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Optimization of Propellant Tanks Supported by One or Two Optimized Laminated Composite Skirts

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The propellant tank is a shell of revolution completely filled with liquid hydrogen (LH2). This propellant tank is to be launched into space. During launch it is subjected to high axial and lateral accelerations. The tank is supported by one or two conical skirts each of which consists of five segments: two short segments near each end of the skirt and a central long segment that has a laminated composite wall. Each of the short segments nearest the ends of the skirt has an isotropic one-layered wall with tapered thickness. Each short segment next to each short end segment is multi-layered with the extreme layers consisting of tapered isotropic material and the remaining internal, contained layers consisting of the same laminated composite wall as the long central segment. This tank/skirt system is optimized via GENOPT/BIGBOSOR4 in the presence of two loading cases: (1) 10 g axial acceleration and 0 g lateral acceleration and (2) 0 g axial acceleration and 10 g lateral acceleration. In addition to the g-loading the tank has 25 psi internal ullage pressure, the tank wall is 200 degrees cooler than the wall of the launch vehicle from which it is supported by the conical skirt(s), and there exists axisymmetric meridionally non-uniform cooling of the skirts. In the BIGBOSOR4 modal vibration model the mass of the propellant is "lumped" into the tank wall, a conservative model. The tank/skirt system, a multi-segment branched shell of revolution, is optimized in the presence of the following constraints: (1) the minimum modal vibration frequency of the tank/skirt(s) system must be greater than a given value; (2) five stress components in each ply of the laminated composite wall of the conical skirt(s) shall not exceed five specified allowables; (3) the conical skirt(s) shall not buckle; (4) the maximum effective (von Mises) stress in the tank wall shall not exceed a specified value; (5) the tank wall shall not buckle. The objective to be minimized is in general a weighted combination of the normalized mass of the empty tank plus the normalized conductance of the support system: Objective = W x (normalized empty tank mass) + (1-W) x (normalized strut conductance), in which W is a user-selected weight between 0.0 and 1.0. Two propellant tank/skirt systems are optimized: (1) a long tank with only one supporting skirt joined to the tank at the midlength of the tank and (2) the same long tank with two supporting skirts, an aft skirt and a forward skirt. It is emphasized that the tank/skirt(s) combination is optimized as a single branched shell of revolution. The flexibility of the launch vehicle to which the tank/skirt(s) system is attached is neglected: the ends of the supporting skirt(s) attached to the launch vehicle are assumed to be attached to rigid "ground". Linear theory is used throughout. Predictions for the optimized tank/skirt designs obtained here are compared with those from the general-purpose finite element code, STAGS. The agreement between the predictions of GENOPT/BIGBOSOR4 and STAGS qualifies the use of GENOPT/BIGBOSOR4 for preliminary design in the particular cases studied here.

Section 1. INTRODUCTION AND SOME GEOMETRICAL DETAIL

This paper is analogous to the paper on optimization of propellant tanks supported by struts [1] and is an updated and shortened version of the full unpublished report in [2]. Please read [1] and the documents referenced there and in [2] for additional background information.

In this work the **generic** case is called "**tank2**" and the two **specific** cases optimized here are called "**oneskirt**" and **“twoskirt”**. (See [1] and below for the meaning of “generic case” and “specific cases”.) The variable names, definitions, roles and properties established by the GENOPT user via the GENOPT processor, GENTEXT, for the generic case called “tank2” are listed in Table 1. The "oneskirt" specific case has only one supporting skirt (Figs. 1 – 7 and Tables 2 and 3). The specific “twoskirt” case has two supporting skirts, one aft and the other forward (Figs. 8 – 17 and Tables 4 – 7). In order to permit the optimization of tanks with two skirts, aft and forward, GENOPT [3] had to be modified rather extensively to permit more than 50 decision variable candidates.

Much of the technology on which this work is based is described in [1] and in the papers, reports and files referenced in [1] and [2]. The overall aspect of the geometry of the propellant tank is the same as that described for what is called “the long propellant tank” in [1]. (See Figs. 1a-1c in [1].) In the “oneskirt” case the conical skirt is attached to the tank at the axial coordinate that corresponds to the midlength of the cylindrical part of the propellant tank (Fig. 1a in this paper). In the "twoskirt" case the conical skirts are attached to the propellant tank at the aft and forward junctions of the cylindrical part of the tank with the aft and forward ellipsoidal end domes (Figs. 8a, 9 and 11 in this paper).

The geometry of each tank/skirt junction is similar, but not the same, as that shown in the sketch near the beginning of [1]. (Henceforth the name, “tank/skirt” becomes “tank2/skirt”.) That sketch in [1] is repeated here with modification of the geometry at the tank2/skirt junction. **The tank end of the skirt coincides with the reference surface of the propellant tank shell wall**, not with the centroid of the external propellant tank support ring as is the case for the tank supported by struts. The word, “strut”, is replaced by the word, “skirt”.

```
   .---tank shell reference surface
  .
  |
|||
|||\
||| \<---tapered external doubler
|||  \
||------------
||      .    | <--discrete ring of rectangular cross section
||\\----------
|||\\/
||| \\
|||/ \\<--aft skirt at its propellant tank end. The last nodal
|||   \\  point on the reference surface of the aft skirt lies
|||    \\ on the reference surface of the propellant tank.
|||
```

|

Sketch of the propellant tank wall with a local reinforcement at the axial location where the **tank-end of the aft conical skirt is attached to the propellant tank reference surface**. Note: in the above sketch the innermost "layer" of the propellant tank, which consists of an orthogrid with "smeared" stringers and rings, is not shown. (See Fig.1c of [1] for a sketch that shows this innermost orthogrid “layer”.)

The conical **aft** skirt consists of five segments as shown below and in Figs. 1a and 8a. If there exist two conical skirts, in the **forward** skirt the segment numbering is reversed, with the first nodal point in segment 1 of the forward skirt attached to the forward part of the propellant tank reference surface and the last nodal point in segment 5 of the forward skirt attached to “ground” (the launch vehicle). The forward conical skirt slants the other way, that is, the forward propellant tank support ring is at the lowest axial coordinate of the forward skirt (small-diameter end of the forward skirt) and the “ground” support is at the highest axial coordinate of the skirt (large-diameter end of the forward skirt), as shown in Figs. 8a and 9. In Fig. 8a segment 1 of the forward skirt is segment 34 of the BIGBOSOR4 model of the entire tank2/skirt system, and segment 5 of the forward skirt is segment 38 of the BIGBOSOR4 model of the entire tank2/skirt system. Here is a schematic of the 5-segment **aft** skirt:

```
| <-- propellant tank reference surface to which a ring is attached
 \
  \  <-- aft skirt seg.5 (one tapered isotropic layer)
   \
 ------ <--wall thicknesses of seg.4 & seg.5 match
     \
      \  <--aft skirt seg.4 (n+2 layers; the outer and
       \      inner layers are tapered isotropic and
    -------     the interior n layers are the same as
        \        in segment 3). See Fig.1a.
         \
          \
           \
            \
             \
              \  <--aft skirt seg.3 (n composite plies
               \     as with each strut tube described
                \     in [1])
                 \
                  \
                   \
              ------ <--wall thicknesses of
                  \       segment 2 and segment 3 match
                   \  <--aft skirt seg.2 (n+2 layers,
                    \     n internal composite plies)
                  ------ <-- wall thicknesses of
                      \        seg. 1 and seg. 2 match
                       \ <--aft skirt seg.1 (one tapered
                        \            isotropic layer)
                      ------ "ground" (launch vehicle)
```

The conical **aft** skirt support consists of five segments, two short segments at each end and a central long segment. For a more representative schematic of the aft skirt, see the right-hand side of Fig. 1a and Fig. 1b.

Segment 2 of the **aft** skirt (a segment with tapered “prongs”) has the wall shown schematically here:

```
                   |               |
                   |    Seg. 3     |
Segment 3 ---->    |  n composite  |
                   |     plies     |
                   |               |
                   -----------------
                  /|               |\
Segment 2 ---->  / |               | \
                /  |               |  \
inner tapered  /   |               |   \ <--outer tapered
isotropic     /    |               |    \   isotropic layer
layer-----> /      |  n composite  |     \  called "prong"
called      /      |  plies, as    |      \ below
"prong"    /       |  with Seg.3   |       \
below     /        |               |        \
         /         |               |         \
        ----------------------------------------
        Segment 1: one-layered, tapered isotropic
```

Schematic of wall constructions of **aft skirt** segments 2 and 3. [Also see Figs. 1a and 1b(A).]

Segment 4 of the **aft** skirt (a segment with tapered “prongs”) has the wall shown schematically here:

```
        Segment 5: one-layered, tapered isotropic

          ____________________________________
          \        |               |        /
           \       |               |       /
            \      |  n composite  |      /
             \     |  plies, as    |     /
inner tapered \    |  with Seg.3   |    / <--outer tapered
isotropic layer\   |               |   /    isotropic layer
                \  |               |  /     called "prong"
Segment 4 ---->  \ |               | /      below
                  \|               |/
                   -----------------
                   |               |
                   |    Seg. 3     |
Segment 3 ---->    |  n composite  |
                   |    plies      |
```

Schematic of wall constructions of **aft skirt** segments 3 and 4. [Also see Figs. 1a and 1b(B).]

The shape of the propellant tank (left-hand side of Fig. 1a) is the same as that of the long propellant tank displayed in Figs. 1a and 4 of [1]. The propellant tank has three parts: aft ellipsoidal dome, middle cylindrical part and forward ellipsoidal dome. In the particular cases explored here the ellipsoidal domes have a ratio 2:1 major axis/minor axis. As listed in Table 2, the overall dimensions of the propellant tank (length and radius) are the same as those listed in Table 2 of [1]. The wall of the propellant tank is modeled as consisting of three layers as described in [1]: the inner layer represents the internal orthogrid, the middle layer represents the skin, and the outer layer represents the tapered external doublers that reinforce the tank2/skirt junction(s). The geometry of the reinforcement of the propellant tank at tank2/skirt junctions is the same as that displayed in Figs. 1b and 1c of [1] (with different segment numbering from that in Fig. 1b of [1]). The same material properties in the tank wall and skirt are used here as those in the tank wall and struts described in [1]. See Table 2. The two loading cases are the same here as those used in [1]. The run stream used to obtain the optimized configuration is analogous to that described in [1] and listed in [2].

The decision variable candidates are the same as those described in [1] except that in the generic case called "**tank2**" there exist the following additional decision variable candidates (Fig. 1b of this paper):

1. tank-end length of one-layered skirt part: LNGTNK1(i)
2. tank-end thickness of tapered skirt part: THKTNK1(i)
3. tank-end length of tapered prongs: LNGTNK2(i)
4. tank-end maximum thickness of one tapered prong: THKTNK2(i)
5. "ground" end length of one-layered skirt part: LNGVEH1(i)
6. "ground"-end thickness of tapered skirt part: THKVEH1(i)
7. "ground"-end length of tapered prongs: LNGVEH2(i)
8. "ground"-end maximum thickness of one tapered prong: TNKVEH2(i)

in which i = 1 for the aft skirt and i = 2 for the forward skirt, if any. These decision variable candidates are indicated in Fig. 1b. **A "decision variable candidate" is a problem variable associated with Role No. 1**. See Table 1.

If there exists only one skirt (as in the specific case called "oneskirt") or in the aft-most skirt in the "twoskirt" configuration, the "tank-end" quantities correspond to skirt segments 4 and 5. (See the right-hand side of Fig. 1a.) The "ground-end" quantities correspond to skirt segments 1 and 2. By "length" is meant "slant length", that is, the length measured along the meridian of the conical reference surface as shown in Fig. 1b. The extreme layers of Segments 2 and 4 are of the same isotropic material (aluminum in the particular cases treated in this paper) as that of Segments 1 and 5, and the propellant tank is also made of that same material. The maximum thicknesses and slant lengths of the metallic tapered "prongs" at the outer and inner conical shell wall surfaces in a given skirt segment are the same. The "prongs" in skirt Segment 2 may have different dimensions from the "prongs" in skirt Segment 4 (Figs. 1a and 1b). The purpose of the tapered metallic prongs is to contain the composite laminate at either end of the skirt. This geometry has been used previously in successful Lockheed Martin projects (e.g. the WISE project [9]).

