: (EIG2MN(1,ISEG),ISEG=1,5)=
  3.8769E+00  6.4317E+00  2.9783E+00  4.5277E+00  3.2721E+00
```

and the minimum load factor among those (the minimum minimum):

Buckling load factor corresponding to the initial loss of either meridional or circumferential tension, EIGMIN= 2.9783E+00

The bifurcation buckling load factor (eigenvalue) from BIGBOSOR4 for N = 0 circumferential waves is: **3.6727E+00**(0). This value is very close to the load factor corresponding to initial loss of **meridional** tension in Segment No. 38 (module no. 8): **3.6024E+00.** (Segment No. 38 is one of the straight segments of the inner wall, the thickness of which is TFINNR.) The minimum bifurcation buckling load factor from BIGBOSOR4 corresponds to N = 64 circumferential waves and is: **3.1851E+00**(64). This critical (minimum) value is above and fairly close to the minimum load factor, **2.9783E+00**, that corresponds in this particular case to initial loss of **circumferential** tension in Segment No. 38 (module no. 8). It is emphasized that the search for a critical (minimum) buckling load factor from BIGBOSOR4 is conducted **only if ITYPE = 2**, that is, analysis of a "fixed" design. **In order to save computer time during optimization runs (ITYPE = 1) and in order to avoid the use of spurious buckling data (Figs.30,31) only the bifurcation buckling load factor corresponding to N = 0 circumferential waves is computed.** Since the minimum buckling load factor (3.1851E+00 in this particular case) is somewhat lower than the buckling load factor corresponding to N=0 (3.6727E+00 in this particular case), one might think that optimum designs obtained with this strategy might be somewhat unconservative. However, recall that now there exists the behavioral constraint, TENLOS/TENLOSF, in which TENLOS/TENLOSF in this particular case is 2.9783E+00/3.0, which is somewhat lower than the minimum (critical) bifurcation buckling ratio computed from BIGBOSOR4: 3.1851E+00/3.0. Therefore, the optimum design should be somewhat conservative in spite of the lack of a search over circumferential wave number during optimization cycles.

Section 7.11 From Item 7k of Table 8:

ITEM 7k. In the case of the spherical balloon there is no need use a temperature, DELTAT, in order to generate the proper stress resultants normal to the plane of the paper in Fig.1 of [1], for example. Therefore, if ISHAPE = 2 (spherical balloon) DELTAT is set equal to zero in SUBROUTINE BEHX1 of the behavior.balloon library. The circumferential stress resultants, that is, the stress resultants normal to the plane of the paper in Fig. 1, are computed by BIGBOSOR4 in the case of the spherical balloon.

Section 7.12 From Item 7m of Table 8:

ITEM 7m. In the case of cylindrical balloons junctions between "shell" segments in the BIGBOSOR4 model are hinged, that is, a typical junction constraint condition is as follows:

```
  1      $ At how may stations is this segment joined to previous segs.?
 31      $ INODE = node in current segment (ISEG) of junction, INODE(  1)
  1      $ JSEG  = segment no. of lowest segment involved in junction
```

```
       31      $ JNODE = node in lowest segmnt (JSEG) of junction
        1      $ IUSTAR= axial displacement (0=not slaved, 1=slaved)
        1      $ IVSTAR= circumferential displacement (0=not slaved, 1=slaved)
        1      $ IWSTAR= radial displacement (0=not slaved, 1=slaved)
        0      $ ICHI  = meridional rotation (0=not slaved, 1=slaved) ← NOTE!
```

In the case of spherical balloons the typical segment junction condition is as follows:

```
        1      $ At how may stations is this segment joined to previous segs.?
       31      $ INODE = node in current segment (ISEG) of junction, INODE(  1)
        1      $ JSEG  = segment no. of lowest segment involved in junction
       31      $ JNODE = node in lowest segmnt (JSEG) of junction
        1      $ IUSTAR= axial displacement (0=not slaved, 1=slaved)
        1      $ IVSTAR= circumferential displacement (0=not slaved, 1=slaved)
        1      $ IWSTAR= radial displacement (0=not slaved, 1=slaved)
        1      $ ICHI  = meridional rotation (0=not slaved, 1=slaved) ← NOTE!
```

In the above two lists a “1” for IUSTAR, IVSTAR, IWSTAR, ICHI means “displacement component is restrained”. A “0” means “displacement component is free”.

Section 7.13 From Item 8 of Table 8:

In the case of the spherical balloons, for which the pre-buckled state is different in every module and at every nodal point within each shell segment, the output included in the *.OPM file (with NPRINT = 2) is as follows (in a particular case with 8 modules over 90 degrees of the meridian of the spherical balloon with radial webs):

```
 MAXIMUM PREBUCKLING MERIDIONAL STRESS RESULTANTS
                "fixed"       "total"
              Load Step 1   Load Step 2
 Seg.J  Node I  N1FIX(I,J)    N1VAR(I,J)
   26      1   3.20114E+02   2.47268E+02
    7      2   1.22921E+03   1.14633E+03
   23      1   1.26495E+03   9.22114E+02
    9      1   1.12449E+03   1.11431E+03
    5      1   1.87964E+03   1.75817E+03

