STRUCTURAL ANALYSIS OF GENERAL SHELLS

VOLUME III

(preliminary and incomplete)

EXAMPLE CASES FOR STAGSC

January 1977

by

B. O. Almroth

E. Meller

Section I

INTRODUCTION

(in preparation)

Section 2

EXAMPLE CASES

2.1 General

The example cases presented in this section are intended to serve as a complement to the instructions given in Volume 2. Whenever a user finds the instructions unclear, it is hoped that he will find an example here that clarifies the point in question. The sample cases can also be used for the purpose of training personnel in the use of the code.

After definition of a case, the input cards are shown together with selected parts of the output. The following cases are included:

1. Paraboloid with meridional stiffening. Bifurcation buckling for axial compression with fixed internal pressure.

2. Elliptic cylinder. Bifurcation buckling and vibrations with nonlinear prestress. Axial compression.

3. Cylindrical panel with free edges. Vibrations with nonlinear prestress (point load).

4. Conical shell with end ring. Linear stress analysis. Pressure load.

5. Plates of variable thickness, connected with torsion springs. Free vibrations.

6. Spherical cap with gravity load. Triangular elements at apex. Linear analysis.

7. Quadrilateral plate with discrete stiffening (off and on grid lines). Geometrically linear inelastic analysis.

8. Cylinder with one rectangular cutout. Bifurcation buckling under axial compression.

9. Bent tube (cylinder and torus). Bifurcation buckling analysis under bending.

10. Spherical shell roof with square plan form. Buckling with nonlinear prestress under gravity load. Beam and spring elements.

11. Cylindrical shell with ellipsoidal head. Vibration analysis and transient response analysis with forced displacement history.

12. Two connected paraboloids. Bifurcation buckling under thermal loading.

13. Cylindrical shell with two rectangular cutouts. Cutout covers are attached at eight points. Bifurcation buckling under axial load.

14. Cylinder with two circular cutouts. Isogrid stiffening.
Bifurcation buckling analysis under axial compression.

Table 2.1 summarizes the different features of the example cases. Throughout the example cases the SI system of units has been used as follows:

Quantity	Unit	Symbol
Distance	millimetres	mm
Force	newtons	N
Pressure Stress Modulus	mega pascals	MPa
Lineload	newtons/millimetre	N/mm
Mass	kilogram	kg
Temperature	degree Celsius	^{o}C
Time	second	s
Velocity	millimetre/second	mm/s
Acceleration	millimetre/second/second (Gravitational acceleration = 9807 mm/s^2)	mm/s^2
Density	Kilogram/millimetre cube	kg/mm^3

Angles are usually given in degrees rather than in the SI unit.

2.2 Example Case 1: Paraboloid with Meridional Stiffening

2.2.1 Case Description

Geometry:

A paraboloid shell with meridional stiffening is considered. The generator is defined by the equation $\eta^2 = 4 \times 6.35 \times \xi$. Dimensions are indicated in Fig. 2.1.