The layup(s) of the walls of the laminated composite skirt(s) is (are) analogous to that (those) of the laminated composite strut tubes described in [1]. (See the integers, LAYTYP(i,1), listed in Table 2 of [1] and in Table 2 of this paper; the 12-layered composite laminate is symmetric.) The behaviors accounted for during optimization cycles are analogous to those described in [1] except there is no “BEHAVIOR (3)” of [1]: buckling of the skirt as a column; there is no “BEHAVIOR (7)” of [1]: maximum force in a skirt during the launch-hold phase of a mission; and for “BEHAVIOR (5)” of [1] and “BEHAVIOR (6)” of [1] the tank2/skirt(s) system is always analyzed as a single branched multi-segment shell of revolution, that is, there are no models in which the skirt(s) is (are) replaced by loads applied by the skirt(s) to the propellant tank as is the case for the strut supports described in [1]. In the “tank2” BIGBOSOR4 model the skirts are **always** included as part of the entire tank2/skirt structure, which is a branched, segmented shell of revolution. There are no springs in the tank2/skirt model. Therefore, the tank2/skirt system is much simpler than the tank/strut system of [1].

As with the tank/strut system [1], there are two load cases that are identical (with one exception described in the next paragraph) to those specified in the short section in [1] entitled, “Section 8. TWO LOAD CASES”: Load Case 1 = 10g axial acceleration plus 25 psi ullage pressure plus 200-degree propellant tank cool-down, and Load Case 2 = 10g lateral acceleration plus 25 psi ullage pressure plus 200-degree propellant tank cool-down. As in [1] the acceleration is in Load Set A (“eigenvalue” loading) and the ullage pressure plus thermal loading are in Load Set B (“fixed” loading, that is, loading not to be multiplied by the eigenvalue).

There is one significant difference in the thermal loading in the tank2/skirt model described here from that in the tank/strut model described in [1]. In the tank2/skirt model there exists, in addition to the uniform 200-degree propellant tank cool-down, an axisymmetric, **meridionally non-uniform**, cool-down of each skirt. This non-uniform skirt cool-down is –200 degrees in the two short segments nearest the propellant tank and decays to 0 degrees along the meridian of Segment 3 of the skirt(s). This additional thermal loading in the tank2/skirt system is part of Load Set B. Input data for BIGBOSOR4 pertaining to this additional axisymmetric non-uniform thermal loading of the aft and forward skirts are listed in Appendix 3.

Section 2. INFORMATION ABOUT THE GENERIC CASE CALLED "tank2"

In the GENOPT universe there are two types of cases:

(1) **A generic case** (called **"tank2"** in this paper)

(2) **Specific cases** that fit within the generic set. These specific cases are called **“oneskirt”** (Fig. 1a) and **“twoskirt”** (Fig. 8a) in the work reported here.

Corresponding to each of the two classes of case, generic and specific, there are possibly different users. The role of the **GENOPT user** is to create the software for setting up the GENERIC environment **("tank2")**. The **End user** exercises the GENERIC environment, "**tank2**", for SPECIFIC cases, such as the cases called **“oneskirt”** and **“twoskirt”**. The GENOPT user and the End user in the work reported here are the same person: the first author of this paper.

The following files pertain to the GENERIC case, **"tank2"**. The files listed next are contained in the compressed "tar" file, tank2.tar.gz, which is contained in the bigger compressed “tar” file, …/genopt/case/tank/tanktank2.tar.gz, which is part of the very big compressed “tar” file that can be downloaded from the “Downloads” page of the “shellbuckling.com” website [2]. Some of the files contained in the “tank2.tar.gz” file are as follows:

```
-----------------------------------------------------------------
FILES RELATED TO THE GENOPT USER'S "GENERIC CASE" PHASE
OF THIS PROJECT

  86604 Feb 18 2013 behavior.tank2                ("fleshed-out" version)
 180559 Nov  4 2012 bosdec.tank2                  ("permanent" version)
  27044 Feb 18 2013 struct.tank2                  ("fleshed-out" version)
  37971 Feb 18 2013 tank2.DEF                     (general information)
 121887 Feb 18 2013 tank2.INP                     (input for GENTEXT)
  47462 Feb 18 2013 tank2.PRO  (Table 2 of [2]) (prompting file)
   8725 Feb 18 2013 tank2.glossary (Table 1)    (glossary of variables)
 681340 May 18 2012 addbosor4.tank2.density.var ("temporary" BIGBOSOR4)
 680079 Jun 27 2012 addbosor4.regular           ("permanent" BIGBOSOR4)
 181111 Nov  4 2012 bosdec.tank2.density.var    ("temporary" version)
-------------------------------------------------------
```

The purposes of these files are analogous to the purposes of the analogous files described in [1].

The first sentence in the abstract reads, “The propellant tank is a shell of revolution completely filled with liquid hydrogen (LH2).” However, in the GENOPT/TANK2 software the tank can be filled with any fluid. The effect of the fluid is introduced by means of its weight density, which is called “DENPRP” in Tables 1 and 2.

As in [1], the mass of the propellant is "lumped" into the middle layer of the three-layered propellant tank. In a general shell of revolution the amount of propellant mass to be "lumped" into the propellant tank shell wall middle layer at a nodal point in a shell segment depends on the radius from the axis of revolution and on the rate of change of this radius with meridional arc length [2]. Hence, in the aft and forward ellipsoidal domes the effective density of the middle tank layer varies along the meridian of each dome shell segment. However, the “permanent” (stand alone) version of BIGBOSOR4 cannot handle shell segments with meridionally varying material density within a single shell segment. Therefore, "temporary" versions of BIGBOSOR4 and BOSDEC were created that are valid only for the generic case, "tank2". As reported in [2], optimized designs were evaluated with either the "temporary" or the "permanent" versions of BIGBOSOR4 and BOSDEC. In the work reported in this paper only the “temporary” versions of BIGBOSOR4 and BOSDEC were used for optimization. These “temporary” versions of BIGBOSOR4 and BOSDEC are embodied in the files called “addbosor4.tank2.density.var” and “bosdec.tank2.density.var”, respectively [2]. The "stand-alone" version of BIGBOSOR4 was not changed in any way. More complete explanations are given in [1] and [2]. Directions for how to establish the proper files and for how to run the generic case, tank2, and a specific case (twoskirt) are given in Appendix 1. Directions for how to generate plots of vibration modes such as those displayed in Figs. 4 and 9 are given in Appendix 2.

Section 3. DECISION VARIABLE CANDIDATES AND THE OPTIMIZED “oneskirt” DESIGN

Decision variable candidates, that is, problem variables associated with Role 1 in Table 1, are listed here for the case called **“oneskirt”**. In this case the laminated composite section of the skirt has a symmetric angle-ply layup, as is the case for each strut tube described in [1]. As in [1], for each laminated composite skirt there is only one ply thickness that is a decision variable. All other ply thicknesses are linked to that ply thickness in the particular configurations described in this paper. The values of the decision variable candidates, behaviors, design margins, and objective listed next are those corresponding to the best optimum design determined after two successive executions of the GENOPT processor called "SUPEROPT", which is described in [1]. Figures 2 and 3 show the evolution of the objective during the SUPEROPT process.

The following lists of optimum design, behaviors, margins and objective are part of the complete and rather long file called "oneskirt.OPM". The oneskirt.OPM file is included in the compressed file, tank2.tar.gz [2], that contains many other files that document the generic case called "tank2" and the specific cases called "oneskirt" and "twoskirt".

Decision variable candidates for the optimized design of the specific case, "oneskirt"

```
================================================================================
  VALUES OF DESIGN VARIABLES CORRESPONDING TO BEST FEASIBLE DESIGN
 VAR.   CURRENT
 NO.     VALUE            DEFINITION
  1    1.749E-01  thickness of the tank aft dome skin: THKAFT
  2    5.716E-02  thickness of the tank cylinder skin: THKMID
  3    8.477E-02  thickness of the forward tank dome skin: THKFWD
  4    4.708E+00  spacing of the tank orthogrid stringers: STRSPC
  5    4.708E+00  spacing of the tank orthogrid rings: RNGSPC
  6    5.657E-01  thickness of the tank orthogrid stringers: STRTHK
  7    4.000E-01  height of the tank orthogrid stringers: STRHI
  8    5.657E-01  thickness of the tank orthogrid rings: RNGTHK
  9    4.000E-01  height of the tank orthogrid rings: RNGHI
 10    3.000E+02  global axial coordinate of tank support ring: ZTANK(1 )
 11    2.000E+01  global axial coordinate of "ground": ZGRND(1 )
 12    3.000E+01  axial length of the propellant tank doubler: DUBAXL(1 )
 13    4.072E-02  max.thickness of the propellant tank doubler: DUBTHK(1 )
 14    5.901E-02  thickness of the tank reinforcement ring: TRNGTH(1 )
 15    2.951E-01  height of the tank reinforcement ring: TRNGHI(1 )
 16    2.000E+00  tank-end length of one-layered skirt part: LNGTNK1(1 )
 17    1.000E-01  tank-end thickness of tapered skirt part: THKTNK1(1 )
 18    2.000E+00  tank-end length of tapered prongs: LNGTNK2(1 )
 19    3.000E-02  tank-end thickness of one tapered prong: THKTNK2(1 )
 20    2.000E+00  "ground" end length of one-layered skirt part: LNGVEH1(1 )
 21    1.000E-01  "ground"-end thickness of tapered skirt part: THKVEH1(1 )
 22    2.000E+00  "ground"-end length of tapered prongs: LNGVEH2(1 )
 23    3.000E-02  "ground"-end thickness of one tapered prong: THKVEH2(1 )
 24    2.911E-02  thickness of a lamina: THICK(1 )
 25    2.911E-02  thickness of a lamina: THICK(2 )
 26    2.911E-02  thickness of a lamina: THICK(3 )
 27    2.911E-02  thickness of a lamina: THICK(4 )
 28    2.911E-02  thickness of a lamina: THICK(5 )
 29    2.911E-02  thickness of a lamina: THICK(6 )
 30    8.000E+01  layup angle: ANGLE(1 )
 31   -8.000E+01  layup angle: ANGLE(2 )
 32    3.058E+01  layup angle: ANGLE(3 )
 33   -3.058E+01  layup angle: ANGLE(4 )
 34    8.000E+01  layup angle: ANGLE(5 )
 35   -8.000E+01  layup angle: ANGLE(6 )
================================================================================
```