 MAXIMUM PREBUCKLING CIRCUMFERENTIAL STRESS RESULTANTS
                "fixed"       "total"
              Load Step 1   Load Step 2
 Seg.J  Node I  N2FIX(I,J)    N2VAR(I,J)
    6      2   2.04872E+02   1.74672E+02
    7      2   1.11425E+03   9.87126E+02
    8      1   9.84081E+02   7.50764E+02
```

9	1	1.05412E+03	8.84845E+02
5	1	1.64854E+03	1.36057E+03

MINIMUM EIGENVALUE FOR LOSS OF MERIDIONAL TENSION

			"fixed" Load Step 1	"total" Load Step 2
Seg.J	Node I	EIGEN1(I,J)	N1FIX(I,J)	N1VAR(I,J)
11	23	4.16945E+00	2.27137E+02	1.72660E+02
7	31	1.03148E+01	8.40298E+02	7.58833E+02
38	30	**3.60258E+00**	1.21699E+03	8.79178E+02
29	31	3.73586E+01	7.33646E+02	7.14008E+02
40	31	1.27584E+01	1.62095E+03	1.49390E+03

MINIMUM EIGENVALUE FOR LOSS OF CIRCUMFERENTIAL TENSION

			"fixed" Load Step 1	"total" Load Step 2
Seg.J	Node I	EIGEN2(I,J)	N2FIX(I,J)	N2VAR(I,J)
16	16	3.87690E+00	1.28506E+02	9.53595E+01
12	9	6.43172E+00	7.52156E+02	6.35211E+02
38	17	**2.97831E+00**	4.29922E+02	2.85571E+02
39	17	4.52765E+00	3.97527E+02	3.09727E+02
5	13	3.27211E+00	3.86260E+00	2.68214E+00

Notice that the lowest positive "eigenvalue" for the loss of MERIDIONAL tension is EIGEN1 = **3.60258.** The initial loss of **MERIDIONAL** tension is predicted to occur at Nodal Point 30 in "shell" Segment No. 38. Segment 38 is in Module No. 8. Segment 38 corresponds to the inner wall with straight segments of thickness, TFINNR. This "eigenvalue" is very close to the actual eigenvalue for axisymmetric buckling (N = 0 circumferential waves) computed by BIGBOSOR4 from an actual stability (bifurcation buckling) analysis:

BUCKLING LOAD FACTORS AND MODES FROM BIGBOSOR4 (BEHX1)

3.6727E+00(	0)	<--axisymmetric buckling (0 circ.waves,
3.4612E+00(	16)	**Figure 18**)
3.2674E+00(	32)	
3.1965E+00(	48)	
3.1851E+00(	64)	<--critical buckling (64 circ.waves,
3.2020E+00(	80)	**Figure 19**)
3.2379E+00(	96)	
3.2909E+00(	112)	
3.3580E+00(	128)	
3.4376E+00(	144)	
3.5291E+00(	160)	

Critical buckling load factor, BUCKB4= 3.1851E+00
Critical number of circumferential full-waves, NWVCRT= 64

The local buckling mode corresponding to the eigenvalue, **3.6727,** obtained from BIGBOSOR4 is displayed in **Figure 18.** Local buckling corresponding to N = 0 circumferential waves predicted from the actual BIGBOSOR4 eigenvalue analysis occurs in the same shell segment as predicted by the analysis of initial loss of pre-buckling **MERIDIONAL** tension.

The same relationship between predictions from the analysis of initial loss of tension and the bifurcation buckling load factor from the BIGBOSOR4 bifurcation buckling (eigenvalue) analysis holds for the minimum load factor corresponding to the initial loss of **CIRCUMFERENTIAL** tension. This load factor, called "EIGEN2" above, is equal to **2.97831** and occurs at nodal point 17 in "shell" segment 38. The "initial loss of CIRCUMFERENTIAL tension" load factor, **2.97831,** is fairly close to the critical buckling load factor obtained from BIGBOSOR4, **3.1851,** corresponding to 64 circumferential waves. The local buckling mode corresponding to the eigenvalue, **3.1851,** obtained from BIGBOSOR4 is displayed in **Figure 19.** Local buckling corresponding to N = 64 circumferential waves predicted from the actual BIGBOSOR4 eigenvalue analysis occurs in the same shell segment as predicted by the analysis of initial loss of pre-buckling **CIRCUMFERENTIAL** tension.

Section 7.14 From Item 9 of Table 8:

ITEM 9. At the date of this writing it has not been possible to obtain a general buckling mode for the spherical balloon. As with the cylindrical balloon, the local buckling eigenvalues are clustered closely together. However, in the case of the cylindrical balloon the general buckling mode is sometimes the mode that corresponds to the lowest eigenvalue. That seems always to be so for optimized cylindrical balloons with radial webs. In the case of the spherical balloon the general buckling mode as of this writing has corresponded to an eigenvalue that must be higher than at least 50 eigenvalues all of which correspond to local buckling. For example, for the optimized spherical balloon with 8 modules over 90 meridional degrees and with radial webs, the following 50 eigenvalues, all of which correspond to local buckling with N = 0 circumferential waves (axisymmetric buckling), are:

3.6727, 3.6789, 3.6817, 3.6818, 3.6823, 3.6842, 3.6951, 3.6958, 3.7016, 3.7037,
3.7158, 3.7172, 3.7190, 3.7237, 3.7322, 3.7328, 3.7379, 3.7388, 3.7449, 3.7506,
3.7521, 3.7558, 3.7671, 3.7703, 3.7725, 3.7764, 3.7767, 3.7825, s.7929, 3.8004,
3.8024, 3.8052, 3.8120, 3.8157, 3.8164, 3.8251, 3.8292, 3.8337, 3.8413, 3.8524,

```
3.8555, 3.8661, 3.8727, 3.8772, 3.8854, 3.8856, 3.8899, 3.8921, 3.8961, 3.9024
```

In order to obtain more than one eigenvalue for a given number of circumferential waves it was necessary temporarily to change behavior.new and bosdec.src. SUBROUTINE BEHX1 of the behavior.new library was temporarily changed by commenting out the following statements:

```
        IF (ITYPEX.EQ.2) THEN
          NMAXB = 20*NMODUL
          INCRB = NMAXB/10
          IF (INCRB.LT.1) INCRB = 1
        ENDIF
```

SUBROUTINE BOSDEC was temporarily changed by setting NVEC = 50 instead of NVEC = 1 .

A plot of the buckling mode corresponding to the 45th eigenvalue for N = 0 circumferential waves (axisymmetric buckling) is given in **Figure 20.**

SECTION 8 OPTIMIZATION OF SPHERICAL BALLOONS OVER A RANGE IN THE NUMBER OF MODULES, NMODUL, IN 90 DEGREES OF MERIDIAN (Material = polyethylene terephthalate)

Table 12 and Figs. 23 – 27 pertain to this section.

Figure 23 contains plots of the total weights of the spherical balloons versus the number of modules over 90 degrees of meridian. The lowest weight of the balloon with truss-like (slanted) webs and the lowest weight of the balloon with radial webs both correspond to a model with 35 modules.

Table 12 lists some of the output from the try4.OPM files corresponding to the models with 35 modules.

Figures 24 – 27 show axisymmetric and non-axisymmetric buckling modes corresponding to the optimized spherical balloons with 35 modules over 90 degrees of the meridian.

SECTION 9 OPTIMIZATION OF SPHERICAL BALLOONS OVER A RANGE IN THE NUMBER OF MODULES, NMODUL, IN 90 DEGREES OF MERIDIAN (Material = fictitious carbon fiber cloth)

Tables 13 – 17 and Figs. 28 – 50 pertain to this section.

Section 9.1 Material properties and optimized weights

Table 13 lists the material properties of the fictitious carbon fiber cloth. Note especially that the tensile stress allowable, 75600 psi for "tension along the fibers", is much higher than the stress allowable for polyethylene terephthalate, 10000 psi. Also, the weight density of the fictitious carbon fiber cloth, 0.057 lb/in^3, is not much more than half the weight density used here and in [1] for the polyethylene terephthalate: 0.100 lb/in^3. These two major differences in material properties give rise to the much smaller optimum weights of the balloons made of fictitious carbon fiber cloth than those of the balloons made of polyethylene terephthalate.

Figure 28 contains plots of the total weights of the spherical balloons versus the number of modules over 90 degrees of meridian. The lowest weight of the balloon with truss-like (slanted) webs corresponds to a model with 30 modules. The lowest weight of the balloon with radial webs corresponds to a model with 55 modules, which is about the same weight as that for the model with 50 modules. Fifty-five modules is the maximum number of modules permitted by the "balloon" software. For more modules than 55 both the maximum number of "shell" segments permitted by BIGBOSOR4 and the maximum number of degrees of freedom permitted by BIGBOSOR4 are exceeded. Compare Fig. 28 with Fig. 23. The minimum weights of the spherical balloons made of fictitious carbon fiber cloth is close to a factor of 20 lighter than the minimum weights of the spherical balloons made of polyethylene terephthalate.

Section 9.2 Real and spurious buckling modes

Section 9.2.1 Real buckling mode, spherical shell with radial webs

The "shell" segments of the optimized spherical balloons made of fictitious carbon fiber cloth are much thinner than those of the optimized spherical balloons made of polyethylene terephthalate. This property leads to the production of spurious bifurcation buckling modes from BIGBOSOR4 for N > 0 circumferential waves, that is, for non-axisymmetric bifurcation buckling. Table 14 lists some results from the optimized spherical balloon with 35 modules over 90 degrees of meridian and with **radial webs**. In particular, note the results listed under the heading, "**`BUCKLING LOAD FACTORS AND MODES FROM BIGBOSOR4 (BEHX1)`**". Figure 29 shows the axisymmetric (N = 0 circumferential waves) buckling mode. This is a real buckling mode, not a spurious buckling mode. The corresponding buckling load factor listed in Table 14 is **`2.9541`**, which produces Margin No. 1, -0.01530, listed near the end of Table 14. **NOTE: Only buckling with N = 0 circumferential waves is included in the computation of the buckling constraint, BUCKB4, during optimization cycles and in the computation of the buckling margin (Margin No. 1 listed in Table 14) during the analysis of a fixed design (ITYPE = 2).** Because of the frequent occurrence of spurious buckling modes for N > 0 circumferential waves, the buckling load factors (eigenvalues) obtained by BIGBOSOR4 for N > 0 are not considered in the computation of the buckling constraint, BUCKB4. Non-axisymmetric buckling is "covered", however, by means of the initial-loss-of-tension constraint, TENLOS, because the initial loss of tension in any segment of the balloon corresponds to the onset of either axisymmetric or non-axisymmetric wrinkling of the skin. Wrinkling is a type of buckling. Therefore, ignoring the eigenvalues as computed by BIGBOSOR4 for N > 0 circumferential waves does not lead to the generation of unconservative designs resulting from the neglect of non-axisymmetric bifurcation buckling.

Section 9.2.2 Spurious buckling modes

Section 9.2.2.1 spherical balloon with radial webs:

Figures 30 and 31 show one of the spurious non-axisymmetric buckling modes of the optimized spherical balloon with 35 modules and with radial webs: the spurious buckling mode corresponding to N = 35 circumferential waves. The corresponding buckling load factor (eigenvalue) listed in Table 14 is **`0.35645`**, which would produce a significantly negative margin [margin = (buckling load factor)/(factor of safety) – 1.0 = 0.35645/3.0 -1.0 = -0.8818] if non-axisymmetric buckling (N > 0 circumferential waves) as predicted by BIGBOSOR4 were to be included in the computation of the buckling margin, Margin No. 1. According to the spurious prediction by BIGBOSOR4, the second "shell" segment in the third module of the 35-module model buckles very locally near the beginning of the segment, as demonstrated in Figs. 30 and 31. The buckling mode has a "zig-zag" characteristic typical of spurious buckling modes. This mode would disappear if many more nodal points were used in each of the "shell" segments of the 35-module model. Unfortunately, many more nodal points cannot be used in each "shell" segment because BIGBOSOR4 has a limit on the total number of degrees of freedom permitted in any model. This limit would be exceeded for configurations that correspond to the lightest optimized designs: those with many modules.

Section 9.2.2.2 spherical balloon with truss-like (slanted) webs:

Figure 32 shows a spurious buckling mode that occurs in an optimized spherical balloon with 35 modules and with truss-like (slanted) webs. Spurious buckling occurs in one of the slanted webs. As in the case of the spherical balloon with 35 modules and radial webs, results for which are described in the previous paragraph, the spurious buckling mode in the spherical balloon with truss-like webs shown in Fig. 32 has a "zig-zag" appearance and is associated with an eigenvalue, **2.8134** (not listed in any table in this paper), that is too low. This type of spurious local buckling mode would not exist were many more nodal points used in each of the webs. However, many more nodal points cannot be used because the maximum number of degrees of freedom permitted in BIGBOSOR4 would then be exceeded for models with many modules.

Section 9.3 Real and spurious axisymmetric pre-buckling behavior: spherical balloon with radial webs

Figures 33 – 44 show results for the optimized spherical balloon with 50 modules over 90 degrees of meridian and radial webs. The try7.OPM file corresponding to the optimized design is listed in Table 15.

The axisymmetric pre-buckling solution for the optimized 50-module spherical balloon exhibits some "zig-zag" characteristics analogous to those of the spurious buckling mode shown in Figs. 30 and 31. Figure 33a shows an enlarged view of Segment No. 34 (Segment No. 34 = Segment No. 4 of Module No. 7) as deformed under Load B, the "fixed", non-eigenvalue loading: PINNER = 0 psi, PMIDDL = 60 psi, POUTER = 0 psi, which corresponds to Load Step 11 listed in Table 15. The axisymmetric pre-buckling solution in Segment No. 34 has a "zig-zag" appearance near its junctions with other segments of the model. Figures 33b,c,d are analogous to Fig. 33a. The spurious "zig-zag" component of the axisymmetric pre-buckling deformation diminishes with increasing number of nodal points in Segment No. 34.

Figure 34 shows the axisymmetric pre-buckling deformation of the part of the 50-module model of the spherical balloon in the region near 45 degrees. Notice that the radial webs exhibit a small "zig-zag" (spurious) component of deformation.

The prediction of the circumferential stress resultant, n2, is significantly affected by the spurious "zig-zag" component of axisymmetric pre-buckling deformation. Figure 35 displays the distribution of meridional stress resultant, n1, and the distribution of circumferential stress resultant, n2, over the arc length of Segment No. 34 corresponding to the distribution of nodal points shown in Fig. 33a. There is very little "zig-zag" of n1.