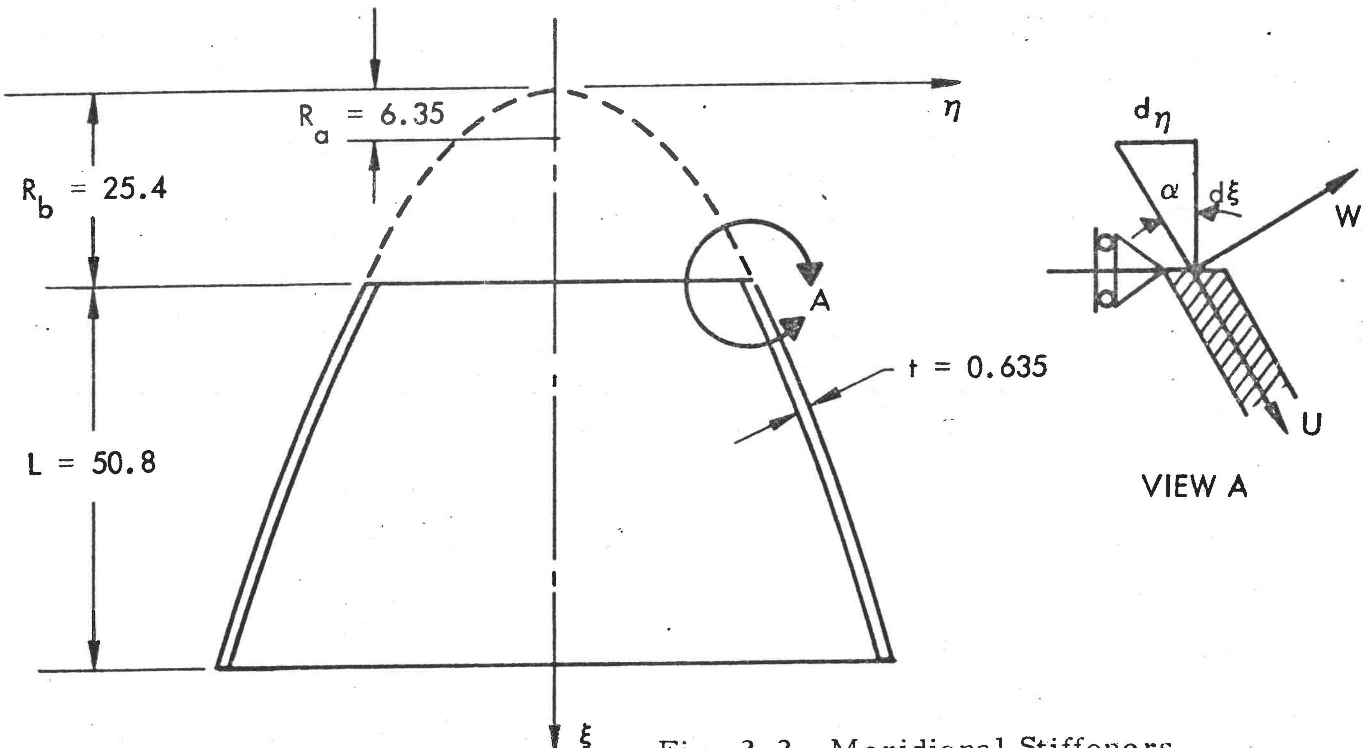

Fig. 2.2 Meridional Stiffeners

Meridional stiffeners, with rectangular cross-section as shown in Fig. 2.2 are attached with 6° of spacing.

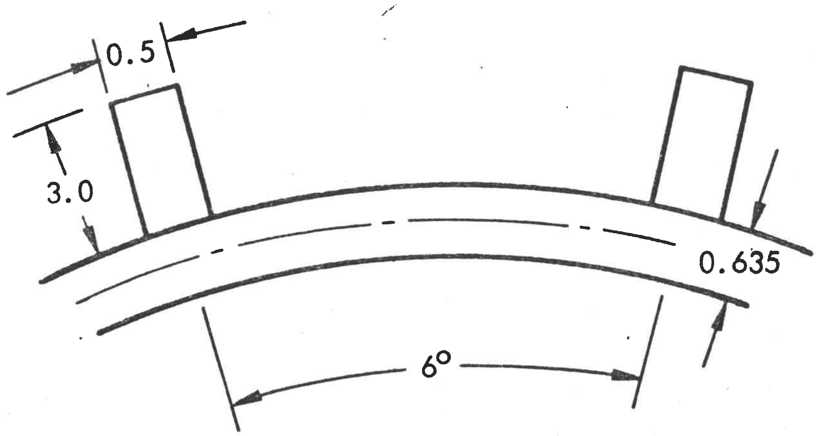

Fig. 2.2 Meridional Stiffeners

Both edges are clamped in rigid end plates.