Behaviors, design margins and objective of the optimized tank2/skirt system for the specific case, "oneskirt" (Critical and almost critical design margins are listed in bold face.)

```
================================================================================

***** RESULTS FOR LOAD CASE NO.  1  ****** (10 g axial, 0 g lateral, etc.)
PARAMETERS WHICH DESCRIBE BEHAVIOR (e.g. frequencies, stress, buckling load)
 BEH.   CURRENT
 NO.     VALUE            DEFINITION
  1    2.354E+01   modal vibration frequency (cps): FREQ(1 ,1 ) (n=0, eig. no. 1)
  2    1.245E+01   modal vibration frequency (cps): FREQ(1 ,2 ) (n=1, eig. no. 1)
  3    3.148E+01   modal vibration frequency (cps): FREQ(1 ,3 ) (n=0, eig. no. 2)
  4    1.640E+01   modal vibration frequency (cps): FREQ(1 ,4 ) (n=1, eig. no. 2)
  5    3.354E+04   maximum stress in the propellant tank: TNKSTR(1 ,1 )
  6    3.354E+04   maximum stress in the propellant tank: TNKSTR(1 ,2 )
  7    6.789E+00   propellant tank buckling load factor: TNKBUK(1 ,1 )
  8    6.789E+00   propellant tank buckling load factor: TNKBUK(1 ,2 )

***** RESULTS FOR LOAD CASE NO.  1  ****** (10 g axial, 0 g lateral, etc.)
 MARGINS CORRESPONDING TO CURRENT DESIGN (F.S.= FACTOR OF SAFETY)

 MARGIN CURRENT
 NO.     VALUE            DEFINITION
  1    9.617E-01  (FREQ(1 ,1 )/FREQA(1 ,1 )) / FREQF(1 ,1 )-1; F.S.=  1.20
  2    3.748E-02  (FREQ(1 ,2 )/FREQA(1 ,2 )) / FREQF(1 ,2 )-1; F.S.=  1.20
  3    1.623E+00  (FREQ(1 ,3 )/FREQA(1 ,3 )) / FREQF(1 ,3 )-1; F.S.=  1.20
  4    3.668E-01  (FREQ(1 ,4 )/FREQA(1 ,4 )) / FREQF(1 ,4 )-1; F.S.=  1.20
  5   -6.101E-03  (TNKSTRA(1 ,1 )/TNKSTR(1 ,1 )) / TNKSTRF(1 ,1 )-1; F.S.=  1.50
  6   -6.101E-03  (TNKSTRA(1 ,2 )/TNKSTR(1 ,2 )) / TNKSTRF(1 ,2 )-1; F.S.=  1.50
  7    2.394E+00  (TNKBUK(1 ,1 )/TNKBUKA(1 ,1 )) / TNKBUKF(1 ,1 )-1; F.S.=  2.00
  8    2.394E+00  (TNKBUK(1 ,2 )/TNKBUKA(1 ,2 )) / TNKBUKF(1 ,2 )-1; F.S.=  2.00


***** RESULTS FOR LOAD CASE NO.  2  ****** (0 g axial, 10 g lateral, etc.)
PARAMETERS WHICH DESCRIBE BEHAVIOR (e.g. frequencies, stress, buckling load)
BEH.   CURRENT
 NO.     VALUE            DEFINITION
  1    2.354E+01   modal vibration frequency (cps): FREQ(2 ,1 ) (n=0, eig. no. 1)
  2    1.245E+01   modal vibration frequency (cps): FREQ(2 ,2 ) (n=1, eig. no. 1)
  3    3.148E+01   modal vibration frequency (cps): FREQ(2 ,3 ) (n=0, eig. no. 2)
  4    1.640E+01   modal vibration frequency (cps): FREQ(2 ,4 ) (n=1, eig. no. 2)
  5    3.333E+04   maximum stress in the propellant tank: TNKSTR(2 ,1 )
  6    3.248E+04   maximum stress in the propellant tank: TNKSTR(2 ,2 )
  7    1.992E+00   propellant tank buckling load factor: TNKBUK(2 ,1 )
  8    1.994E+00   propellant tank buckling load factor: TNKBUK(2 ,2 )

***** RESULTS FOR LOAD CASE NO.  2  ****** (0 g axial, 10 g lateral, etc.)
 MARGINS CORRESPONDING TO CURRENT DESIGN (F.S.= FACTOR OF SAFETY)
 MARGIN CURRENT
 NO.     VALUE            DEFINITION
```

```
  1     9.617E-01  (FREQ(2 ,1 )/FREQA(2 ,1 )) / FREQF(2 ,1 )-1; F.S.=  1.20
  2     3.748E-02  (FREQ(2 ,2 )/FREQA(2 ,2 )) / FREQF(2 ,2 )-1; F.S.=  1.20
  3     1.623E+00  (FREQ(2 ,3 )/FREQA(2 ,3 )) / FREQF(2 ,3 )-1; F.S.=  1.20
  4     3.668E-01  (FREQ(2 ,4 )/FREQA(2 ,4 )) / FREQF(2 ,4 )-1; F.S.=  1.20
  5     5.007E-06  (TNKSTRA(2 ,1 )/TNKSTR(2 ,1 )) / TNKSTRF(2 ,1 )-1; F.S.=  1.50
  6     2.618E-02  (TNKSTRA(2 ,2 )/TNKSTR(2 ,2 )) / TNKSTRF(2 ,2 )-1; F.S.=  1.50
  7    -3.754E-03  (TNKBUK(2 ,1 )/TNKBUKA(2 ,1 )) / TNKBUKF(2 ,1 )-1; F.S.=  2.00
  8    -2.950E-03  (TNKBUK(2 ,2 )/TNKBUKA(2 ,2 )) / TNKBUKF(2 ,2 )-1; F.S.=  2.00
================================================================================
```

NOTE: After the results listed above were obtained the strategy with regard to the computation of the vibration frequencies was modified. Instead of computing the first and second eigenvalues corresponding to n = 0 and n = 1 circumferential waves, the “tank2” subroutines for modal vibration frequencies were modified to compute only the first eigenvalue for n = 0 and 1 and 2 and 3 or 4 circumferential waves (whichever eigenvalue is smaller for n = 3 and n = 4 circumferential waves). For example, for Load Case 1 Behaviors 1 – 4 are now as follows:

```
BEH.    CURRENT
 NO.      VALUE            DEFINITION
  1     2.354E+01   modal vibration frequency (cps): FREQ(1 ,1 ) (n=0, eig. no. 1)
  2     1.245E+01   modal vibration frequency (cps): FREQ(1 ,2 ) (n=1, eig. no. 1)
  3     1.570E+01   modal vibration frequency (cps): FREQ(1 ,3 ) (n=2, eig. no. 1)
  4     1.509E+01   modal vibration frequency (cps): FREQ(1 ,4 ) (n=3, eig. no. 1)
```

Load Case 1 Behaviors 1 and 2 are the same as before, but Behaviors 3 and 4 have different values because they now correspond to different vibration modes. Load Case 1 Design margins 3 and 4 are now correspondingly different, as follows:

```
MARGIN  CURRENT
 NO.      VALUE            DEFINITION
  3     3.083E-01  (FREQ(1 ,3 )/FREQA(1 ,3 )) / FREQF(1 ,3 )-1; F.S.=  1.20
  4     2.572E-01  (FREQ(1 ,4 )/FREQA(1 ,4 )) / FREQF(1 ,4 )-1; F.S.=  1.20
```

(end of NOTE)

Notice that in this "oneskirt" case the modal vibration frequencies are independent of the loading, and the margins corresponding to maximum stress in the tank2/skirt system, TNKSTR(i,j) are critical for both Load Case 1 and Load Case 2. (i=Load Case; j = meridian selected for stress) Also, buckling of the tank2/skirt system, TNKBUK(i,j), is critical for Load Case 2 but not critical for Load Case 1. The name of the buckling variable, TNKBUK, remains the same as in the generic case called "tank" [1]. However, in the generic case called "tank2" TNKBUK signifies the buckling of the entire tank2/skirt systems, not buckling of just the propellant tank under the loads applied by the skirts to the tank plus the externally applied loads: acceleration, internal pressure, and thermal. In other words, in the generic case, "tank2", the tank/skirt system is a single shell of revolution; buckling may occur either in one or more of the segments of the propellant tank shell wall or in one or more of the segments of the supporting skirt(s). It turns out that for the optimized designs presented here buckling of the supporting skirt(s) is always the most critical. Also, **notice from Table 2 that in the work reported in this paper the factors of safety associated with TNKSTR(i,j) are TNKSTRF(i,j) =1.5 and the factors of safety associated with TNKBUK(i,j) are TNKBUKF(i,j) = 2.0, whereas in Table 2 of [1] these factors of safety are equal to 1.0.**

```
******************** DESIGN OBJECTIVE *******************
CURRENT VALUE OF THE OBJECTIVE FUNCTION:
 VAR.   CURRENT
```

```
 NO.     VALUE             DEFINITION
  1    2.466E+00  WGTxTOTMAS/TNKNRM +(1-WGT)xCONDCT/CONNRM: CONDCT

in which

WGT, TOTMAS, TNKNRM, CONDCT, CONNRM =
  5.0000E-01  2.3794E+01  1.0000E+01  5.1062E-03  2.0000E-03
```

Section 4. ADDITIONAL RESULTS FOR THE GENERIC CASE, "tank2", AND THE SPECIFIC CASE, "oneskirt"

The glossary of variable names and definitions for the generic case called "tank2" is given in Table 1 of this paper. Table 2 of [2] lists the prompting file, tank2.PRO, generated automatically by the GENOPT processor called "GENTEXT" with use of the GENOPT user's input listed in the tank2.INP file (input for GENTEXT [1, 2]).

Input data for the specific case, "oneskirt", are listed in Table 2 of this paper and in Tables 4, 5 and 8 of [2]. Results from the optimized design are listed in Table 6 of [2], and the optimum design is archived in the “oneskirt.OPM” file (Table 7 of [2]). The discretized BIGBOSOR4 model of the optimized "oneskirt" design is displayed on the left-hand side of Fig. 1a. Figure 2 shows the evolution of the objective versus design iterations during the first execution of the GENOPT processor, SUPEROPT, with use of the “temporary” versions of bosdec (bosdec.tank2.density.var [2]) and addbosor4 (addbosor4.tank2.density.var [2]). Figure 3 shows the same during the second execution of SUPEROPT. The starting design for the second execution of SUPEROPT is the same as the “best” design determined after completion of the first execution of SUPEROPT.

Figure 4 shows vibration modes and frequencies from the optimized “oneskirt” design obtained with use of both the “permanent” and “temporary” versions of bosdec and addbosor4 [2]. The modal vibration frequencies corresponding to the two shell deformation modes [Figs. 4(E) and 4(F)] are essentially the same with use of either the “permanent” or “temporary” versions of bosdec and addbosor4 because there is little motion of the two end domes in these two vibration modes, and the two end domes are the only parts of the structure for which the “effective” material density of the middle layer of the shell wall varies along the meridian. (“effective” density = density from the lumped propellant mass plus density of the actual shell wall material.)

Figures 5 (A, B) show “oneskirt” prebuckled states from (A): Load Case 1 (10g axial acceleration plus 25 psi internal pressure plus thermal loading) and (B): Load Case 2 (10g lateral acceleration plus 25 psi internal pressure plus thermal loading), and Fig. 5(C) shows the critical buckling mode and load factor for Load Case 2 **corresponding to the prebuckling loading along the meridian at circumferential coordinate, theta = 0.** As explained in more detail below, the critical buckling mode and load factor for Load Case 2 corresponding to the prebuckling loading along the meridian at circumferential coordinate, theta = 90 degrees, (where the in-plane shear loading of the skirt is maximum) is not shown because it is obtained from a PANDA-type of analysis [4], not from a BIGBOSOR4 model. Nonetheless, this theta-=-90-degree critical buckling margin is computed and is represented by Margin No. 8 under Load Case 2 in the list above included as part of Section 3:

```
8   -2.950E-03  (TNKBUK(2 ,2 )/TNKBUKA(2 ,2 )) / TNKBUKF(2 ,2 )-1; F.S.=  2.00
```

Figures 6 and 7 each show buckling modes predicted by two different "oneskirt" BIGBOSOR4 models, (A) and (B), of the tank2/skirt system corresponding respectively to Load Case 1 (Fig. 6) and Load Case 2 (Fig. 7). In the first model for each load case, (A), the nodal point spacing is concentrated near both ends of Segment 3 of the skirt, as displayed on the left-hand side of Fig. 1a. [Fig. 6(A) and Fig. 7(A)]. In the second model, (B), the nodal point spacing is constant in Segment 3 of the skirt [Fig. 6(B) and Fig. 7(B)].

The buckling mode shapes and load factors given in both Figs. 6 and 7 correspond to the prebuckled state along the meridian at circumferential coordinate, theta = 0. **In Load Case 2 (lateral acceleration) the lowest buckling load factor actually corresponds to the prebuckling conditions along the meridian at circumferential coordinate, theta = 90 degrees, where in-plane shear loading of the skirt is maximum.** However, BIGBOSOR4 cannot compute buckling of shells under in-plane shear loading. Nor does BIGBOSOR4 include the effects of anisotropy (the D16 and D26 terms in the 6 x 6 integrated constitutive matrix) or transverse shear deformation (t.s.d.). Therefore, an approximate PANDA-type model [4], which does include these loadings, properties and behavior, is introduced as described next.