However, there is significant “zig-zag” of n2. The red curve shows the distribution of n2 that is used in the “balloon” software, SUBROUTINE BEHX1 (Table 5). This red curve is obtained by averaging the values of n2 at neighboring nodal points in the model shown in Fig. 33a, starting with the fifth nodal point in Segment 34 and ending with the fifth-from-last nodal point in Segment 34.

Figure 36 shows the distributions of the circumferential stress resultant, n2, corresponding to the nodal point distributions in Segment 34 displayed in Figs. 33a-d. The red curve in Fig. 36 is the same as the red curve in Fig. 35. The maximum value of n2 in Segment 34 corresponds to the value at the beginning of the red curve: n2 = 187.357 lb/in (Table 15). This is the result that corresponds to the nodal point distribution shown in Fig. 33a, that is, the nodal point distribution that is used for optimization in the “balloon” software, SUBROUTINE BEHX1 (Table 5). This value, n2 = 187.357 lb/in, is somewhat higher than the maximum value of n2 in Segment 34 corresponding to the converged solution displayed in Fig. 36 (the solution obtained from the model with 99 nodal points in Segment 34). Since the distribution of n2 in Segment 34 is typical of the distributions of n2 in the other analogous segments of the model of the spherical balloon (inner wall with curved segments), the relatively crude model shown in Fig. 33a overestimates the circumferential stress in Segment 34 and in the other analogous segments. Therefore, the relatively crude model used for optimization (Fig. 33a) is conservative.

Figures 37a-d, 38, and 39 are analogous to Figures 33a-d, 35, and 36, respectively. Figures 37a-d, 38, and 39 pertain to Segment No. 22 of the 50-module model of the spherical balloon with the radial webs. Segment No. 22 is Segment 2 of Module No. 5, one of the segments of the outermost wall of the spherical balloon. The discussion in the previous paragraph applies to the results for Segment 22 also, except that in Segment 22 and the other analogous segments of the model of the spherical balloon (outer wall with curved segments) the maximum value of the circumferential resultant, n2, along the red curve in Fig. 39 occurs near the end of segment 22 instead of near the beginning of segment 22. Again, the relatively crude model used for optimization (Fig. 37a) is conservative.

Section 9.4 Axisymmetric meridional stress concentrations at segment junctions

Figure 40 is analogous to Figs. 33a – d. In this case Segment 24 has many more nodal points than it does in the model displayed for Segment 34 in Fig. 33a, and the nodal points are concentrated near the beginning of Segment 24 where it is joined to previous segments in the 50-module model. Where the nodal points are concentrated there is no visible “zig-zag” variation of the pre-buckling displacements. However, the “zig-zag” pattern is pronounced at either end of the portion of Segment 24 with the widely spaced nodal points. As we have seen from Section 9.3, the “zig-zag” component of the pre-buckling axisymmetric displacement pattern is spurious: it disappears in regions where nodal points are concentrated and appears in regions where the nodal point spacing changes abruptly and where the nodal points are sparse in the neighborhood of a junction with other segments in the model.

Figures 41, 42, and 43 are analogous to Figs. 14, 16, and 17, respectively. Figure 41 corresponds to the nodal point distribution shown for a different segment, Segment 34, in Fig. 33a. Figure 42 corresponds to the nodal point distribution shown in Fig. 40. Figure 43 shows the distributions of extreme fiber meridional stress in Segment 24 in the neighborhood of the junction of Segment 24 with previous segments in the model of the spherical balloon. Note that the amplitude of the extreme fiber meridional stress at the beginning of Segment 24 is dramatically reduced by the concentration of nodal points there. Figure 44 shows in red the same distribution of extreme fiber meridional stress as that displayed in Fig. 42 and in red in Fig. 43. The green curve in Fig. 44 shows the distribution of extreme fiber meridional stress obtained from a model in which the nodal point density has been further increased by a factor of about three from that used for the production of Fig. 42. Since there appears to be reasonable convergence of the extreme fiber meridional stress in the neighborhood of the beginning of Segment 24, the stress concentration displayed in Fig. 44 is probably real, with little or no spurious component from a "zig-zag" pre-buckling displacement pattern analogous to that shown for a different segment, Segment 34, in Fig. 33a. **In the fabrication of a balloon the seams along segment junctions must be locally reinforced in order to avoid premature failure caused by excessive local extreme fiber meridional stress there.**

Section 9.5 Convergence of predictions with respect to the number of nodal points in each segment of the model

Tables 16 and 17 and Figs. 45 – 50 pertain to this sub-section. The spherical balloon has 15 modules over 90 degrees of meridian and radial webs. There are two models:

Model 1: There are 31 nodal points [NODSEG = 31 in SUBROUTINE BOSDEC (Table 7)] in each "shell" segment of the 15-module model (Table 16, Figs. 45, 47, 49).

Model 2: There are 97 nodal points [NODSEG = 97 in SUBROUTINE BOSDEC (Table 7)] in each "shell" segment of the 15-module model (Table 17, Figs. 46, 48, 50).

The spherical balloon was optimized with the use of Model 1. Table 16 lists part of the try7.OPM file generated from Model 1 for the optimized design. Table 17 lists part of the try7.OPM file generated from Model 2 for the same optimum design, that is, the optimized design that was generated with the use of Model 1. The following conclusions may be drawn from this convergence study:

Item 1. The buckling load factors (eigenvalues) and the load factors that correspond to the initial loss of tension in the skin of the balloon are hardly affected at all by the change in the number of nodal points in each segment from 31 to 97. Compare Table 16 with Table 17:

From Table 16 (NODSEG = 31 in SUBROUTINE BOSDEC, Table 7):
```
1         3.057343      buckling load factor from BIGBOSOR4: BUCKB4(1 )
2         2.978578      load factor for tension loss: TENLOS(1 )
```
From Table 17 (NODSEG = 97 in SUBROUTINE BOSDEC):
```
1         3.058383      buckling load factor from BIGBOSOR4: BUCKB4(1 )
2         3.022104      load factor for tension loss: TENLOS(1 )
```

Item 2. The pre-buckling deformation corresponding to Model 1 exhibits a small component of spurious "zig-zag" displacement (Fig. 45). This "zig-zag" component of deformation is absent in Model 2 (Fig. 46).

Item 3. A few of the maximum stresses are significantly affected by the change in the number of nodal points from 31 to 97. Compare Table 16 with Table 17:

From Table 16 (NODSEG = 31 in SUBROUTINE BOSDEC, Table 7):

MAXIMUM PREBUCKLING **MERIDIONAL** MEMBRANE STRESSES

		"fixed"	"total"	
		Load Step 1	Load Step 2	
Seg.J	Node I	STRS1F(I,J)	STRS1V(I,J)	
71	5	1.06192E+05	7.47788E+04	
7	**5**	**8.24318E+04**	**7.65506E+04**	**←compare with Table 17**
8	5	1.08390E+05	7.65014E+04	
9	5	7.82563E+04	7.77236E+04	
10	5	8.12339E+04	7.54485E+04	

MAXIMUM PREBUCKLING **CIRCUMFERENTIAL** MEMBRANE STRESSES

		"fixed"	"total"	
		Load Step 1	Load Step 2	
Seg.J	Node I	STRS2F(I,J)	STRS2V(I,J)	
1	**5**	**9.22487E+04**	**7.90948E+04**	**←compare with Table 17**
12	29	5.22069E+04	4.28726E+04	
3	5	9.96337E+04	7.13944E+04	
24	5	5.16460E+04	4.09893E+04	
70	29	4.08609E+04	3.30235E+04	

From Table 17 (NODSEG = 97 in SUBROUTINE BOSDEC):

MAXIMUM PREBUCKLING **MERIDIONAL** MEMBRANE STRESSES

		"fixed"	"total"	
		Load Step 1	Load Step 2	
Seg.J	Node I	STRS1F(I,J)	STRS1V(I,J)	
71	5	1.04627E+05	7.40116E+04	
7	**5**	**8.81015E+04**	**8.18400E+04**	**←compare with Table 16**
8	5	1.06792E+05	7.58427E+04	
9	5	8.09579E+04	8.04490E+04	
10	5	8.17748E+04	7.61409E+04	

MAXIMUM PREBUCKLING **CIRCUMFERENTIAL** MEMBRANE STRESSES

		"fixed"	"total"	
		Load Step 1	Load Step 2	
Seg.J	Node I	STRS2F(I,J)	STRS2V(I,J)	
1	**5**	**8.42247E+04**	**7.28184E+04**	**←compare with Table 16**
2	95	6.55898E+04	5.59135E+04	
3	5	9.42940E+04	6.79018E+04	
19	5	6.33767E+04	4.76153E+04	
15	5	5.29311E+04	3.79004E+04	

Item 4. Although the buckling load factors (eigenvalues) obtained from BIGBOSOR4 are affected only slightly by the change from 31 to 97 nodal points in each segment, there are significant differences in the critical buckling mode shapes. Compare Fig. 47 with Fig. 48 for axisymmetric (n=0 circumferential waves) buckling, and compare Fig. 49 with Fig. 50 for non-axisymmetric (n=45 circumferential waves) buckling. These significant differences in critical local buckling mode shapes are not important, however, because there exist

many very closely spaced local buckling eigenvalues for each number of circumferential waves, and the critical (lowest) local buckling eigenvalue is insensitive to the number of circumferential waves, as listed in Table 16, for example:

```
BUCKLING LOAD FACTORS AND MODES FROM BIGBOSOR4 (BEHX1)
        3.0573E+00(     0)
        3.0438E+00(    15)  [eigenvalue(number of circumferential waves)]
        3.0294E+00(    30)
        3.0216E+00(    45)   (These are all real modes;
        3.0245E+00(    60)    no spurious modes in this
        3.0336E+00(    75)    particular model which has
        3.0474E+00(    90)    only 15 modules.)
        3.0604E+00(   105)
        3.0694E+00(   120)
        3.0783E+00(   135)
        3.0824E+00(   150)
```

Item 5. This convergence study in which the number of nodal points is increased from 31 to 97 **in every segment** of the 15-module model is possible only for models of the spherical shell with relatively few modules (15 modules or less). This is because the number of degrees of freedom permitted by BIGBOSOR4 is limited. It is not possible to conduct an analogous convergence study for the example discussed in sub-section 9.3, for example. In that sub-section the number of nodal points was increased from that used by the "balloon" software, SUBROUTINE BOSDEC (Table 7), that is, NODSEG = 31, in only a single segment of the 50-module model of the spherical shell, not in every segment of that model.

Section 9.6 Difficulty finding a "global" optimum design for spherical balloons with a large number of modules (more than 30 modules) and with truss-like webs

Figures 51a,b and 52a,b pertain to this sub-section. These "a,b" figures are analogous to Figs. 21 and 22, respectively. The difficulty finding a "global" optimum design arises from frequent failure of convergence of the Newton method to solve the nonlinear equilibrium equations for the axisymmetric pre-buckled state of the spherical balloon as loaded by a very small fraction of Load B: PINNER = 0 psi, 0.002 x PMIDDL = 0.002 x 60 psi, POUTER = 0 psi. This failure of convergence occurs because the stiffness matrix of the membrane-like structure is almost singular, especially for configurations with a large number of modules for which the thicknesses, TOUTER, TINNER, TFOUTR, TFINNR, TFWEBS, are very small for optimized configurations.

As described in Item 7d of Table 8 and in Section 7 above, a strategy has been introduced into the "balloon" software, SUBROUTINE BEHX1 (Table5), to ensure that SUPEROPT keeps running in spite of a failure of convergence in an attempt to solve the nonlinear pre-buckling equilibrium equations. This strategy involves a setting of the buckling, loss of tension, and stress constraints equal to very large numbers in order artificially to produce huge behavioral gradients that guarantee that the current design will not change during design iterations until the next execution of the GENOPT processor called AUTOCHANGE. (See the part of Section 7 entitled, "**From Item 7d of Table 8**" in which the objective corresponding to design iterations 221 – 225 are listed.) Also, the objective, WEIGHT, is set equal to a very large number if the index, IMODX = 0 (current design as opposed to perturbed design, for which IMODX = 1) so that the current design will not be judged superior to previously derived FEASIBLE or ALMOST FEASIBLE designs in the same SUPEROPT run.

If this strategy is frequently called upon during an execution of SUPEROPT, there will not be enough exploration of design space to “guarantee” the evolution of a “global” optimum design. The words, “guarantee” and “global” are enclosed in quotation marks because SUPEROPT cannot actually determine a global optimum design rigorously but only come close to a global optimum design by exploring design space from many different starting points.

Figures 51a,b are plots of the objective versus design iteration obtained during a partial execution of SUPEROPT. (SUPEROPT was terminated on purpose at design iteration number 175 in order to save calendar time.) Figure 51a is a plot of the objective versus design iteration number that includes two regions where the objective is very, very large. In these two regions the strategy described above was invoked for the **current** design (index, IMODX = 0). Only when IMODX = 0 (analysis of the current design) is the objective set equal to a large number at design iterations for which the strategy described above is invoked. Figure 51b shows the plot of objective versus design iterations obtained after editing out the very large values of the objective from the file called try7.PL5. There are 13 points indicated in Fig. 51b where the strategy described above and in Item 7d of Table 8 was invoked, preventing a more thorough exploration of design space than would have occurred in the absence of failure of pre-buckling Newton convergence. The numbers in black correspond to occasions when the strategy described above was invoked for **perturbed** designs, that is, when the index, IMODX = 1. During the SUPEROPT run a try7.OPM file is generated at Iteration No. 8 that contains text analogous to the following:

```
********* WARNING **********
 IMODX=0 means current design, IMODX=1 means perturbed design: IMODX= 1
 Decision variable candidates, HEIGHT,RINNER,ROUTER,TINNER,TOUTER=
  2.658000E+01  2.695000E+00  3.630000E+00  4.213000E-03  3.609900E-03
 TFINNR,TFOUTR,TFWEBS=  1.0900E-02  4.0290E-03  2.6280E-03

 Failed to find the pre-buckling "fixed" load B solution.
 The maximum displacement in sub-step no. 1 or 2 is zero.
 This is probably caused by a nearly singular matrix.
 Raise the values of HEIGHT and the thicknesses, and
 also raise the lower bounds of these quantities, and
 try the run again.
 IABORT is now set to 1: IMODX= 1

 WEIGHT and TOTMAS and EIGMIN are being set equal to large
 numbers. If you want a plot of the objective edit the *.PL5
 file by removing all entries with very large numbers, then
 execute DIPLOT.
 ********************************

 Absolute values of maximum constraint gradients for each active
constraint:
  6.6667E+17  6.6667E+17  1.5120E+16  1.5120E+16  1.5120E+16
  1.5120E+16  1.5120E+16
```

```
  ADS DID NOT CHANGE THE DESIGN
------------------------------------------------------------------------------------------------------------------
```

Figures 52a,b are analogous to Figs. 51a,b. In this case the SUPEROPT execution was allowed to continue until completion. (About 470 design iterations are processed for a complete execution of SUPEROPT.) The frequent occurrence of the invoking of the "keep-running-in-spite-of-convergence-failure" strategy demonstrates the difficulty that SUPEROPT has of determining a "global" optimum design due to probable lack of adequate coverage of design space during a single execution of SUPEROPT. Of course, SUPEROPT can be executed many times for a more extensive search for a "global" optimum design. However, in this case, the SUPEROPT computer times are very, very long. For example, the case of the spherical shell with 45 modules over 90 degrees of meridian and with truss-like webs required about 72 hours of computer time for a single complete execution of SUPEROPT. Therefore, it was decided to forgo multiple complete executions of SUPEROPT.

Section 9.7 Re-optimization of spherical balloon with 50 modules and radial webs

Tables 18 and 19 pertain to this sub-section. The starting design is the optimum design listed in Table 15. Table 18 lists the new input data for the GENOPT processor called DECIDE. There are two main differences in the input for DECIDE:

1. The lower bounds of all the thicknesses have been changed from 0.002 inch to 0.001 inch.
2. Two inequality relationships have been introduced. These inequalities involve RINNER and ROUTER:

RINNER > (RADIUS x pi/2)/(2 x NMODUL)
ROUTER > [(RADIUS + HEIGHT) x pi/2]/(2 x NMODUL)

in which RADIUS is the radius to the inner wall as shown in Fig. 1 (RADIUS=120 inches), and NMODUL is the number of modules in the BIGBOSOR4 model (NMODUL = 50).

Table 19 lists the results of the re-optimization. The re-optimized weight is 408.9 lb. That new weight is indicated in Fig. 28. The decrease from the previously optimized weight, 421.5 lb, is small. During the re-optimization one of the executions of SUPEROPT aborted early with the following message:

```
************* ABORT **********
 0.5 x FLOUTR is greater than ROUTER
 0.5 x FLOUTR =  2.2484E+00; ROUTER =  2.2313E+00; IMODX= 1
 Put a higher lower bound on ROUTER.
 The run is now aborting.
 ********************************
```

An acceptable optimum design had already been determined. Therefore, no action was taken with regard to the "ABORT" message.

Section 9.8 Design sensitivity of the re-optimized spherical balloon with 50 modules and radial webs with respect to HEIGHT, RINNER, and ROUTER

Figures 53 – 55 pertain to this sub-section. Each figure corresponds to an execution of the GENOPT processor, OPTIMIZE, with use of the analysis type indicator, ITYPE = 3, in the *.OPT file, which contains the input data for the GENOPT processor, MAINSETUP. With ITYPE = 3 GENOPT performs a design sensitivity analysis. Design margins are computed over a range of a user-selected decision variable. The values of all the other decision variables are held constant; only the decision variable selected by the user as having a range of values changes over that range of values during the execution of OPTIMIZE.

Figure 53 was generated over the user-specified range, 15 inches < HEIGHT < 34.6 inches; figure 54 was generated over the user-specified range, 1.886 < RINNER < 2.5 inches; figure 55 was generated over the user-specified range, 2.21 inches < ROUTER < 2.7 inches. The baseline design is the optimum design listed in Table 19. As is typical for optimized designs, the design sensitivity analysis reveals that many design margins are critical or almost critical at the optimum design.

SECTION 10 OPTIMIZATION OF CYLINDRICAL BALLOONS OVER A RANGE IN THE NUMBER OF MODULES, NMODUL, IN 90 DEGREES OF CIRCUMFERENCE (Material = fictitious carbon fiber cloth)

Tables 20 – 22 and Figs. 56 – 67 pertain to this section. This section on cylindrical vacuum chambers is introduced in this paper because the material, fictitious carbon fiber cloth, was not introduced in [1] and because of the many changes in the "balloon" software made since [1] was written.

Section 10.1 Material properties and optimized weights

The material properties are listed in Table 13. Figure 56 shows the weight/length of the optimized cylindrical balloons versus the number of modules over 90 degrees of circumference. Compare Fig. 56 with Fig. 25 of [1]. The lightest design of the cylindrical balloon made of fictitious carbon fiber cloth is about 17 times lighter than the lightest design of the cylindrical balloon made of polyethylene terephthalate. Table 20 lists results corresponding to the lightest design, which has 40 modules over 90 degrees of circumference (Fig. 56).

Section 10.2 Convergence of predictions with respect to the number of nodal points in each segment of the model

This sub-section is analogous to sub-section 9.5, which pertains to the spherical balloons. Only cylindrical balloons are involved here, however.

Tables 21 and 22 and Figs. 57 – 62 pertain to this sub-section. The cylindrical balloon has 15 modules over 90 degrees of circumference and truss-like (slanted) webs. There are two models:

Model 1: There are 31 nodal points [NODSEG = 31 in SUBROUTINE BOSDEC (Table 7)] in each "shell" segment of the 15-module model.

Model 2: There are 97 nodal points [NODSEG = 97 in SUBROUTINE BOSDEC (Table 7)] in each "shell" segment of the 15-module model.

The cylindrical balloon was optimized with the use of Model 1. Table 21 lists part of the try7.OPM file generated from Model 1 for the optimized design. Table 22 lists part of the try7.OPM file generated from Model 2 for the same optimum design, that is, the optimized design that was generated with the use of Model 1. The following conclusions may be drawn from this convergence study:

Item 1. The buckling load factors (eigenvalues) and the load factors that correspond to the initial loss of tension in the skin of the cylindrical balloon are hardly affected at all by the change in the number of nodal points in each segment from 31 to 97. Compare Table 21 with Table 22

From Table 21 (NODSEG = 31 in SUBROUTINE BOSDEC, Table 7):