Material:

The single layer shell wall as well as the stiffeners are made of aluminum with

$$E = 70\,000 \text{ MPa}$$

$$\nu = 0.3$$

Load:

The shell is subjected to a uniform internal pressure of 0.2 MPa. In addition axial compression is applied at the center of the end plates until buckling occurs (uniform end shortening).

Mode of Analysis:

A bifurcation buckling analysis is made. It is assumed that the shell will buckle in 5 circumferential waves. It is sufficient to consider a shell segment corresponding to one quarter of a wave, i.e. the analysis is applied to a segment covering 18^{o} in the circumferential direction. Stiffeners are smeared (see Vol. 2).

Grid:

A reasonably accurate analysis should be obtained by use of 11 points in the axial and 5 points in the circumferential directions.

2.2.2 Note

The structure defined here is a shell of revolution with axisymmetric load. For such a configuration it is known that buckling will occur in a mode that is sinusoidal in the circumferential direction. Computer programs for shells of revolution, such as BOSOR, are specialized for this purpose by taking advantage of this fact. Therefore, STAGS is not the most suitable program for analysis of this particular case.

2.2.3 Input Preparation

Stiffener spacing:

Since the circumference of the shell varies with the axial coordinate, the stringer spacing is variable. However, for smeared stiffeners, the spacing is defined by use of the surface coordinate values. In this case the spacing is given in degrees and regular data cards can be used.

The two end plates are free to move relative to one another in the axial direction. We can consider the shell clamped at the large end ($\xi = 76.2$). At the other end the shell edge is constrained from motion in the radial direction.

This condition must be enforced by use of a user written UCONST since the condition involves two displacement components, U and W (see Fig. 2.1). Constraints against rotation and tangential displacement V can be introduced by the regular boundary condition cards.

We have

$$d\eta/d\xi = 2\sqrt{6.35}\ \frac{1}{2}\ \xi^{-\frac{1}{2}} = \sqrt{6.35/\xi}$$

i.e. at the shell edge

$$d\eta/d\xi = \sqrt{6.35/25.4} = 0.5$$

i.e. $tg\,\alpha = 0.5$

We have the constraint $u \sin\alpha + w \cos\alpha = 0$ or $u + 2w = 0$

In order to obtain better balance between the size of the coefficients in the final equation system, it is recommended that the equation is multiplied by the shell wall modulus.

The lineload at the small end will correspond to 1. N/mm if we apply a loads of $2/\sqrt{5}$ cos α in the U-direction. and $-1/\sqrt{5}$ $-\sin\alpha$ in the W-direction.

Data cards and user written subroutine UCONST and WALL are shown in Table 2.1.

2.2.4 Output

Table 2.1a

Data Cards for Example Case 1

```
EXAMPLE CASE 1 , PARABOLOID WITH STIFFENERS
C  THE FOLLOWING CARDS ARE INCLUDED : B-1, C-1, C-2, E-1, F-2, F-3, I-1, I-2,
C  J-1, K-1, K-2, K-3, L-1, L-2, L-2, L-1, L-2, M-1, M-2, M-4, O-1, P-1,
C  P-2A, P-2B, P-5
    1    1    1    0    0    5    0    0
   11    5
    1    0    0    0    1    0    2
 1.                                1.
    1    2   30    0
 0.        0.        0.            2
   10    2    0    0    0    0
 0.        50.8      0.          18.        6.35       25.4
    0    0
    0    4    2    4    1
    1    0    1    0
    2    4    2    5
    2    0
-.446         2    3    1    0
 .892         2    1    1    0
    1    0
 .2           4    3    0    0
    1    1    2    0 70000.    26900.     0.         0.
 .3175     0.        3.3175    0.
 .5        3.        0.        1.8175
    0    1    1    1    0
    3    1
 0.           1
 .635      70000.    .3
    1    1 6.
```