Buckling load factors are obtained from both the BIGBOSOR4 model and an approximate PANDA-type model [4]. The approximate PANDA-type model [4] is described in Tables 5 and 6, both of which pertain to the "twoskirt" configuration. However, although the numbers are different for the "twoskirt" configuration from those for the "oneskirt" configuration, the characteristics of the PANDA-type model govern both the "oneskirt" and "twoskirt" configurations. As seen from Figs. 6 and 7, for the optimized "oneskirt" design the approximate PANDA-type model corresponding to the prebuckled state along the meridian at circumferential coordinate, theta = 0 degrees predicts lower buckling load factors than does the BIGBOSOR4 model. **The GENOPT/tank2 software computes the design margins corresponding to buckling of the tank2/skirt system as the more critical (smallest) of the buckling load factors predicted by the BIGBOSOR4 model of the entire tank2/skirt system and the PANDA-type model of Segment 3 of the aft skirt by itself, loaded as if it were embedded in the BIGBOSOR4 model of the entire tank2/skirt system.**

Load Case 2 (lateral acceleration) gives rise to significant prebuckling in-plane shear loading in the skirt that is maximum at circumferential coordinate, theta = 90 degrees. As mentioned above, BIGBOSOR4 cannot handle buckling under in-plane shear loading, anisotropic shell walls, or transverse shear deformation. The approximate PANDA-type model [4] does include these loadings, properties and effects. Corresponding to Load Case 2 and the prebuckled state along the meridian at circumferential coordinate, theta = 90 degrees, the output file, oneskirt.OPM, for the optimized "oneskirt" design listed in Section 3 includes the following lines:

```
PANDA2-type buckling predictions corresponding to the
 prebuckled state along the skirt meridian at theta=  9.0000E+01
degrees...
Buckling load factor predicted from a PANDA2-type model of
 the aft skirt for Load Case  2:
 Average axial resultant over meridian length,NXAVE =  2.2641E-04
 Average hoop  resultant over meridian length,NYAVE = -4.0780E-06
 Average in-plane shear resultant,            NXYAVE=  7.3884E+02
 Average Load Set B prebuckling resultants used in the PANDA2 model:
 Average axial resultant over meridian length,NXFIX = -3.4180E-03
 Average hoop  resultant over meridian length,NYFIX =  6.5536E-01
 Length of the equivalent cylindrical shell,  FLEFF =  2.8443E+02
 Radius of the equivalent cylindrical shell,  RAVE  =  1.2698E+02
```

```
Critical buckling mode:
Number of axial half-waves over shell length,MSKIN =   1
Number of circ. half-waves over 180 degrees, NSKIN =  10
Slope of the buckling nodal lines,           SLOPE =  2.7372E-01
SLOPE=dy/dx in Fig. 9(b) of 1987 "Theoretical Basis paper [4].
"Bump-up" factor to adjust for milder imperfection
 sensitivity factor when in-plane shear is significant  1.5000E+00
The actual buckling load factor,EIGLOC*FKNOCK/RATIO=  1.3294E+00
```

In the approximate PANDA-type model [4] the conical skirt is treated as an **"equivalent" cylindrical shell** with length, FLEFF, (close to 284 inches) and radius, RAVE, (close to 127 inches) and subjected to **uniform** stress resultants from Load Set A ("eigenvalue" loads), NXAVE, NYAVE, NXYAVE (lb/in), and Load Set B ("fixed" loads), NXFIX and NYFIX (lb/in). The buckling load factor predicted from the approximate PANDA-type theory is **1.3294**, significantly lower than the buckling load factors displayed in Fig. 7, which correspond to prebuckling loading along the meridian at circumferential coordinate, theta = 0. It is this "shear" buckling at theta = 90 degrees that gives rise to the eighth design margin listed in Section 3 under Load Case 2 and repeated here:

8   -2.950E-03 (TNKBUK(2 ,2)/TNKBUKA(2 ,2)) / TNKBUKF(2 ,2)-1; F.S.= 2.00

in which the meanings of "i" and "j" in TNKBUK(i, j) are: i = 2 means "Load Case 2"; j = 2 means "prebuckling conditions along the tank2/skirt wall Meridian No. 2" (circumferential coordinate, theta = 90 degrees in this case). That eighth margin is computed as follows:

(buckling load factor) x ("bump-up" factor)/(factor of safety) – 1.0 = 1.3294 x 1.5/2.0 – 1.0 = -0.00295

The purpose of the "bump-up" factor is given in the footnote to Table 6, which pertains to the "twoskirt" case.

Table 3 lists the maximum stress components from Load Case 1 and Load Case 2 in the specific case called "oneskirt". The following comments apply:

1. The "oneskirt" BIGBOSOR4 shell-of-revolution model shown on the left-hand side of Fig. 1a has 33 segments. Segments 1 – 5 are in the skirt; Segments 6 – 17 are in the aft ellipsoidal dome of the tank; Segments 18 – 21 are in the cylindrical part of the tank; Segments 22 – 33 are in the forward ellipsoidal dome of the tank. The skirt segments 1 – 5 are identified in Fig. 1a and schematically in one of the sketches in the section entitled, "Section 1. INTRODUCTION AND SOME GEOMETRICAL DETAIL". Segment 19 is the very short (15-inch axial length) cylindrical shell segment that includes the lower half of the external tapered doubler (layer 3 of the propellant tank wall), and Segment 20 is the very short (15-inch axial length) cylindrical shell segment that includes the upper half of this external tapered doubler. The entire 30-inch-wide external tapered doubler, the axial midlength of which corresponds to the midlength of the cylindrical part of the propellant tank in this "oneskirt" model, is depicted in the first sketch in the section entitled, "Section 1. INTRODUCTION AND SOME GEOMETRICAL DETAIL". Segments 18 and 21 represent most of the axial length of the cylindrical shell where there is no external doubler.

2. The stress component labeled "effect. Stress" (meaning "von Mises effective stress") corresponds to the isotropic skin and external tapered doubler of the tank and to the material of the skirt that is isotropic, such as Segments 1 and 5 and the extreme layers of Segments 2 and 4 that represent the tapered metallic prongs (right-hand side of Fig. 1a).

3. "matl=3" pertains to the smeared internal orthogrid (represented by layer 1 of the propellant tank wall).

4. In Table 3 the letter, " A ", means "Load Set A" ("eigenvalue" load), "z" is the thickness coordinate, negative for a point inside of the shell reference surface, and "FS" means "Factor of Safety".

5. Notice that for all stress components the factor of safety is 1.5. In the tank2/skirt system this means that the factor of safety for stresses in the propellant tank is higher than it is in the case of the tank/strut system [1], in which the factor of safety for stress in the propellant tank is specified as 1.0 (Table 2 of [1]). This significant difference in the two models, tank/strut and tank2/skirt, means that the optimized objectives should not be compared in the particular cases explored in this paper with regard to whether it is best to support the tank with struts or with skirt(s).

Section 5. STILL MORE RESULTS FOR THE SPECIFIC CASE, "oneskirt"

The optimum design listed under the heading, "**Section 3. Optimized 'variable density' design of the specific case, 'oneskirt'**", includes the following lines:

```
VALUES OF DESIGN VARIABLES CORRESPONDING TO BEST FEASIBLE DESIGN
 VAR.   CURRENT
 NO.     VALUE            DEFINITION
 11    2.000E+01  global axial coordinate of "ground": ZGRND(1 )
```

In this particular case, at the optimum design the decision variable, ZGRND(1), is at its lower bound: 20 inches. The corresponding optimized objective is equal to 2.466. If the lower bound of ZGRND(1) is set much lower, –200 inches, for example, and if the tank2/skirt system is re-optimized via the GENOPT processor, SUPEROPT, the new optimum design has ZGRND(1) = -2.851 inches and the new optimized objective is very slightly less than 2.466: 2.458. (The other decision variables in the re-optimized tank2/skirt system have also changed from those listed above, of course. They are not given in this paper.)

It is of interest to perform several "oneskirt" optimizations, each optimization with use of a different fixed value of the variable, ZTANK(1). In the baseline optimum design listed above under the heading, "**Section 3. Optimized 'variable density' design of the specific case, 'oneskirt'**", the variable, ZTANK(1), defined as "global axial coordinate of tank support ring", is equal to 300 inches. ZTANK(1) is a decision variable candidate but is usually not a decision variable in this particular case. The value, ZTANK(1) = 300 inches, corresponds to the mid-length of the cylindrical part of the propellant tank. The following "oneskirt" optimized results are obtained for various values of **ZTANK(1) = "global axial coordinate of tank support ring**":

ZTANK(1) (inches)	Objective (normalized)
150	3.221 ←ZTANK(1) is at the aft dome/cylinder junction
300	2.458 ←ZTANK(1) is at the midlength of the propellant tank
365.8	**2.456 ← In this instance ZTANK(1) was chosen as one of the decision variables.**
425	2.632 ←ZTANK(1) is 25 inches below the forward dome/cylinder junction
450	2.774 ←ZTANK(1) is at the forward dome/cylinder junction

In all the “oneskirt” cases just listed the single skirt support slants as shown in Fig. 1a, that is, the axial coordinate of the propellant tank support ring, ZTANK(1), is always greater than the axial coordinate of the support at rigid ground, ZGRND(1). In fact, in the GENOPT/tank2 model the aft skirt, skirt no. 1, **always** slants as shown in Fig. 1a [ZTANK(1), ZGRND(1)], and the forward skirt, skirt no. 2 (if any) **always** slants in the opposite sense, as shown in the top left portion of Fig. 8a, for example [ZTANK(2), ZGRND(2)].

Section 6. RESULTS FOR THE SPECIFIC CASE, "twoskirt"

The left-hand side of Fig. 8a shows the BIGBOSOR4 model of the optimized “twoskirt” tank2/skirt system, with shell segment numbering as indicated. Input data for the specific case, "twoskirt" are listed in the files, twoskirt.BEG, twoskirt.DEC, and twoskirt.OPT of [2]. As with the specific case, “oneskirt”, the optimum “twoskirt” design is obtained with use of the “temporary” versions of bosdec (bosdec.tank2.density.var [2]) and addbosor4 (addbosor4.tank2.density.var [2]). Figure 8b shows the evolution of the objective during the first execution of the GENOPT processor, SUPEROPT.

Figure 9 shows vibration modes and frequencies from the GENOPT/BIGBOSOR4 model of the optimized “twoskirt” design obtained with use of both the “permanent” and “temporary” versions of bosdec and addbosor4 [2]. The modal vibration frequencies corresponding to the three shell deformation modes [Figs. 9(E), 9(F) and 9(G)] are essentially the same with use of either the “permanent” or “temporary” versions of bosdec and addbosor4 because there is little “rigid-body-like” motion (rolling, axial motion, lateral-tilting motion) of the propellant tank in these three “shell deformation” vibration modes, and the modal deformations in the two end domes are very small.

