

**MEMORANDUM M-675**

**Computational Mechanics at the Faculty of  
Aerospace Engineering, Delft University  
of Technology**

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# COMPUTATIONAL MECHANICS AT THE FACULTY OF AEROSPACE ENGINEERING, DELFT UNIVERSITY OF TECHNOLOGY

By E. Riks

## INTRODUCTION

Mechanics is a branch of Physics that studies the motion and deformation of matter under the influence of forces. Computational mechanics is a relatively young (about 30 years old) sub-branch of mechanics that pursues to study the problems of mechanics by means of computers. It can therefore be described as a numerical approach to the solution of the governing equations of mechanics whereby the solution and interpretation of the results are conducted on computers.

Apart from the aspect of simulation, which involves the visualization of the flow or the deformations as a function of the time, computational mechanics is also strongly identified with the capability to cope with the nonlinearity of the phenomena studied. This is in striking contrast to the classical analytical methods of applied mechanics which are usually severely restricted to the extent in which nonlinear behavior can be taken into account.

At the present time, it is customary to divide computational mechanics into two parts, a division that also exists in theoretical mechanics and applied mechanics. This is the division into Fluid and Gas Dynamics which studies the flow of fluids and gases, and, Solid Mechanics and Structural Mechanics which studies the motion and deformation of solids and structures. The applications discussed in this memorandum belong exclusively to the domain of Structural Mechanics.

## APPLICATIONS TO AIRCRAFT STRUCTURAL DESIGN

At the basis of the development of computational tools for solid mechanics stands the finite element method (Refs. 1 - 9). Finite element methods are now widely used in the design offices and their participation in the design process is growing. They are used for stress analysis, vibration analysis, the analysis of stress concentrations in fasteners, the analysis of cracks just to name a few topics. Their great potential is due to their versatility, i.e., they can be applied to a great class of problems without the need to change the implementation of the method. Hence, the advent of finite element computer codes that can deal with the analysis of a great variety of structures.

As mentioned in the introduction, one of the most notable features of the methods of computational mechanics is that they can be applied to nonlinear problems. It is in this area that