Table 2.1b

User Written Subroutine for Example Case 1

```
      SUBROUTINE UCONST
C     EXAMPLE CASE 1.
C     BOUNDARY CONDITIONS LINE 1
      DIMENSION IBRNCH(20),IX(20),IY(20),ID(20),CC(20)
      E=70000.
      IX(1)=1
      IX(2)=1
      ID(1)=1
      ID(2)=3
      CC(1)=E
      CC(2)=2.*E
      IBRNCH(1)=1
      IBRNCH(2)=1
      DO 10 I=1,5
      IY(1)=I
      IY(2)=I
   10 CALL CONSTR (2,IBRNCH,IX,IY,ID,CC)
      RETURN
      END
```

2.3 Example Case 2: Elliptic Cylinder

2.3.1 Case Description

Geometry:

An elliptic cylinder with dimensions as shown in Fig. 2.3 is considered.

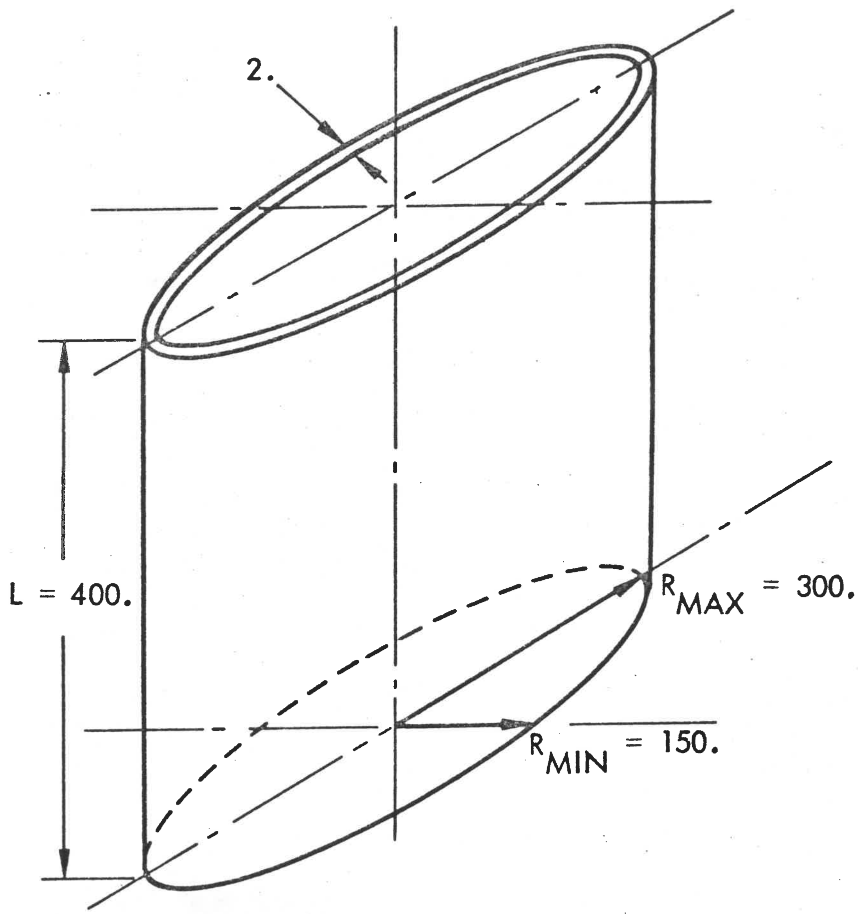

Fig. 2.3 Elliptic Cylinder

Both ends are simply supported (the shell is free to rotate at edge supports).

Material:

The single layer shell wall is made of steel with

$$E = 210\,000 \text{ MPa}$$

$$\nu = 0.3$$

and the density

$$\rho = 0.8\ 10^{-5} \text{ kg/mm}^3$$

Load:

A uniform axial line load is applied at one end.