Figure 10a shows (A) prebuckled states from Load Case 1 (10g axial acceleration plus 25 psi internal pressure plus thermal loading) and (B) Load Case 2 (10g lateral acceleration plus 25 psi internal pressure plus thermal loading) and (C) the critical buckling mode and load factor for Load Case 2 **as loaded by the distribution of prebuckling stress resultants along the meridian at circumferential coordinate, theta = 0.**

Table 4 lists the maximum stress components from Load Case 1 and Load Case 2 as predicted by the GENOPT/BIGBOSOR4 “twoskirt” model. The comments about Table 3 (the “oneskirt” example) apply also to Table 4, except there are more shell segments in the “twoskirt” case than in the “oneskirt” case because there are two supporting skirts (Fig. 8a) rather than only one supporting skirt (Fig. 1a). Also, the nature of the walls of the shell segments differs in the cylindrical part of the tank and in the segments in the aft and forward domes adjacent to the cylindrical part of the tank. In the BIGBOSOR4 “twoskirt” shell-of-revolution model, which is displayed on the left-hand side of Fig. 8a, Segments 1 – 5 are in the aft skirt; Segments 6 – 17 are in the aft ellipsoidal dome of the tank; Segments 18 – 21 are in the cylindrical part of the tank; Segments 22 – 33 are in the forward ellipsoidal dome of the tank; Segments 34 – 38 are in the forward skirt. In the “twoskirt” model the 30-inch-wide external tapered doublers are approximately centered on the aft and forward dome/cylinder junctions. (See Figs. 1b and 1c in [1] for the geometry but not for the segment numbering.) Segment 17 (the shell segment in the aft ellipsoidal dome that contains the equator of the aft ellipsoidal dome) includes the lower less-than-15-inch meridional arc length of the aft external tapered doubler, and Segment 18 (the shell segment in the cylindrical part that is adjacent to the equator of the aft ellipsoidal dome) includes the upper 15-inch axial length of the aft external tapered doubler. Similarly, Segment 21 (the shell segment in the cylindrical part that is adjacent to the equator of the forward ellipsoidal dome) includes the lower 15-inch axial length of the forward external tapered doubler, and Segment 22 (the shell segment in the forward ellipsoidal dome that contains the equator of the forward ellipsoidal dome) includes the upper less-than-15-inch meridional arc length of the forward external tapered doubler. (See Fig. 1b of [1] for the geometry, not for the segment numbering. The meridional arc lengths of the parts of the tapered doublers in the ellipsoidal shell segments, 17 and 22, are the same and are equal to 7.15 inches.) Segments 19 and 20, the middle two long segments of the cylindrical part of the tank, represent the lower and upper long sections of the cylindrical part of the propellant tank where there is no tapered doubler.

The optimized “twoskirt” design, behaviors, margins, and objective are as follows:

Optimized decision variable candidates for the case called "twoskirt"

```
 ==========================================================================
VALUES OF DESIGN VARIABLES CORRESPONDING TO BEST ALMOST FEASIBLE DESIGN
 VAR.   CURRENT
 NO.     VALUE            DEFINITION
  1    1.2983E-01  thickness of the tank aft dome skin: THKAFT
  2    1.1820E-01  thickness of the tank cylinder skin: THKMID
  3    8.9186E-02  thickness of the forward tank dome skin: THKFWD
  4    5.1335E+00  spacing of the tank orthogrid stringers: STRSPC
  5    5.1335E+00  spacing of the tank orthogrid rings: RNGSPC
  6    3.1920E-01  thickness of the tank orthogrid stringers: STRTHK
  7    4.0000E-01  height of the tank orthogrid stringers: STRHI
  8    3.1920E-01  thickness of the tank orthogrid rings: RNGTHK
  9    4.0000E-01  height of the tank orthogrid rings: RNGHI
 10    1.5000E+02  global axial coordinate of tank support ring: ZTANK(1 )
 11    4.5000E+02  global axial coordinate of tank support ring: ZTANK(2 )
 12    2.0000E+01  global axial coordinate of "ground": ZGRND(1 )
 13    6.0231E+02  global axial coordinate of "ground": ZGRND(2 )
 14    3.0000E+01  axial length of the propellant tank doubler: DUBAXL(1 )
 15    5.4026E-01  max.thickness of the propellant tank doubler: DUBTHK(1 )
 16    1.3335E-01  thickness of the tank reinforcement ring: TRNGTH(1 )
 17    6.6675E-01  height of the tank reinforcement ring: TRNGHI(1 )
 18    2.8160E+00  tank-end length of one-layered skirt part: LNGTNK1(1 )
 19    2.0000E+00  tank-end length of one-layered skirt part: LNGTNK1(2 )
 20    1.6617E-01  tank-end thickness of tapered skirt part: THKTNK1(1 )
 21    1.7310E-01  tank-end thickness of tapered skirt part: THKTNK1(2 )
 22    3.2489E+00  tank-end length of tapered prongs: LNGTNK2(1 )
 23    2.0000E+00  tank-end length of tapered prongs: LNGTNK2(2 )
 24    3.4118E-02  tank-end thickness of one tapered prong: THKTNK2(1 )
 25    1.1634E-02  tank-end thickness of one tapered prong: THKTNK2(2 )
 26    2.0000E+00  "ground" end length of one-layered skirt part: LNGVEH1(1 )
 27    2.0000E+00  "ground" end length of one-layered skirt part: LNGVEH1(2 )
 28    1.5309E-01  "ground"-end thickness of tapered skirt part: THKVEH1(1 )
 29    1.0000E-01  "ground"-end thickness of tapered skirt part: THKVEH1(2 )
 30    2.0000E+00  "ground"-end length of tapered prongs: LNGVEH2(1 )
 31    2.0000E+00  "ground"-end length of tapered prongs: LNGVEH2(2 )
 32    1.0000E-02  "ground"-end thickness of one tapered prong: THKVEH2(1 )
 33    1.1497E-02  "ground"-end thickness of one tapered prong: THKVEH2(2 )
 34    8.9935E-03  thickness of a lamina: THICK(1 )
 35    8.9935E-03  thickness of a lamina: THICK(2 )
 36    8.9935E-03  thickness of a lamina: THICK(3 )
 37    8.9935E-03  thickness of a lamina: THICK(4 )
```

```
 38    8.9935E-03  thickness of a lamina: THICK(5 )
 39    8.9935E-03  thickness of a lamina: THICK(6 )
 40    9.5511E-03  thickness of a lamina: THICK(7 )
 41    9.5511E-03  thickness of a lamina: THICK(8 )
 42    9.5511E-03  thickness of a lamina: THICK(9 )
 43    9.5511E-03  thickness of a lamina: THICK(10)
 44    9.5511E-03  thickness of a lamina: THICK(11)
 45    9.5511E-03  thickness of a lamina: THICK(12)
 46    7.7875E+01  layup angle: ANGLE(1 )
 47   -7.7875E+01  layup angle: ANGLE(2 )
 48    1.0000E+01  layup angle: ANGLE(3 )
 49   -1,0000E+01  layup angle: ANGLE(4 )
 50    1.0000E+01  layup angle: ANGLE(5 )
 51   -1.0000E+01  layup angle: ANGLE(6 )
 52    8.0000E+01  layup angle: ANGLE(7 )
 53   -8.0000E+01  layup angle: ANGLE(8 )
 54    1.0000E+01  layup angle: ANGLE(9 )
 55   -1.0000E+01  layup angle: ANGLE(10)
 56    1.0000E+01  layup angle: ANGLE(11)
 57   -1.0000E+01  layup angle: ANGLE(12)
==================================================================
```

Behaviors and design margins of the optimized tank2/skirt system for the specific case, "twoskirt" (Critical and almost critical design margins are listed in bold face.)

```
==================================================================

***** RESULTS FOR LOAD CASE NO.  1  (axial acceleration of 10g) ******
PARAMETERS WHICH DESCRIBE BEHAVIOR (e.g. stress, buckling load)
 BEH.   CURRENT
 NO.     VALUE           DEFINITION
  1    2.422E+01   modal vibration frequency (cps): FREQ(1 ,1 )
  2    1.851E+01   modal vibration frequency (cps): FREQ(1 ,2 )
  3    5.055E+01   modal vibration frequency (cps): FREQ(1 ,3 )
  4    2.394E+01   modal vibration frequency (cps): FREQ(1 ,4 )
  5    3.358E+04   maximum stress in the propellant tank: TNKSTR(1 ,1 )
  6    3.358E+04   maximum stress in the propellant tank: TNKSTR(1 ,2 )
  7    5.630E+00   propellant tank buckling load factor: TNKBUK(1 ,1 )
  8    5.630E+00   propellant tank buckling load factor: TNKBUK(1 ,2 )

***** RESULTS FOR LOAD CASE NO.  1  (axial acceleration of 10g) ******
 MARGINS CORRESPONDING TO CURRENT DESIGN (F.S.= FACTOR OF SAFETY)
 MARGIN CURRENT
 NO.     VALUE           DEFINITION
  1    1.019E+00  (FREQ(1 ,1 )/FREQA(1 ,1 )) / FREQF(1 ,1 )-1; F.S.=  1.20
  2    5.424E-01  (FREQ(1 ,2 )/FREQA(1 ,2 )) / FREQF(1 ,2 )-1; F.S.=  1.20
  3    3.213E+00  (FREQ(1 ,3 )/FREQA(1 ,3 )) / FREQF(1 ,3 )-1; F.S.=  1.20
  4    9.954E-01  (FREQ(1 ,4 )/FREQA(1 ,4 )) / FREQF(1 ,4 )-1; F.S.=  1.20
  5   -7.441E-03  (TNKSTRA(1 ,1 )/TNKSTR(1 ,1 )) / TNKSTRF(1 ,1 )-1; F.S.=  1.50
  6   -7.441E-03  (TNKSTRA(1 ,2 )/TNKSTR(1 ,2 )) / TNKSTRF(1 ,2 )-1; F.S.=  1.50
  7    1.815E+00  (TNKBUK(1 ,1 )/TNKBUKA(1 ,1 )) / TNKBUKF(1 ,1 )-1; F.S.=  2.00
  8    1.815E+00  (TNKBUK(1 ,2 )/TNKBUKA(1 ,2 )) / TNKBUKF(1 ,2 )-1; F.S.=  2.00
```

```
***** RESULTS FOR LOAD CASE NO.  2  (lateral acceleration of 10g) ******
PARAMETERS WHICH DESCRIBE BEHAVIOR (e.g. stress, buckling load)
 BEH.   CURRENT
 NO.     VALUE           DEFINITION
  1    2.422E+01   modal vibration frequency (cps): FREQ(2 ,1 )
  2    1.851E+01   modal vibration frequency (cps): FREQ(2 ,2 )
  3    5.055E+01   modal vibration frequency (cps): FREQ(2 ,3 )
  4    2.394E+01   modal vibration frequency (cps): FREQ(2 ,4 )
  5    3.341E+04   maximum stress in the propellant tank: TNKSTR(2 ,1 )
  6    3.463E+04   maximum stress in the propellant tank: TNKSTR(2 ,2 )
  7    2.185E+00   propellant tank buckling load factor: TNKBUK(2 ,1 )
  8    1.947E+00   propellant tank buckling load factor: TNKBUK(2 ,2 )

***** RESULTS FOR LOAD CASE NO.  2  (lateral acceleration of 10g) ******
 MARGINS CORRESPONDING TO CURRENT DESIGN (F.S.= FACTOR OF SAFETY)
 MARGIN CURRENT
 NO.     VALUE           DEFINITION
  1    1.019E+00  (FREQ(2 ,1 )/FREQA(2 ,1 )) / FREQF(2 ,1 )-1; F.S.=  1.20
  2    5.424E-01  (FREQ(2 ,2 )/FREQA(2 ,2 )) / FREQF(2 ,2 )-1; F.S.=  1.20
  3    3.213E+00  (FREQ(2 ,3 )/FREQA(2 ,3 )) / FREQF(2 ,3 )-1; F.S.=  1.20
  4    9.954E-01  (FREQ(2 ,4 )/FREQA(2 ,4 )) / FREQF(2 ,4 )-1; F.S.=  1.20
  5   -2.425E-03  (TNKSTRA(2 ,1 )/TNKSTR(2 ,1 )) / TNKSTRF(2 ,1 )-1; F.S.=  1.50
  6   -3.750E-02  (TNKSTRA(2 ,2 )/TNKSTR(2 ,2 )) / TNKSTRF(2 ,2 )-1; F.S.=  1.50
  7    9.239E-02  (TNKBUK(2 ,1 )/TNKBUKA(2 ,1 )) / TNKBUKF(2 ,1 )-1; F.S.=  2.00
  8   -2.673E-02  (TNKBUK(2 ,2 )/TNKBUKA(2 ,2 )) / TNKBUKF(2 ,2 )-1; F.S.=  2.00
==============================================================
```

Note: The footnote under the "oneskirt" case applies also to this "twoskirt" case, with different values for Behaviors 1 - 4 and Design Margins 3 and 4 from those values listed in the footnote for the "oneskirt" case.