```
1         2.988862      buckling load factor from BIGBOSOR4: BUCKB4(1 )
2         2.998502      load factor for tension loss: TENLOS(1 )
```

From Table 22 (NODSEG = 97 in SUBROUTINE BOSDEC):

```
1         3.009818      buckling load factor from BIGBOSOR4: BUCKB4(1 )
2         3.009218      load factor for tension loss: TENLOS(1 )
```

Item 2. The pre-buckling deformation corresponding to Model 1 exhibits a small component of spurious "zig-zag" displacement (Fig. 57 and 58). This "zig-zag" component of deformation is absent in Model 2 (Fig. 59).

Item 3. The maximum stresses are NOT significantly affected by the change in the number of nodal points from 31 to 97. This result differs from the analogous finding for the spherical balloon with 15 modules, in which there are a few fairly significant changes in maximum stress components. Compare Table 21 with Table 22:

From Table 21 (NODSEG = 31 in SUBROUTINE BOSDEC, Table 7):

```
PREBUCKLING STRESS RESULTANTS IN THE FIRST MODULE
           "fixed" from Load Step No. 1 total from Load Step No. 2
 Seg.J  Node I  N1FIX(I,J)   N2FIX(I,J)   N1VAR(I,J)   N2VAR(I,J)
    1      1   5.6203E+02   2.0575E+02   4.0705E+02   2.0208E+02
    2      1   5.5239E+02   2.5039E+02   5.0351E+02   2.4923E+02
    3      1   2.1292E+03   7.3509E+02   1.4191E+03   7.1826E+02
    4      1   3.7778E+02   2.1762E+02   3.7790E+02   2.1763E+02
    5      1   4.9727E+02   2.2892E+02   4.5975E+02   2.2803E+02
    6      1   4.9551E+02   2.2887E+02   4.5849E+02   2.2800E+02