Mode of Analysis:

The purpose of the analysis is to determine the collapse load (or shortening). While the prebuckling displacement pattern is symmetric about a plane containing the shell axis and the shortest radius, the collapse mode is either symmetric or antisymmetric about that plane. For simplicity we assume that the collapse mode is symmetric about the mid-plane and about a plane containing the shell axis and the longest radius. One possible way to determine the lowest collapse load is to consider a segment covering 180^o and half the length of the cylinder. A small imperfection is introduced to trigger antisymmetric deformation. A nonlinear analysis is performed under gradually increasing load until the load displacement curve approaches a point with a horizontal tangent. The other possibility, chosen here, is to analyze a 90^o half length segment only. In the nonlinear analysis symmetry conditions prevail on all sides except the loaded edge. A nonlinear analysis then will reveal only the load corresponding to symmetric collapse. An additional run is made in which bifurcation buckling into an antisymmetric mode is considered (nonlinear pre-buckling). Bifurcation buckling into a symmetric mode does not represent a rigorous solution, since the precritical deformation pattern contains a component of the buckling mode. However, for this case it is known to give results reasonably close to the collapse load (limit point). If we consider bifurcation from some point above zero load from the nonlinear configuration the approximation should be better. There, we will also attempt to determine a number of bifurcation buckling loads corresponding to antisymmetric modes at the different levels of the stress state. Such buckling loads may indicate that the critical load is being approached. This can also be indicated by the vanishing of the vibration frequency corresponding to the critical mode. We will perform four different analysis

1) Nonlinear symmetric analysis, save data on file
2) Buckling analysis, symmetric modes
   (The reason for this analysis is explained in the discussion of output)
3) Buckling analysis, antisymmetric modes
4) Vibration analysis, symmetric modes

Grid:

We will use a grid with 9 gridlines in each direction.

2.3.2 Input Preparation

In order to get some idea about the collapse load let us consider an equivalent cylinder with the radius of curvature equal to the maximum radius of curvature of the elliptic cylinder. The elliptic cylinder will probably collapse at a somewhat higher value of the line load. With a uniform shortening, rather than uniform load, the difference presumably would be bigger. The critical load (bifurcation) for the equivalent cylinder is 1680 N/mm. This corresponds to a stress that is well into the inelastic range. However, we chose to ignore this fact, rather than to use another example which would require the use of a finer grid.

With a base load of 1700 N/mm (L-2) we would want to interrupt the analysis if a load factor larger than 12. (card E-1) is reached within the allotted time. This may indicate errors in input data. We allow a total of 3 cuts of stepsize in order to prevent the load step to become smaller than 0.125 in which case we would like to interrupt the computations and reconsider the strategy.

Input data cards for the case are shown in Table 2.2.

2.3.3 Output

Table 2.2a

Data Cards for Example Case 2

```
EXAMPLE CASE 2 , ELLIPTIC CYLINDER   , NONLINEAR
C THE FOLLOWING CARDS ARE INCLUDED : B-1,C-1,E-1,E-2,E-3,F-1,I-1,I-2,J-1,K-1,
C L-1,L-2,O-1,O-2,P-1,P-2A,P-2B
    4    1    0    0    0    0    0 9.807
    6    9
 1.        1.        12.
  14     0
 1.        2.        3.        4.        5.        6.        7.        8.
 9.        9.5       10.       10.5      11.       11.5
    0  110    3   10    1
    9    1    0    0    0    0
 0.        .125      0.        90.       .3        .15       .3
    0    0    0
    1    4    4    4
    1    0
 1700.        2    1    1    0
    0    4    2    4    1    1
    3    9    9
    3    0
 0.           1
 2.        21   E+ 6 .3                                      8    E- 5

ELLIPTIC CYLINDER , SYMMETRIC BUCKLING
   REVISE CARDS A-1, E-2, O-1, ADD CARDS F-2, F-3, ELIMINATE CARD O-2,
    10    3
    0    1    1    0    0    1
    1    2  100
 0.        0.        0.        1
ELLIPTIC CYLINDER , ANTISYMMETRIC BUCKLING
   REVISE CARDS A-1, K-1,
    1    5    4    4
ELLIPTIC CYLINDER , VIBRATION
   REVISE CARDS A-1, B-1, F-3, K-1,
    5    1    0    0    0    0    0 9807.
 0.        0.        0.        2
    1    4    4    4
```