Optimized objective:

```
===============================================================

******************* DESIGN OBJECTIVE ******************
CURRENT VALUE OF THE OBJECTIVE FUNCTION:
 VAR.   CURRENT
 NO.     VALUE           DEFINITION
  1    2.824E+00  WGTxTOTMAS/TNKNRM +(1-WGT)xCONDCT/CONNRM: CONDCT

in which

WGT, TOTMAS, TNKNRM, CONDCT, CONNRM =
  5.0000E-01  1.7310E+01  1.0000E+01  7.8326E-03  2.0000E-03
 ==============================================================
```

Figure 11 shows (A) buckling of the entire tank2/skirt system under Load Case 1 according to the BIGBOSOR4 model of the optimized structure and (B) buckling of the aft skirt modeled by itself under the axial compression experienced by the aft skirt as embedded in the entire tank2/skirt system. For Load Case 1 the BIGBOSOR4 prediction agrees well with the prediction from the PANDA-type model [4]. From Fig. 11(B) it is seen that there is very little difference between the BIGBOSOR4 predictions from linear and nonlinear theory. Also, there is very little difference between the prediction for buckling from the BIGBOSOR4 model of the entire tank2/skirt system and the prediction from the BIGBOSOR4 model of the aft skirt by itself. The two different BIGBOSOR4 models, (A) and (B), are both included here because, for Load Case 1, STAGS is capable of obtaining a non-spurious buckling mode and load factor only for a nonlinear finite element model of the aft skirt by itself, as shown in Fig. 14b. STAGS predictions are described in the next section.

Tables 5 and 6 describe the PANDA-type model [4] and give predictions for Load Case 1 (Table 5) and Load Case 2 (Table 6) from the PANDA-type model of the specific case, twoskirt. Prediction of the critical buckling load factor, 5.946, from the BIGBOSOR4 model is also listed for Load Case 1 near the beginning of Table 5.

Section 7. COMPARISON WITH PREDICTIONS FROM STAGS FOR THE SPECIFIC CASE CALLED "TWOSKIRT"

Modal vibration

The modal vibration predictions from the BIGBOSOR4 model of the tank2/skirt system shown in the top row of Fig. 9 should be compared with the predictions from STAGS ([5 – 8], Appendix 2 of [1]) displayed in Fig. 12a (the STAGS finite element model) and Fig. 12b, in which are displayed the four vibration modes and frequencies predicted by STAGS that, except for (B), involve significant "rigid-body-like" motions of the propellant tank [Fig12b(A) tank rolling and (C, D) two tank lateral-pitching motions]. STAGS predictions of vibration modes analogous to those displayed in the lower row of Fig. 9 are not given in this paper.

Stress in the propellant tank skin and at the tips of the internal orthogrid stringers

The prediction from the BIGBOSOR4 model for Load Case 1 [Fig. 10a(A)] should be compared with the prediction from STAGS displayed in Figs. 13a and 13b. The prediction from the BIGBOSOR4 model for Load Case 2 [Fig. 10a(B)] should be compared with the prediction from STAGS displayed in Figs. 13c and 13d. According to the STAGS model, the highest **compressive** stresses at the tips of the internal orthogrid stringers (narrow blue bands in Figs. 13b and 13d) occur at the junctions between the end domes and the ends of the cylindrical part of the propellant tank. The GENOPT/BIGBOSOR4 model does not exhibit these high **compressive** stresses at the orthogrid stringer tips at that location because in that model the external doubler is tapered with maximum thickness, DUBTHK = 0.54 inch, whereas the external doubler in the STAGS model is of constant thickness = 0.27 inch. The constant-thickness doubler was used in the STAGS application in order to avoid the need to introduce user-written subroutines.

Tapered doublers with maximum thickness = 0.54 inch versus doublers of constant average thickness = 0.27 inch

In the STAGS model there are no tapered thicknesses, whereas in the GENOPT/BIGBOSOR4 model the external doublers are doubly tapered as shown in the sketch near the beginning of this paper. Figure 10b shows the prebuckling deformation under **Load Case 2** (10 g lateral acceleration plus 25 psi internal pressure plus thermal loading) of the top part of the propellant tank with two different GENOPT/BIGBOSOR4 models: (A) model with the doubly tapered external doubler approximately centered on the forward dome/cylinder junction and (B) model with an external doubler of constant thickness equal to the average thickness of the tapered doubler. **Model (B) simulates the STAGS model of the propellant tank shown in Fig. 13d**. Notice that there is much more local meridional bending of the propellant tank wall at the forward dome/cylinder junction shown in Fig. 10b(B) than that shown in Fig. 10b(A). The maximum **compressive** stress at the tips of the internal orthogrid stringers computed from the GENOPT/BIGBOSOR4 Model (B) is almost three times that shown in Model (A): –43000 psi for Model (B) versus only –16000 psi for Model (A). The GENOPT/BIGBOSOR4 Model (B) prediction is in reasonably good agreement with the maximum **compressive** stress predicted by STAGS and displayed in Fig. 13d. **The dramatic differences in deformation displayed in Figs. 10b(A) versus 10b(B) with the associated dramatic difference in the prediction of maximum compressive stress at the tips of the stringers of the internal orthogrid "layer" of the shell wall strongly indicate the need for the introduction of tapered doublers into the STAGS model.**

Various comments about the STAGS model and STAGS predictions
Table 7 and Figs. 12a, 12b, 13a – 13d, Fig. 14a and Fig. 14b, and Figs. 15 - 17 contain:

1. the STAGS finite element model (Fig. 12a). The STAGS 410 finite element is used throughout. The internal orthogrid is represented by smeared stringers and rings via an option in which the internal orthogrid is modeled as a shell wall "layer" with appropriate thickness, material stiffnesses and density as was done in [1]. The STAGS literal "smeared stiffener" input option is not used.

2. the STAGS prediction of vibration modes and frequencies that correspond to modes in which there is overall "rigid-body-like" movement of the propellant tank (Fig. 12b). There is very good agreement between the predictions of STAGS and GENOPT/BIGBOSOR4 with use of the temporary versions of addbosor4 (addbosor4.tank2.density.var [2]) and bosdec (bosdec.tank2.density.var [2]) in the GENOPT/BIGBOSOR4 model. There is significant "rigid-body-like" movement of the propellant tank in (A) (rolling motion), (C) (lateral/pitching mode 1) and (D) (lateral/pitching mode 2). In (B) there appears to be very little tension/compression of the skirts, that is, very little approximately rigid body axial motion of the propellant tank. Instead, the vibration mode in (B) consists mostly of axisymmetric deformation of the end domes with significant local deformation in the neighborhoods of the aft (local outward deformation) and forward (local inward deformation) dome/cylinder junctions. This mode is similar to that predicted by GENOPT/BIGBOSOR4 and displayed in Fig. 9(B).

3. the STAGS prediction of stress in the skin of the propellant tank under Load Case 1 (Fig. 13a). Compare with the GENOPT/BIGBOSOR4 predictions given in Fig. 10a(A) and in the top part of Table 4.

4. the STAGS prediction of stress at the internal orthogrid stringer tips of the propellant tank under Load Case 1 (Fig. 13b). Compare with the GENOPT/BIGBOSOR4 predictions given in Fig. 10a(A) and in the top part of Table 4 that correspond to Material Type 3 (matl=3). There is reasonably good agreement between STAGS and GENOPT/BIGBOSOR4 predictions of maximum **tensile stress** at the stringer tips in the knuckle regions of the ellipsoidal domes (red bands). However, STAGS predicts much higher maximum **compressive stress** at the tips of the stringers of the internal orthogrid at the dome/cylinder junctions (narrow blue bands in Fig. 13b) than does GENOPT/BIGBOSOR4 because the constant thickness doubler used in the STAGS model has half the maximum thickness of the tapered doubler used in the GENOPT/BIGBOSOR4 model at the dome/cylinder junctions. This difference in modeling leads to far more local meridional bending at the dome/cylinder junctions in the STAGS model than in the GENOPT/BIGBOSOR4 model, giving rise to much higher and very local **compressive** stress at the orthogrid stringer tips predicted by the STAGS model.

5. the STAGS prediction of stress in the skin of the propellant tank under Load Case 2 (Fig. 13c). Compare with the GENOPT/BIGBOSOR4 predictions given in Fig. 10a(B) and in the bottom part of Table 4.

6. the STAGS prediction of stress at the internal orthogrid stringer tips of the propellant tank under Load Case 2 (Fig. 13d). Compare with the GENOPT/BIGBOSOR4 predictions given in Fig. 10a(B) and in the bottom part of Table 4. The same comments apply here as those just given in Item 4. Figure 10b shows the deformations of the forward part of the propellant tank predicted by the GENOPT/BIGBOSOR4 model with (A) the tapered doubler and (B) the doubler of constant thickness (thus simulating the STAGS model). The dramatic difference in local meridional bending in the neighborhood of the dome/cylinder junction is obvious. The prediction of maximum compressive stress at the tips of the internal orthogrid stringers from the GENOPT/BIGBOSOR4 model with a constant thickness doubler [close to –43000 psi obtained from the model shown in Fig. 10b(B)] is significantly more than the prediction from STAGS (close to –35000 psi as shown in Fig. 13d) because the axial component of the finite element mesh density in the STAGS model is not sufficient to capture accurately the axially very local compressive stress concentrations at the internal orthogrid stringer tips at the aft and forward dome/cylinder junctions (extremely thin blue lines in Fig. 13d). Experiments with the GENOPT/BIGBOSOR4 model with the constant thickness doubler demonstrate that whether the doubler is tapered or of constant thickness has negligible effect on the modal vibration frequencies and buckling load factors. Nonetheless, a STAGS model should be constructed with tapered doublers and with tapered ends of the supporting skirts. In all the STAGS models processed during this project all the tapered thicknesses are replaced by parts of constant thickness in which that constant thickness is equal to half the maximum thickness of the corresponding tapered parts of the propellant tank and its supporting skirt(s). This was done in order to avoid the need to introduce user-written subroutines in the STAGS models.

7. a table of STAGS predictions of stresses in Segments 3 of the aft and forward skirts from STAGS and GENOPT/TANK2 (Table 7). Skirt Segment 3 of each conical support is the long laminated composite segment shown on the right-hand side of Fig. 1a, for example. The prediction of maximum stress in the laminated composite parts of the skirts from GENOPT/BIGBOSOR4 is about 15 per cent higher than that from STAGS for Load Case 1 and about 8 per cent higher for Load Case 2.

8. a STAGS spurious buckling mode from Load Case 1 (Fig. 14a). The presence of spurious buckling modes such as this made it impossible to determine buckling mode shapes and load factors for the optimized “twoskirt” design with the use of the linear bifurcation buckling option (INDIC = 1) in STAGS. Therefore, the nonlinear equilibrium and nonlinear bifurcation buckling options (INDIC = 4) were used. Also, it was not possible for Load Case 1 (axial acceleration) to use a dense enough finite element mesh in order to determine with required accuracy the buckling of the entire tank2/skirt system. Therefore, **for Load Case 1 only the aft skirt was included in the STAGS model.** The prebuckling loading on the model of the aft skirt by itself was determined from a complete model in which the aft skirt is embedded in the entire tank2/skirt model.

9. two STAGS finite element plots of the deformed aft skirt obtained from a nonlinear equilibrium analysis of the aft skirt modeled by itself under the axial compression experienced by the aft skirt from Load Case 1 (Fig. 14b). The top frame (A) shows the development of incipient non-axisymmetric deformation (buckling) at load factors, PA = 5.869 and PB = 1.0, in which PA = load factor associated with axial acceleration and PB = load factor associated with the 25 psi ullage pressure and thermal loading. The lower frame (B) in Fig.14b shows the development of much more non-axisymmetric deformation at a very slightly higher load factor, PA = 5.9. Therefore, it appears from Fig. 14b that buckling occurs very close to PA = 5.869, which is in good agreement with the BIGBOSOR4 predictions of 5.888 and 5.946 listed in Fig. 11(A) and with the BIGBOSOR4 prediction of 5.884 listed in Fig. 11(B). However, note that the buckling theory incorporated into BIGBOSOR4 neglects the effects of shell wall anisotropy (D16 and D26 terms in the 6 x 6 integrated constitutive matrix) and transverse shear deformation (t.s.d). Therefore, the very good agreement between STAGS and BIGBOSOR4 may be something of a coincidence.