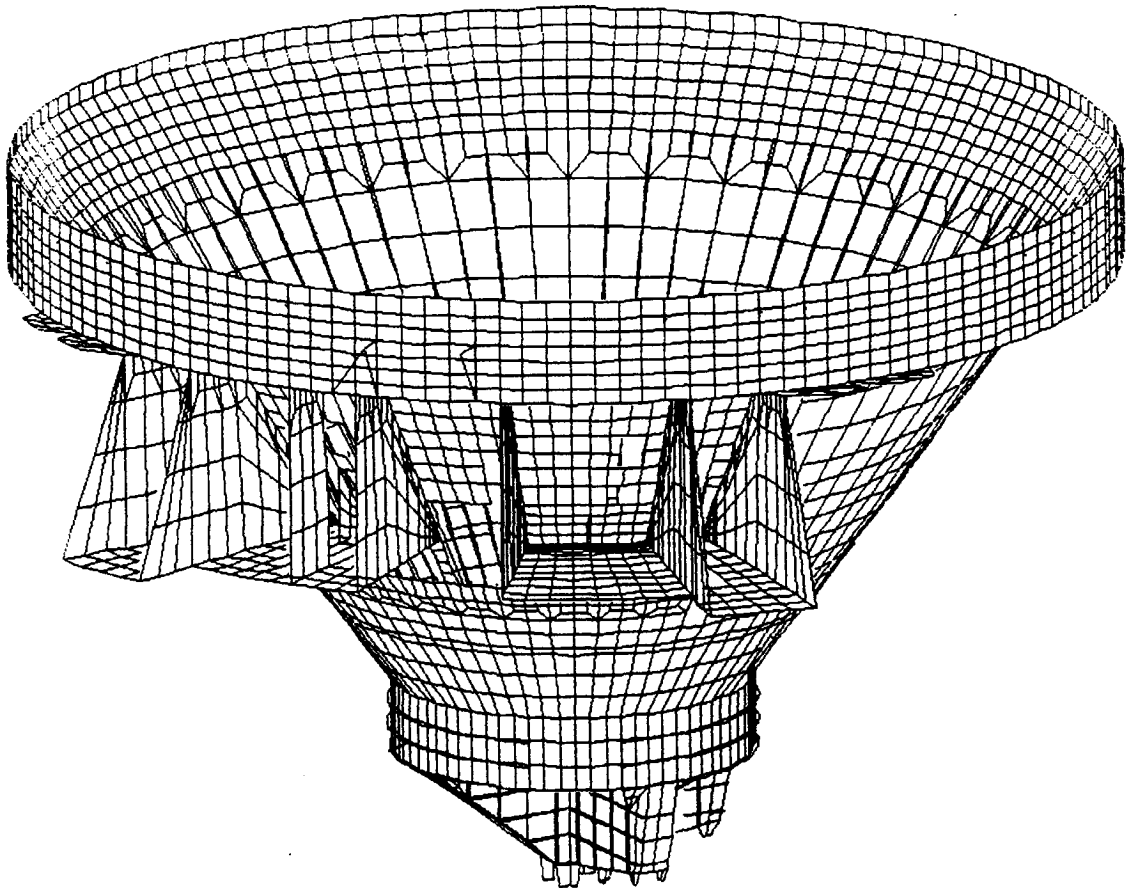
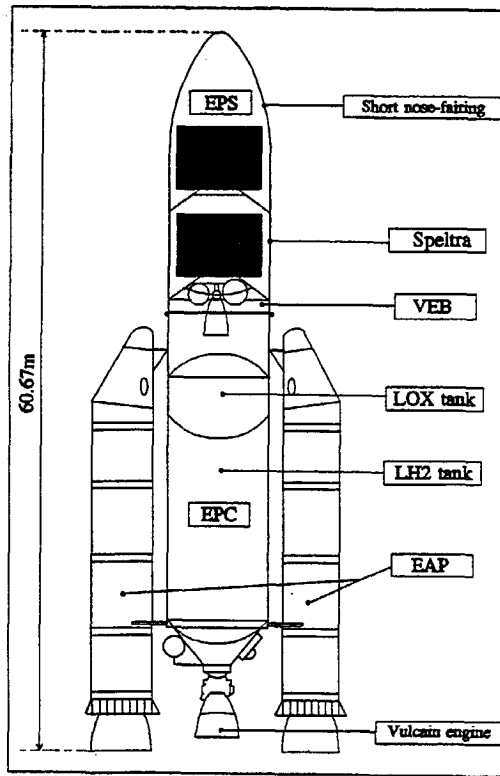


Figure 1 Collapse Analysis Vulcain Engine Support

the development of numerical solution techniques is particularly rewarding. Therefore, in this paper we focus exclusively on the nonlinear problems that are encountered in aircraft and space craft structural design.

Of the nonlinear problems that are encountered there are two types that are very particular to the mechanical properties of thin walled plate and shell panels, the structural elements that are used in the construction of space- and aircraft structures.

Plate and shells are to some degree optimal forms, i.e. their thickness is very small with respect to the other dimensions because they are used to carry loads in the plane of the plate or shell only. If a shell is loaded in compression, this optimal form promotes the occurrence of buckling. This is a behavior that is described by nonlinear equations. Buckling usually leads to loss of stiffness or worse, collapse. Buckling problems play therefore an important role in space- and aircraft design.

Another form of degradation of the strength of shells is damage caused by cracks. Cracks develop in metal sheet panels when they are loaded in tension whereby the loading is applied in cycles over a prolonged period of time. Many crack problems in plates and stiffened panels can be formulated in terms of linear equations. But for a longitudinal crack in a pressurized fuselage the problem is nonlinear, and, in this case, only the finite element method is able to obtain accurate solutions.

Thus the danger of buckling and the danger of uncontrolled crack propagation in space and aircraft structures creates two types of nonlinear problems that require thorough investigation. This is the background to the research and code developments described in the remaining part of this paper.

## **STABILITY ANALYSIS OF SHELLS**

A basic strength problem in aircraft structural design is the determination of the load carrying capacity of the wing box. It turns out that the load carrying capacity of these type of structures is almost always determined by stability criteria. The same applies to the load carrying capacity of rockets and boosters.

To illustrate how a stability analysis can be carried out in these cases, we describe here an actual example (Ref. 10). The engine mount of the Ariane second stage rocket is required to carry a vectored thrust during part of the launch of the rocket. The structure is a stiffened conical shell which has the function to transfer the engine thrust to the main body of the second stage of the rocket ( a cylindrical shell filled with fuel). The dominant question during the design phase of the structure is: "Can the structure sustain the required thrust with a sufficient margin of safety"? This looks like is a very simple question, but the determination of the answer turns out to be far from trivial.