2.4 Example Case 3: Cylindrical Panel with Free Edges. Transient Response

2.4.1 Case Description

Geometry:

A shallow cylindrical panel with geometry as shown in Fig. 2.5 is considered. The longitudinal edges are free and the curved edges are simply supported.

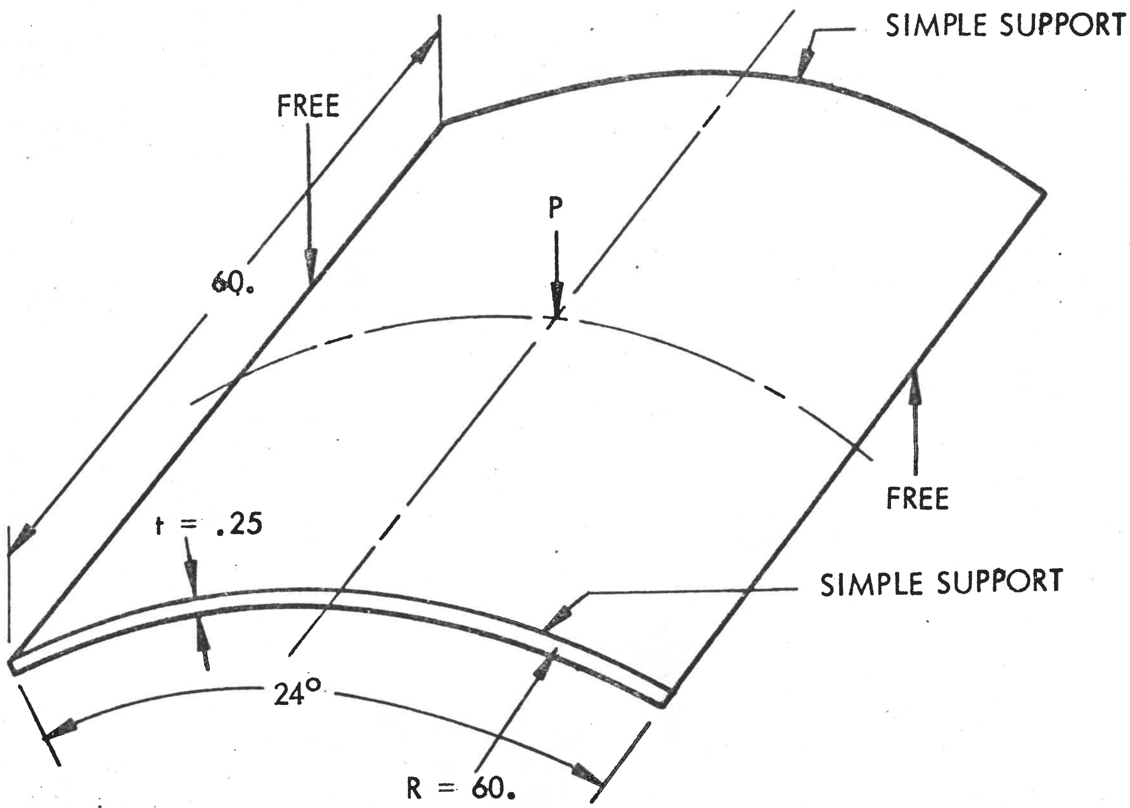

Fig. 2.4 Cylindrical Shell Panel

Material:

The single layer shell wall is made of aluminum with

$$E = 70\,000 \quad \text{MPa}$$

$$\nu = 0.3$$

$$\rho = 0.26\ 10^{-5}\ \text{kg/mm}^3$$

Load:

The shell is subjected to impulsive loading. The load is applied at the center of the panel and directed towards the cylinder axis (see Fig. 2.4). The size of the impulse is 10^{-3} N sec.