10. the STAGS prediction of maximum shear stress resultant in the aft skirt versus the Load Set A component (loading from axial acceleration) of Load Case 1 (Fig. 15). There is a rather steep divergence from an approximately horizontal trend starting from Load Step A (PA) = 5.5, a value that is very close to the prediction of aft skirt buckling from the approximate PANDA-type models [4]. The PANDA-type buckling load factors are 5.56 and 5.63, as listed in Fig. 11(A). Unlike the BIGBOSOR4 model, the PANDA-type model does include the effects of shell wall anisotropy and transverse shear deformation. On the other hand, the PANDA-type model is approximate because it treats the conical skirt as an "equivalent" cylindrical shell and the prebuckling resultants as uniform, as listed in Table 5. Therefore, the very good agreement between the STAGS prediction and the PANDA-type prediction may be something of a coincidence.

11. nonlinear bifurcation buckling of the aft skirt from a STAGS model in which the aft skirt is modeled by itself and subjected to the loads in Load Case 1 as if the skirt were embedded in the entire tank2/skirt model (Fig. 16). The buckling mode predicted by STAGS is displayed in Fig. 16. This buckling mode resembles that from BIGBOSOR4 shown in Fig. 11(A). The STAGS buckling load factor, PA = 5.7333, agrees well with those from the PANDA-type models: 5.56 and 5.63 listed in Fig. 11(A), and with the BIGBOSOR4 models: 5.888 and 5.946 and 5.884 listed in Figs. 11(A) and 11(B).

12. nonlinear bifurcation buckling of the entire tank2/skirt system under Load Case 2 (Fig. 17). In this case it was possible to obtain nonlinear bifurcation buckling modes and load factors for the entire tank2/skirt system. However, the use of the INDIC = 1 option in STAGS (linear bifurcation buckling) still generated spurious buckling modes (not shown here). Therefore, the nonlinear bifurcation buckling option (INDIC = 4) had to be used. As seen from Fig. 17 and Table 6, there is very good agreement between the STAGS prediction and the PANDA-type prediction for buckling **at circumferential coordinate, theta = 90 degrees**, both with respect to the buckling mode shape and the buckling load factor. According to STAGS, buckling occurs in the skirts in the neighborhood of the circumferential coordinate, theta = 90 degrees. According to STAGS buckling occurs in the aft skirt at a slightly lower load factor [PA(STAGS) = 1.3005] than in the forward skirt [PA(STAGS) = 1.3107], whereas the approximate PANDA-type Model 1 predicts buckling in the aft skirt at a slightly higher load factor [PA(PANDA) = 1.3022] than in the forward skirt [PA(PANDA) = 1.2977].

13. General conclusion and about varying thicknesses in STAGS models. Except for prediction of the maximum **compressive** stress at the tips of the stringers of the internal orthogrid (Figs. 13b and 13d), the GENOPT/TANK2 predictions are either in very good agreement with those of STAGS or are conservative but not overly conservative. In the STAGS models tapered thicknesses, such as in the external tapered doubler shown in the sketch in Section 1 near the beginning of this paper, in Fig. 1b of [1], and on the right-hand side of Fig. 1a in this paper, are not used in order to avoid the need to introduce user-written subroutines in the STAGS models. Instead, constant average thicknesses of the external doublers and constant average thicknesses for the tapered walls that form parts of the skirt(s) (as shown on the right-hand side of Fig. 8a) are used in the STAGS models. Numerical experiments with the GENOPT/BIGBOSOR4 model indicate that the effect of tapering the external doublers and tapering the metallic parts of the walls of the skirt ends is not significant for the prediction of modal vibration and buckling of the optimized "twoskirt" design. However, as displayed in Fig. 10b, including and neglecting tapering of the external doublers has a huge effect on the prediction of maximum **compressive stress** at the tips of the stringers of the internal orthogrid. **Therefore, STAGS models should be constructed in which tapered thicknesses are included.**

Section 8. CONCLUSIONS

1. The GENOPT/BIGBOSOR4 "tank2" model seems to work. Comparisons of predictions from STAGS and GENOPT/BIGBOSOR4 demonstrate that this application of GENOPT/BIGBOSOR4 can be used for preliminary design.

2. The GENOPT software in the directory ../genopt/sources was significantly modified to permit more than 50 decision variable candidates. The maximum number of decision variable candidates has been raised from 50 to 98.

3. BIGBOSOR4 does not handle the effect of buckling under in-plane shear loading, which is present especially at the circumferential coordinate, theta = 90 degrees in the supporting skirt in Load Case 2 (10g lateral acceleration). In order to compensate for this, an approximate PANDA-type model [4] has been introduced into the “tank2” software in order to predict with reasonable accuracy buckling of the laminated composite skirt(s) under both Load Case 1 and Load Case 2.

4. STAGS models should be constructed that include tapering the thicknesses of the external doubler(s) and tapering the thicknesses of the metallic parts of the skirt support(s) in order to obtain accurate predictions of maximum stress, especially maximum compressive stress at the tips of the stringers in the internal orthogrid.

5. The reader should read [1] and perhaps [2] in order to obtain much background information required for a fuller understanding of how GENOPT/BIGBOSOR4 works.

APPENDIX 1

How to run an existing “tank2/twoskirt” case assuming that the “parent” directory of all of Bushnell’s computer programs is /home/progs:

Part 1 Retrieving the proper files and copying them to their proper locations:
cd /home/progs/genopt/case/tank (go to the directory where the file, tanktank2.tar.gz, is stored)
cp tanktank2.tar.gz /home/progs/work1/. (“work1” is a working directory established by you)
cd /home/progs/work1 (go to your “work1” working directory)
gunzip tanktank2.tar.gz (uncompress the compressed “tar” file, tanktank2.tar.gz)
tar xvf tanktank2.tar (get the files contained in the uncompressed file, tanktank2.tar)
cp tank2.tar.gz /home/progs/work2/. (“work2” is another working directory established by you)
cd /home/progs/work2 (go to your “work2” working directory)
gunzip tank2.tar.gz (uncompress the compressed “tar” file, tank2.tar.gz)
tar xvf tank2.tar (get the files contained in the uncompressed file, tank2.tar)
cp addbosor4.tank2.density.var /home/progs/bosdec/sources/addbosor4.src (most of BIGBOSOR4)
cp bosdec.tank2.density.var /home/progs/bosdec/sources/bosdec.src (bosdec creates BIGB4 input)
cp behavior.tank2 /home/progs/genoptcase/. (behavior.tank2 is the “fleshed out” behavior.new file)
cp struct.tank2 /home/progs/genoptcase/. (struct.tank2 is the “fleshed out” struct.new file)
cp tank2.INP /home/progs/genoptcase/. (tank2.INP is the input file for the GENTEXT processor)

cp twoskirt.BEG /home/progs/genoptcase/. (“twoskirt” input data for the BEGIN processor)
cp twoskirt.DEC /home/progs/genoptcase/. (“twoskirt” input data for the DECIDE processor)
cp twoskirt.OPT /home/progs/genoptcase/. (“twoskirt” input data for the MAINSETUP processor)
cp twoskirt.CHG /home/progs/genoptcase/. (“twoskirt” input data for the CHANGE processor)

Part 2 Running the generic case called “tank2”:
cd /home/progs/genoptcase (go to “genoptcase”. All GENOPT cases are run from “genoptcase”)
genoptlog (activate the GENOPT set of commands)
gentext (execute the GENOPT processor called “genprompt”. Use the generic case name, “tank2”)
cp behavior.tank2 behavior.new (establish the “fleshed out” version of behavior.new)
cp struct.tank2 struct.new (establish the “fleshed out” version of struct.new)
genprograms (compile the program system for the generic case called “tank2”)

Part 3 Running the specific case called “twoskirt”:
begin (run the GENOPT processor called “begin”; generic case=”tank2”, specific case=”twoskirt”)
change (run the GENOPT processor called “change”; the optimized “twoskirt” design=twoskirt.CHG)
decide (run the GENOPT processor called “decide”; the input data for “decide” = twoskirt.DEC)
mainsetup (run the processor called “mainsetup” with twoskirt.OPT; Use NPRINT=2 and ITYPE=2)
optimize (run the GENOPT mainprocessor to generate the file called “twoskirt.OPM” for the “fixed” and previously optimized “twoskirt” design)

If you want to get BIGBOSOR4 predictions and plots for all behaviors (except vibration modes), then do the following. (This particular example is for the BIGBOSOR4 prediction of stresses along meridian no. 1 under Load Case 1 from the file, twoskirt.BEHX811):

cd <working directory for BIGBOSOR4 cases>
bigbosor4log (activate the BIGBOSOR4 set of commands)
cp /home/progs/genoptcase/twoskirt.BEHX811 twoskirt.ALL (BEHX811=stresses under Load Case 1)
bigbosorall (execute BIGBOSOR4)
{inspect the file, twoskirt.OUT [search for the string, “ALLOWABLE STRESS”]}
bosorplot (choose what to plot)

If you want to get BIGBOSOR4 plots of the vibration modes, see Appendix 2 for what to do.

If you want actually to optimize using the previously optimized design as a starting design, then do the following:

cd /home/progs/genoptcase
superopt (with "optimization" option, that is, NPRINT = 0 and ITYPE = 1 in the twoskirt.OPT file)
{Inspect the file, twoskirt.OPP; search for the string, "BEST FEASIBLE" in that file}
chooseplot (generate a file used for obtaining a plot the objective versus design iterations)
diplot (generate a “postscript” file, twoskirt.5.ps, that contains the objective versus design iterations)
gv twoskirt.5.ps (get the plot of objective versus design iterations on your screen using “ghost view”)

APPENDIX 2

How to get BIGBOSOR4 plots of the vibration modes

Part 1 Meaning of the files, bosdec.tank2 and bosdec.tank2.density.var, addbosor4.regular and addbosor4.tank2.density.var and how to retrieve and store them:

The files, bosdec.tank2 and addbosor4.regular, correspond to the model in which the density of Layer No. 2 of each shell segment in the propellant tank is constant and includes the lumped mass of the propellant. The files, bosdec.tank2.density.var and addbosor4.tank2.density.var, correspond to the model in which the density of Layer No. 2 of each shell segment in the propellant tank may vary along the meridian of that segment and includes the lumped mass of the propellant. The file, bosdec.tank2.density.var and addbosor4.tank2.density.var, produce the most accurate predictions of modal vibration frequencies. However, you can get plots of vibration modes from the stand-alone BIGBOSOR4 **only** with use of the files, bosdec.tank2 and addbosor4.regular.