Several computer codes are now available that can model this structure. One of the requirements for the construction of a proper model for the structure is that it should be capable to represent the possible deformations properly. In this particular example, see figure 1,

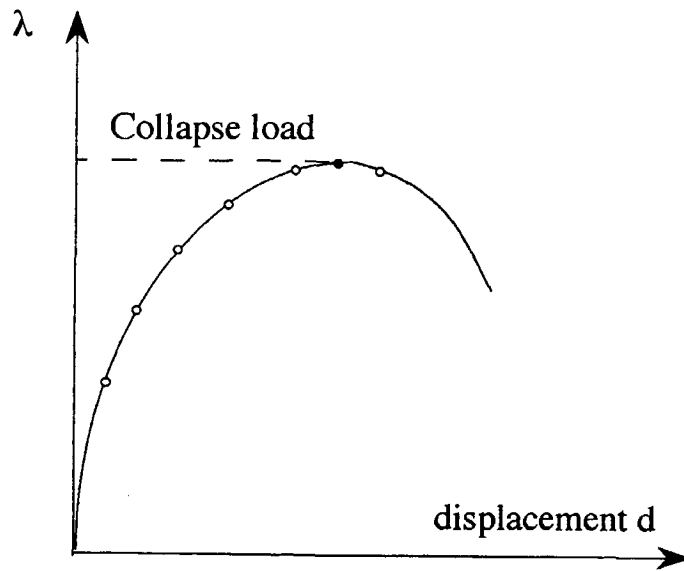


Figure 2 Computation of the collapse load

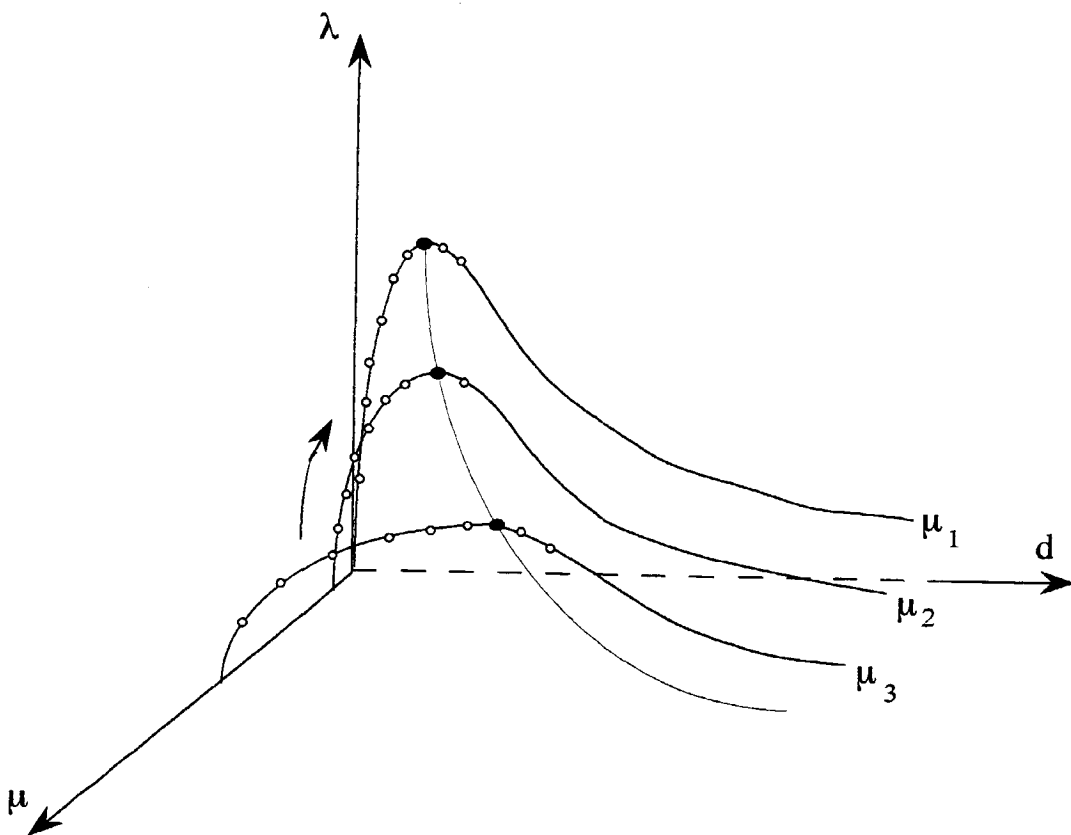


Figure 3 Collapse Analysis Conus

this requirement leads to a mesh that generates approximately 160,000 nonlinear equations with an equal number of degrees of freedom. The response of the structure to the loading that is acting on it is computed in a step by step fashion. In this procedure, which is called path following, each new solution point is obtained by the repeated solution of a system of linearized equations derived from governing set of equations mentioned earlier. The result of such an analysis is pictured in figure 2 where the load intensity  $\lambda$  is plotted against a measure of deformation, for example a displacement of a point of the structure, or some average of the displacements. It is seen that this response curve contains a maximum for the load  $\lambda$ . It is this point which represents the collapse load of the structure for the particular loading system and boundary conditions considered in the computations.

However, the evaluation of the load carrying capacity can hardly ever be restricted to one set of loads and one set of boundary conditions only. It is usually necessary to consider a whole set of variations in the boundary conditions and alternative load systems. If these variations are represented by a parameter  $\mu$ , the analysis has the character of computing a set of curves on a two dimensional surface in the space of the loading  $\lambda$ , the design variables  $\mu$  and the degrees of freedom  $d$  (Figure 3). Each curve for  $\mu = \mu_i$  ( $i = 1,2,3..