 PREBUCKLING MEMBRANE STRESSES IN THE FIRST MODULE COMPUTED FROM
 N1FIX/thickness, N2FIX/thickness, N1VAR/thickness, N2VAR/thickness:
           "fixed" from Load Step No. 1 total from Load Step No. 2
 Seg.J  Node I STRS1F(I,J)  STRS2F(I,J)  STRS1V(I,J)  STRS2V(I,J)
    1      1   1.0425E+05   3.8166E+04   7.5506E+04   3.7484E+04
    2      1   8.3092E+04   3.7664E+04   7.5739E+04   3.7490E+04
    3      1   1.1101E+05   3.8326E+04   7.3989E+04   3.7448E+04
    4      1   6.4623E+04   3.7226E+04   6.4643E+04   3.7227E+04
```

5	1	8.1747E+04	3.7632E+04	7.5579E+04	3.7486E+04
6	1	8.1459E+04	3.7625E+04	7.5373E+04	3.7481E+04

From Table 22 (NODSEG = 97 in SUBROUTINE BOSDEC):

PREBUCKLING STRESS RESULTANTS IN THE FIRST MODULE
"fixed" from Load Step No. 1 total from Load Step No. 2

Seg.J	Node I	N1FIX(I,J)	N2FIX(I,J)	N1VAR(I,J)	N2VAR(I,J)
1	1	5.3560E+02	2.0512E+02	3.8994E+02	2.0167E+02
2	1	5.4833E+02	2.5029E+02	5.0086E+02	2.4917E+02
3	1	2.0354E+03	7.3286E+02	1.3590E+03	7.1683E+02
4	1	3.8110E+02	2.1770E+02	3.7995E+02	2.1768E+02
5	1	4.9593E+02	2.2888E+02	4.5879E+02	2.2800E+02
6	1	4.9564E+02	2.2888E+02	4.5859E+02	2.2800E+02

PREBUCKLING MEMBRANE STRESSES IN THE FIRST MODULE COMPUTED FROM
N1FIX/thickness, N2FIX/thickness, N1VAR/thickness, N2VAR/thickness:
"fixed" from Load Step No. 1 total from Load Step No. 2

Seg.J	Node I	STRS1F(I,J)	STRS2F(I,J)	STRS1V(I,J)	STRS2V(I,J)
1	1	9.9351E+04	3.8049E+04	7.2331E+04	3.7409E+04
2	1	8.2480E+04	3.7649E+04	7.5340E+04	3.7480E+04
3	1	1.0612E+05	3.8210E+04	7.0857E+04	3.7374E+04
4	1	6.5191E+04	3.7240E+04	6.4993E+04	3.7235E+04
5	1	8.1528E+04	3.7627E+04	7.5421E+04	3.7482E+04
6	1	8.1480E+04	3.7626E+04	7.5388E+04	3.7481E+04

Item 4. Although the buckling load factors (eigenvalues) obtained from BIGBOSOR4 are affected only slightly by the change from 31 to 97 nodal points in each segment, there are significant differences in the critical buckling mode shapes. The general buckling mode, which is the critical buckling mode shape obtained from Model 1 and which is displayed in Fig. 60, cannot be found for Model 2. Local buckling mode shapes are found for both Models 1 and 2. Compare Fig. 61 with Fig. 62 for local buckling from Models 1 and 2, respectively.

Item 5. This convergence study in which the number of nodal points is increased from 31 to 97 **in every segment** of the 15-module model of the cylindrical balloon is possible only for models with relatively few modules (15 modules or less). This is because the number of degrees of freedom permitted by BIGBOSOR4 is limited. It is not possible to conduct an analogous convergence study for the optimum design listed in Table 20, for example.

Section 10.3 Pre-buckling and bifurcation buckling behavior of the optimized cylindrical balloon with 40 modules and truss-like (slanted) webs

Figures 63 – 67 pertain to this sub-section. There are 40 modules over 90 degrees of circumference of the cylindrical balloon.

10.3.1 Axisymmetric pre-buckling deformation of the optimized cylindrical balloon under Load B

The axisymmetric pre-buckling deformation under Load B (the "fixed", non-eigenvalues loads, PINNER = 0 psi, PMIDDL = 60 psi, POUTER = 0 psi) is analogous to that displayed in Fig. 57. Figure 63 shows an enlarged view of the axisymmetric pre-buckled deformation in the region near the circumferential angle 0 degrees. This figure is analogous to Fig. 58. There is a similar spurious "zig-zag" component of axisymmetric pre-buckling displacement in both the outermost and innermost walls of the cylindrical balloon.

10.3.2 Local and general buckling mode shapes of the optimized cylindrical balloon

Figures 64 – 67 pertain to the critical local and general buckling mode shapes. Figure 64 shows the local buckling mode shape. This mode shape is the second buckling eigenvector. The first eigenvector is the general buckling mode shape shown in Figs. 65 – 67. The critical general buckling mode displayed in Fig. 65 is unusual in that it has one full circumferential wave over 90 degrees of the circumference rather than the more usual one half circumferential wave as displayed in Fig. 60, for example. The general buckling modes shown in [1] for the cylindrical balloons with truss-like (slanted) webs have only one half circumferential wave over 90 degrees of the circumference (ovalization). (See Figs. 11 and 20 in [1], for examples.)

Figures 66 and 67 show enlarged views of the same general buckling mode as that displayed in Fig. 65. There are pronounced components of spurious "zig-zag" local buckling displacements in both the outermost and innermost walls of the "double-walled" cylindrical balloon near 0 degrees of circumference, and there is a pronounced component of spurious "zig-zag" local buckling displacement in both of the truss-like webs near 22.5 degrees of circumference where the overall normal displacement of the "double-walled" cylindrical balloon is near zero and the overall average rotation of the balloon wall is maximum. Perhaps these spurious components of local buckling displacements superposed on the general buckling mode are responsible for the unexpected overall shape of the general buckling mode displayed in Fig. 65. A search over the lowest 50 eigenvectors of the optimized cylindrical balloon failed to reveal any general buckling mode analogous to that shown in Fig. 60 of this paper and analogous to those in Figs. 11 and 20 of [1].

SECTION 11 CONCLUSIONS

The following conclusions may be drawn:

1. No general buckling mode was ever discovered for the spherical balloons analogous to the general buckling modes found for the cylindrical balloons, such as the modes displayed in Figs. 11 and 20 of [1] and in Figs. 57 and 62 of this paper.

2. In cylindrical balloons the lightest optimized designs are those with truss-like (slanted) webs (Fig. 56). In contrast, with spherical balloons the lightest optimized designs are those with radial webs (Fig. 28).

3. In spite of the fact that the segments of the balloon wall behave like membranes that have no bending stiffness rather than like shells that have a finite bending stiffness, BIGBOSOR4, a shell-of-revolution code, which is designed to handle segmented shell structures with finite bending stiffness, seems to be capable of solving both the nonlinear pre-buckling phase and the linear bifurcation buckling phase of the analysis with adequate accuracy for the purpose of preliminary design. However, read the next item.

4. For the balloons made of carbon fiber cloth, both the axisymmetric pre-buckling deformation (Figs. 33a, 34, 35, 55) and the bifurcation buckling modal deformation (Figs. 30, 32, 60) of optimized balloons may exhibit significant components of spurious local "zig-zag" deformation. The amplitude of this spurious component decreases with increasing number of nodal points in each "shell" segment (Fig. 59). Convergence studies indicate that the presence of these spurious local "zig-zag" components does not seriously affect the overall pre-buckling predictions (Figs. 36, 39, Tables 16, 17, 21, 22) and bifurcation buckling predictions (Tables 16, 17, 21, 22) for the spherical and cylindrical balloons. Therefore, it appears that the optimum designs obtained here are valid even though the number of nodal points used in each "shell" segment in the optimization model (31 nodal points, as specified in SUBROUTINE BOSDEC – Table 7) leads to the generation of significant local "zig-zag" deformations in the optimized balloons made of carbon fiber cloth.

5. The balloons made of the fictitious carbon fiber cloth are a factor of 15 – 20 times lighter than those fabricated with polyethylene terephthalate. For optimized spherical balloons compare Fig. 28 with Fig. 23. For optimized cylindrical balloons compare Fig. 56 with Fig. 25 of [1].

6. A strategy is established by means of which failure of convergence of the nonlinear pre-buckling analysis is minimized. This strategy is described in [1].

7. A strategy is established by means of which failure of convergence of the nonlinear pre-buckling analysis does not cause early termination of an execution of SUPEROPT. This strategy is described in Item 7d of Table 8 and in Section 7. For a spherical balloon with truss-like webs this strategy may be invoked many times during an execution of SUPEROPT (Figs. 52a and 52b), which makes it difficult to find a "global" optimum design.

8. Stress components in the various segments of a module are computed from membrane theory, that is, the stress component is equal to the appropriate stress resultant divided by the thickness of the segment (Item No. 11 in Table 9). This is an unconservative strategy because there exist large meridional stress concentrations in the immediate neighborhoods of segment junctions (Fig. 44). An actual balloon fabricated in a configuration that corresponds to an optimized design developed here by GENOPT/BIGBOSOR4 should therefore have reinforced seams at the junctions between segments. The analysis of balloons with reinforced seams is beyond the scope of this paper.

9. The new "initial loss of tension" behavioral constraint called "TENLOS", which is a predictor of initial wrinkling of the skin in one or more of the segments of the model of the balloon, is close in value to the bifurcation buckling constraint, BUCKB4, as it should be, since wrinkling is a type of buckling. See Item 1 in Section 9.5 for a spherical balloon and Item 1 in Section 10.2 for a cylindrical balloon.

10. The capability to analyze and design multi-walled spherical and cylindrical vacuum chambers (balloons) is established within the GENOPT/BIGBOSOR4 framework. Enough information is provided in this paper and in [1], [3], [6 – 9], and [13] so that researchers can use GENOPT/BIGBOSOR4 to analyze and design other shell structures of a similar nature.

11. The "balloon" software, behavior.balloon and bosdec.balloon, has been modified from that listed in Tables 5 and 7 of [1]. Now both cylindrical and spherical balloons can be handled and each of these types of balloons can have either radial or truss-like (slanted) webs. Tables 5 and 7 of [1] are now out-of-date.

SECTION 12 ACKNOWLEDGMENT

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