Copy the files, bosdec.tank2, addbosor4.regular, bosdec.tank2.density.var and addbosor4.tank2.density.var, to your /home/progs/bosdec/sources directory, as follows:

cd /home/progs/work2 (go to the same “work2” directory mentioned in Part 1 of Appendix 1)
cp bosdec.tank2 /home/progs/bosdec/sources/. (“bosdec” for “constant density along meridian”)
cp bosdec.tank2.density.var /home/progs/bosdec/sources/. (“bosdec” for “varying density”)
cp addbosor4.regular /home/progs/bosdec/sources/. (“addbosor4” for “constant density”)
cp addbosor4.tank2.density.var /home/progs/bosdec/sources/. (“addbosor4” for “varying density”)

Part 2 Getting plots of vibration modes from BIGBOSOR4

cd /home/progs/bosdec/sources (go to the bosdec/sources directory)
cp bosdec.tank2 bosdec.src
cp addbosor4.regular addbosor4.src
cd home/progs/genoptcase (go to the directory where GENOPT cases are always run)
genprograms (re-compile, using the “constant density” model in order to get vibration modes)
optimize (with the "fixed" design option, that is, NPRINT = 2 and ITYPE = 2 in the *.OPT file)
cd <working directory for BIGBOSOR4 cases>
bigbosor4log (activate the BIGBOSOR4 set of commands)
cp /home/progs/genoptcase/twoskirt.BEHX11 twoskirt.ALL (BIGBOSOR4 input for modal vibration)
bigbosorall (execute BIGBOSOR4)
{inspect the file, twoskirt.OUT [search for the string, "EIGENVALUE(", including the left paren]}
bosorplot (choose what to plot)

APPENDIX 3

A list of the input data for BIGBOSOR4 that pertains to the axisymmetric meridionally non-uniform cool-down of the aft and forward skirts of the specific case called “twoskirt” follows:

PART 1: Axisymmetric non-uniform thermal loading in Segment 3 of the aft skirt

```
----------------------------------------------------------------------
      H           $ DISTRIBUTED LOAD INPUT FOLLOWS...
         2        $ IDISAB= indicator (0, 1, 2 or 3) for load set A and B
      H           $ SURFACE LOAD INPUT FOR LOAD SET "B" FOLLOWS
         2        $ NLTYPE=control (0,1,2,3) for type of surface loading
        11        $ NTSTAT= number of meridional callouts for temperature
         1        $ NTGRAD=control for type of thermal gradient thru thickness
         4        $ NTYPEL=index (use 4) for input of nonsymmetric temperature
         1        $ NLOAD(1)=indicator for temperature coef. T1 (0=none, 1=some)
         0        $ NLOAD(2)=indicator for temperature coef. T2 (0=none, 1=some)
         0        $ NLOAD(3)=indicator for temperature coef. T3 (0=none, 1=some)
 -0.2000000E-01   $ T1    = temperature factor at Ith meridional callout, T1(  1)
 -0.2000000       $ T1    = temperature factor at Ith meridional callout, T1(  2)
  -2.000000       $ T1    = temperature factor at Ith meridional callout, T1(  3)
  -10.00000       $ T1    = temperature factor at Ith meridional callout, T1(  4)
  -20.00000       $ T1    = temperature factor at Ith meridional callout, T1(  5)
  -40.00000       $ T1    = temperature factor at Ith meridional callout, T1(  6)
  -80.00000       $ T1    = temperature factor at Ith meridional callout, T1(  7)
  -130.0000       $ T1    = temperature factor at Ith meridional callout, T1(  8)
  -160.0000       $ T1    = temperature factor at Ith meridional callout, T1(  9)
  -180.0000       $ T1    = temperature factor at Ith meridional callout, T1( 10)
  -200.0000       $ T1    = temperature factor at Ith meridional callout, T1( 11)
        12        $ NTHETA= number of circumferential callouts for load
         2        $ NOPT  = control for how g(THETA) is to be input (1,2,or 3)
         1        $ NODD  = control integer for oddness, evenness, of g(THETA)
   0.000000       $ THETA = circumferential coordinate, in degrees, THETA(  1)
   180.0000       $ THETA = circumferential coordinate, in degrees, THETA(  2)
   1.000000       $ YPLUS = value of g(THETA) at THETA(  1)
   1.000000       $ YPLUS = value of g(THETA) at THETA(  2)
      N           $ Do you want to print out output Fourier expansion of load?
         2        $ NTYPE = control for meaning of callout (2=z, 3=r)
   23.73338       $ Z(I)  = axial coordinate of Ith temperature callout, z(  1)
   35.79398       $ Z(I)  = axial coordinate of Ith temperature callout, z(  2)
   47.85458       $ Z(I)  = axial coordinate of Ith temperature callout, z(  3)
   59.91518       $ Z(I)  = axial coordinate of Ith temperature callout, z(  4)
   71.97577       $ Z(I)  = axial coordinate of Ith temperature callout, z(  5)
   84.03637       $ Z(I)  = axial coordinate of Ith temperature callout, z(  6)
   96.09697       $ Z(I)  = axial coordinate of Ith temperature callout, z(  7)
   108.1576       $ Z(I)  = axial coordinate of Ith temperature callout, z(  8)
   120.2182       $ Z(I)  = axial coordinate of Ith temperature callout, z(  9)
   132.2788       $ Z(I)  = axial coordinate of Ith temperature callout, z( 10)
   144.3394       $ Z(I)  = axial coordinate of Ith temperature callout, z( 11)
------------------------------------------------------------------------------
```

PART 2: Axisymmetric non-uniform thermal loading in Segment 3 of the forward skirt

```
-----------------------------------------------------------------------------
      H           $ DISTRIBUTED LOAD INPUT FOLLOWS...
         2        $ IDISAB= indicator (0, 1, 2 or 3) for load set A and B
      H           $ SURFACE LOAD INPUT FOR LOAD SET "B" FOLLOWS
         2        $ NLTYPE=control (0,1,2,3) for type of surface loading
        11        $ NTSTAT= number of meridional callouts for temperature
```

```
        1       $ NTGRAD=control for type of thermal gradient thru thickness
        4       $ NTYPEL=index (use 4) for input of nonsymmetric temperature
        1       $ NLOAD(1)=indicator for temperature coef. T1 (0=none, 1=some)
        0       $ NLOAD(2)=indicator for temperature coef. T2 (0=none, 1=some)
        0       $ NLOAD(3)=indicator for temperature coef. T3 (0=none, 1=some)
  -200.0000     $ T1    = temperature factor at Ith meridional callout, T1(  1)
  -180.0000     $ T1    = temperature factor at Ith meridional callout, T1(  2)
  -160.0000     $ T1    = temperature factor at Ith meridional callout, T1(  3)
  -130.0000     $ T1    = temperature factor at Ith meridional callout, T1(  4)
  -80.00000     $ T1    = temperature factor at Ith meridional callout, T1(  5)
  -40.00000     $ T1    = temperature factor at Ith meridional callout, T1(  6)
  -20.00000     $ T1    = temperature factor at Ith meridional callout, T1(  7)
  -10.00000     $ T1    = temperature factor at Ith meridional callout, T1(  8)
  -2.000000     $ T1    = temperature factor at Ith meridional callout, T1(  9)
 -0.2000000     $ T1    = temperature factor at Ith meridional callout, T1( 10)
 -0.2000000E-01 $ T1    = temperature factor at Ith meridional callout, T1( 11)
        2       $ NTHETA= number of circumferential callouts for load
        2       $ NOPT  = control for how g(THETA) is to be input (1,2,or 3)
        1       $ NODD  = control integer for oddness, evenness, of g(THETA)
   0.000000     $ THETA = circumferential coordinate, in degrees, THETA(  1)
   180.0000     $ THETA = circumferential coordinate, in degrees, THETA(  2)
   1.000000     $ YPLUS = value of g(THETA) at THETA(  1)
   1.000000     $ YPLUS = value of g(THETA) at THETA(  2)
     N          $ Do you want to print out output Fourier expansion of load?
        2       $ NTYPE = control for meaning of callout (2=z, 3=r)
   453.8005     $ Z(I)  = axial coordinate of Ith temperature callout, z(  1)
   468.2714     $ Z(I)  = axial coordinate of Ith temperature callout, z(  2)
   482.7423     $ Z(I)  = axial coordinate of Ith temperature callout, z(  3)
   497.2132     $ Z(I)  = axial coordinate of Ith temperature callout, z(  4)
   511.6841     $ Z(I)  = axial coordinate of Ith temperature callout, z(  5)
   526.1550     $ Z(I)  = axial coordinate of Ith temperature callout, z(  6)
   540.6260     $ Z(I)  = axial coordinate of Ith temperature callout, z(  7)
   555.0969     $ Z(I)  = axial coordinate of Ith temperature callout, z(  8)
   569.5677     $ Z(I)  = axial coordinate of Ith temperature callout, z(  9)
   584.0387     $ Z(I)  = axial coordinate of Ith temperature callout, z( 10)
   598.5096     $ Z(I)  = axial coordinate of Ith temperature callout, z( 11)
--------------------------------------------------------------------------------
```

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[1] David Bushnell, Michael S. Jacoby and Charles C. Rankin, "Optimization of propellant tanks supported by optimized laminated composite tubular struts", unpublished paper to be presented at the AIAA 54th Structures, Structural Dynamics and Materials Conference, Boston, MA, April 2013. See also the long unpublished report with the same title by David Bushnell, January - March 2012, included in [2]. This long report was written before Michael Jacoby and Charles Rankin generated the STAGS models and predictions.

[2] Bushnell, D. and Bushnell, W.D., Shell buckling website, http://shellbuckling.com/, in particular the "GENOPT" page of that website and the two unpublished full reports and their associated figures and tables, "Optimization of Propellant Tanks Supported by Optimized Laminated Composite Tubular Struts", by David Bushnell, January – March 2012, and "Optimization of Propellant Tanks Supported by One or Two Optimized

Laminated Composite Skirts", by David Bushnell, February – June 2012. These two detailed reports and associated figures and tables, written before Michael Jacoby and Charles Rankin generated the STAGS models and predictions, can be downloaded by website visitors. NOTE: These detailed reports give configurations that are now out of date because certain errors were found and other software modifications were made after the detailed reports were written. In spite of this, the detailed reports are instructive because of other information given in them. In order to download the two unpublished full reports and their associated figures and tables, go to the "Downloads" page of the website, http://shellbuckling.com/ , and download a very large compressed "tar" file (about 700 Mbytes) containing the latest versions of the following computer programs/documentation/cases created over many years by David Bushnell: BOSOR4, BIGBOSOR4, BOSOR5, PANDA2 and GENOPT, and a small file, "How to Run on LINUX". Generate the proper directory structure for the five programs; go to the directory, …/genopt/case/tank; copy the file, tanktank2.tar.gz to a working directory; go to that working directory; decompress (gunzip) the file, tanktank2.tar.gz; disassemble the resulting file, tanktank2.tar (tar xvf tanktank2.tar); and uncompress and disassemble three of the resulting files, general.info.tar.gz, tank.tar.gz (strut-supported tank), and tank2.tar.gz (skirt-supported tank). All of the files referred to in this paper can be obtained by this procedure. The two "unpublished full reports" mentioned at the beginning of this reference are located in the "folder" called "tanktank2", which appears in a list of the entities that exist following your command, "tar xvf tanktank2.tar". The "folder" called "tanktank2" contains the two "folders", tank.paper (strut-supported propellant tank) and tank2paper (skirt-supported propellant tank). The tank.paper "folder" contains several "folders", in particular the two "folders" called "tankfigsdoc" (the "tank" figures) and "tanktablesdoc" (the "tank" tables) and the file called "tankpapertext.pdf" (the text of the unpublished full report on the strut-supported tank). The tank2paper "folder" contains several "folders", in particular the two "folders" called "tank2figsdoc" (the "tank2" figures) and "tank2tablesdoc" (the "tank2" tables) and the file called "tank2papertext.pdf" (the text of the unpublished full report on the skirt-supported tank). The word, "folder", which is jargon used on the iMAC computers, can be replaced by the word, "directory", appropriate for workstations.

[3] Bushnell, D., "GENOPT--A program that writes user-friendly optimization code", International Journal of Solids and Structures, Vol. 26, No. 9/10, pp. 2031-380, 1990. The same paper is contained in a bound volume of papers from the International Journal of Solids and Structures published in memory of Professor Charles D. Babcock, Jr, formerly with the California Institute of Technology.

[4] Bushnell, D., "Theoretical basis of the PANDA computer program for preliminary design of stiffened panels under combined in-plane loads", Computers & Structures, Vol. 27, No. 4, pp. 541-563, 1987

[5] B. O. Almroth, F. A. Brogan, "The STAGS Computer Code", NASA CR-2950, NASA Langley Research Center, Hampton, Va.(1978)

[6] C. C. Rankin, P. Stehlin and F. A. Brogan, "Enhancements to the STAGS computer code", NASA CR 4000, NASA Langley Research Center, Hampton, Va, November 1986

[7] Riks, E., Rankin C. C., Brogan F. A., "On the solution of mode jumping phenomena in thin walled shell structures", First ASCE/ASM/SES Mechanics Conference, Charlottesville, VA, June 6-9, 1993, in: Computer Methods in Applied Mechanics and Engineering, Vol.136, 1996.

[8] G. A. Thurston, F. A. Brogan and P. Stehlin, "Postbuckling analysis using a general purpose code", AIAA Journal, 24, (6) (1986) pp. 1013-1020.

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