$ ) can then be seen as one loading simulation. Computations of this type have much in common with the testing of the structure in the laboratory. One significant difference is that the loading and boundary conditions are much more easily changed in the numerical model than in a full scale laboratory experiment.

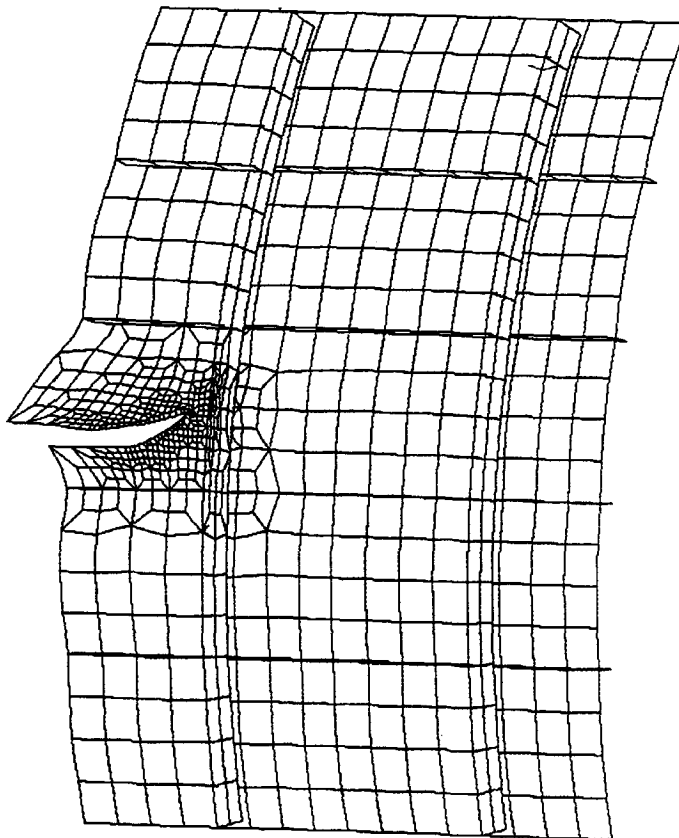
Computations as sketched above have only recently become feasible because it took some time before certain limitations of the method could be removed. These limitations were for example:

1. *The lack of appropriate finite elements that could adequately describe the deformations of the shells during loading process.* The development of shell finite elements that can adequately describe the behavior of a shell under finite displacements and rotations has only recently come to a level that seems stable and final. (See for example (Refs. 3 - 9).
2. *The lack of appropriate solution algorithms that can deal with the special nature of the governing equations.* The development of solution strategies that are specifically suited for stability analysis was initiated in the seventies, (Refs. 11 - 19), but it took considerable time before these techniques found their way in the available codes. Even today it can be said that the solution algorithms presently operational still leave much to be desired. This is especially true on the level of the computer implementation of these techniques.

3. *The absence of computers that were powerful enough to cope with the volume of the computations required for this type of analysis.* An important limiting factor in the analysis of large computer models is the speed by which the computer operates and the capacity of the storage devices that are connected to it. Fortunately, there has been a steady increase in these capabilities, in particular when they are pitted against the cost of these operations. New parallel computing machines are lingering on the horizon and there is every reason to expect that computing power will continue to increase so that voluminous calculations come more and more within the reach of the available budget of the designer.

### **CRACK ANALYSIS IN PRESSURIZED FUSELAGES**

Another area where the application of the (nonlinear) finite element method has turned out to be fruitful is the analysis of the residual strength of aircraft structures that are damaged by cracks. A special topic under this heading is the problem of the residual strength of a damaged fuselage loaded by internal pressure and other external loads produced during the operation of the aircraft. Here the problem is concerned with the prediction of the way a through-crack will propagate this in terms of speed as well as direction (Ref. 20, 22).



B2000/BASPL Version 2.53c

Figure 4 A Crack Propagating in a Fuselage Segment



Complications in the analysis of these problems are the curvature of the shell that constitutes the fuselage, the geometrical non linearity of the governing equations, the presence of stiffeners which have a significant influence on the way the surrounding structure interacts with the crack. In the beginning, the efforts in this area were primarily devoted to the development of methods that are able to compute the stress intensity and its characterization at the tip of through cracks accurately (Refs. 22 - 32). Recently, however, the efforts are being directed toward simulations of a crack that is propagating through the structure (Refs. 29 - 32) (see Figure 4 for an illustration). The ultimate goal is to provide the designer with tools that enable him to simulate crack growth and crack arrest under conditions that compare realistically with the conditions in a real fuselage under service loading.

### **MODE JUMPING AND TRANSIENT ANALYSIS OF SHELLS**

Buckling and collapse are dynamical phenomena although this is not always visible in the treatment of these type of problems. In the classical approach, the inherent dynamical problem of loss of stability is studied with the methods of statics. However, this should not be surprising. The primary point of interest in structural stability analysis is the computation of the collapse load at which the structure fails and it is this information that can be obtained without the need to solve the elasto- dynamic equations of the structure.

In the past history of the stability analysis of shells the need to compute what actually happens when the buckling process sets in has seldom arisen. If there existed such a need, the available computational tools of the day were not capable to obtain the necessary solutions. But, there are at least two reasons for wanting to simulate the buckling process accurately. The first reason is the wish to observe what actually happens when buckling takes place. The second reason is born out of a practical necessity. This necessity arises with so called mode jump problems in stiffened panels where dynamical snaps are experienced during the time the loading is applied and varied. The dynamical snaps are accompanied by marked changes in the deformation pattern or mode, hence the name mode jumping. In these cases, the static methods are not capable to obtain the solutions. Consequently, there is considerable interest in the development of methods that can compute the actual dynamical response of a structure when it reaches a point of instability or when it experiences modal jumps (Ref. 32).

There are, however, also other areas of structural design in aero-and astronautics where transient methods will prove to be very useful. One can think of applications to the deployment of folded antennas in space, to impact problems, to crash and crash worthiness analysis, to frangibility studies etc.. It is expected that the development of transient analysis methods for the study of the dynamic response of shell structures to rapidly varying loads will gain more and more prominence in the near future. This is the reason that the development of a transient response code has recently been initiated (Ref.33).

## CURRENT PROJECTS IN THE FACULTY OF AEROSPACE ENGINEERING

There are two main projects dealing with research and development work on computer codes within the Structures Group of the Faculty of Aerospace Engineering.

(I) --- The first concerns the development of DISDECO (**D**elft **I**nteractive **S**hell **D**esign **C**ode), a project intended to make the accumulated theoretical, numerical and practical knowledge of the last 25 years or so readily accessible to users interested in the analysis of buckling sensitive structures (Refs. 34 - 35).

With this hierarchical, interactive computer code the user can access from his work-station successively programs of increasing complexity. The computational modules currently operational in DISDECO provide the prospective user with facilities to calculate the critical buckling loads of stiffened anisotropic shells under combined loading, to investigate the effect various types of boundary conditions will have on the critical load, and to get a complete picture of the degrading effects the different shapes of possible initial imperfections might cause, all in one session.

Once the design is finalized, its collapse load can be verified by running a large refined finite element model remotely from behind the work station with one of the current 2- dimensional codes referred to earlier (see Refs. 10 and 32) or the B2000 code mentioned below which have advanced capabilities to handle both geometric and material nonlinearities.

(II) ---The second development concerns the Finite Element code B2000 (see the appendix for a short description). The B2000 code (**B**asis **2**000) is a program designed for the development of linear and nonlinear finite element methods for general applications. It is a code that is in use and under development in 4 institutions: The SMR company in Switzerland, the National Aerospace Laboratory NLR in the Netherlands, the German Aerospace Laboratory DLR and the Faculty of Aerospace Engineering of the Technical University of Delft. In the Structures Group of the Faculty of Aerospace Engineering the code is intended to take a central place in all the research and development work that involves *general* finite element applications (as opposed to the more specialized methods used in DISDECO). Ongoing work is at present directed towards:

1. The development of mesh generators, special elements and solution methods for nonlinear crack analysis in stiffened fuselage shells.
3. The development of a nonlinear transient analysis module for applications to crash and crash worthiness simulations.
3. The development of a combination of a static and transient analysis methods for the simulation of dynamic buckling and mode jumping problems.

## CONCLUSION

The computer simulation of the behavior of thin walled shell and plate structures under stationary and non stationary loads provides a powerful means to analyze the adequacy of the design of these structures. In the Structures Group of the Faculty of Aerospace Engineering, the development of these means are presently aimed at two families of problems with many practical applications in aerospace design.

The first direction concerns the nonlinear collapse behavior of plate and shell structures in the broadest possible sense. This development includes quasi-static and dynamic buckling analysis, transient response and crash simulation. The methods vary from classical shell buckling formulas and classical nonlinear analysis in the hierarchical analysis code DISDECO to the general approaches offered by the finite element method, path following techniques and time integration methods in the general finite element code B2000.

The second direction concerns the nonlinear behavior of cracks and other types of damage in pressurized stiffened shells that cannot be analyzed by classical methods and/or linear finite element methods. The methods that are considered here include the classical methods such as the J-integral and mode enrichment technique adapted to the geometrical nonlinear loading conditions encountered in these structures, path following and load relaxation methods, mesh refinement and mesh adaptation methods for self seeking crack growth simulations.

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## APPENDIX

### **B2000: A FEM CODE DEVELOPMENT FACILITY AT THE FACULTY OF AEROSPACE ENGINEERING, TECHNICAL UNIVERSITY OF DELFT**

The **B2000** code (= **Basis 2000**) is a program designed for finite element applications which, at the same time, functions as a programming environment for code development and testing of new methods. The architecture of the code provides an attractive platform for the development of nonlinear numerical models in fluid and solid mechanics.

Originally created in Switzerland by the SMR Company, it is now being developed by NLR, the National Aerospace Laboratory in the Netherlands (since 1984), DLR, the German Aerospace Laboratory (since 1993), and the Faculty of Aerospace Engineering of the University of Delft (since 1991). The code is modular with a modern data structure, equipped with an extensive array of utilities that are commonly used in finite element calculations.

The architecture is database oriented with a data base and data base manager called MEMCOM. MEMCOM is an integrated Memory and data Management system that takes a central place in the code. It is especially designed for finite element/finite difference applications. MEMCOM provides a transparent data representation and makes the storage, retrieval and manipulation of data in the code easy.

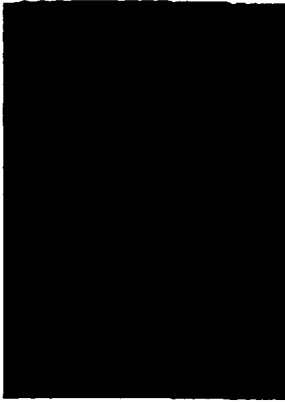
The code is equipped with a post processor BASPL. Interfaces with any other visualization program can easily be provided.

The following features are under development at the present time in the various institutes where B2000 is being developed: Nonlinear shell analysis, Structural vibrations, Acoustic elements, Solution procedures for the analysis of through cracks in shells, Transient response in shells, Optimization of shell structures.

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