Mode of Analysis:

A transient response analysis is carried out over a time span of 10^{-3} seconds. We will use the option of explicit time integration (see Volume 2).

Only a quarter of the panel needs to be considered because of prevailing symmetry conditions.

Grid:

We will use a grid with 6 lines in the axial direction and 5 in the circumferential direction (3^o spacing).

2.4.2 <u>Input Preparation</u>

The velocities used for computation of the time step (Vol. 2,Eq. 6.1) are:

$$C = \sqrt{70\,000 / (0.91 \cdot 0.26 \cdot 10^{-5})}$$

$$= 1.72\ 10^5 \ \text{mm/sec}$$

$$C_S = \sqrt{70\,000 / (2.6 \cdot 0.26 \cdot 10^{-5})}$$

$$= 1.02 \; 10^5 \text{ mm/sec}$$

The grid spacings in the two directions are

$$\Delta\beta = 6 \text{ mm} \qquad \Delta\alpha = \frac{3}{360} \cdot 2\pi \cdot 60 = 3.14 \text{ mm}$$

The time step is

$$\Delta t = \text{Min} \begin{cases} 1/\sqrt{\left(\frac{1.72 \cdot 10^5}{3.14}\right)^2 + \left(\frac{1.02 \cdot 10^5}{6}\right)^2} = 1.73 \cdot 10^{-5} \\ \frac{\sqrt{3}}{0.25 \cdot 1.72 \cdot 10^5} \qquad \frac{1}{(1/6)^2 + (1/3.14)^2} = 31 \cdot 10^{-5} \end{cases}$$

Since the analysis is nonlinear we choose a slightly shorter time step

$$\Delta t = 1.6 \cdot 10^{-5} \text{ sec}$$

The impulsive loading is applied by letting a constant load P act over a period equal to one time step.

$$P = \frac{10^{-3}}{1.6 \cdot 10^{-5}} = 62.5 \text{ N.}$$

The load profile in time will be defined by use of data cards.

$$PA = \begin{cases} 62.5 \text{ if } T < 1.6 \cdot 10^{-5} \\ 0 \text{ if } T \geq 1.6 \cdot 10^{-5} \end{cases}$$

Table 2. 3a

Data Cards for Example Case 3

```
EXAMPLE CASE 3 ,CYLINDRICAL SHELL PANEL
C  THE FOLLOWING CARDS ARE INCLUDED : B-1,C-1,F-1,G-1,G-2,I-1,I-2,J-1,K-1,
C  L-1,L-2,O-1,O-2,P-1,P-2A,P-2B
    6    1    Ø    Ø    Ø    Ø    Ø 98Ø7.
    6    5
    Ø   6Ø
 Ø.        .ØØ1      .ØØØØ16   1.         Ø.         Ø.         Ø.        1.
    1    Ø    Ø    Ø 2.        .ØØØØ16   Ø.         -62.5
    5    1    Ø    Ø    Ø    Ø
 Ø.        .3        Ø.        12.       .6
    Ø    Ø    Ø
    1    4    4    3
    1    Ø
-1.            1    3    6    5
    Ø    5   1Ø    Ø    1    Ø
    3    6    5
    3    Ø
 Ø.            1
 .25       7    E+Ø5 .3                                          26    E-Ø5
```

There is no damping and the parameters SUP and THOLD (G-1 card) are irrelevant for analysis with a constant time step. Input data cards are shown in Table 2.3.

2.5 Example Case 4: Conical Shell with End Ring

2.5.1 Case Description

Geometry:

A conical shell of sandwich construction with a ring attached at one end is considered. Dimensions and support conditions are indicated in Figure 2.6.

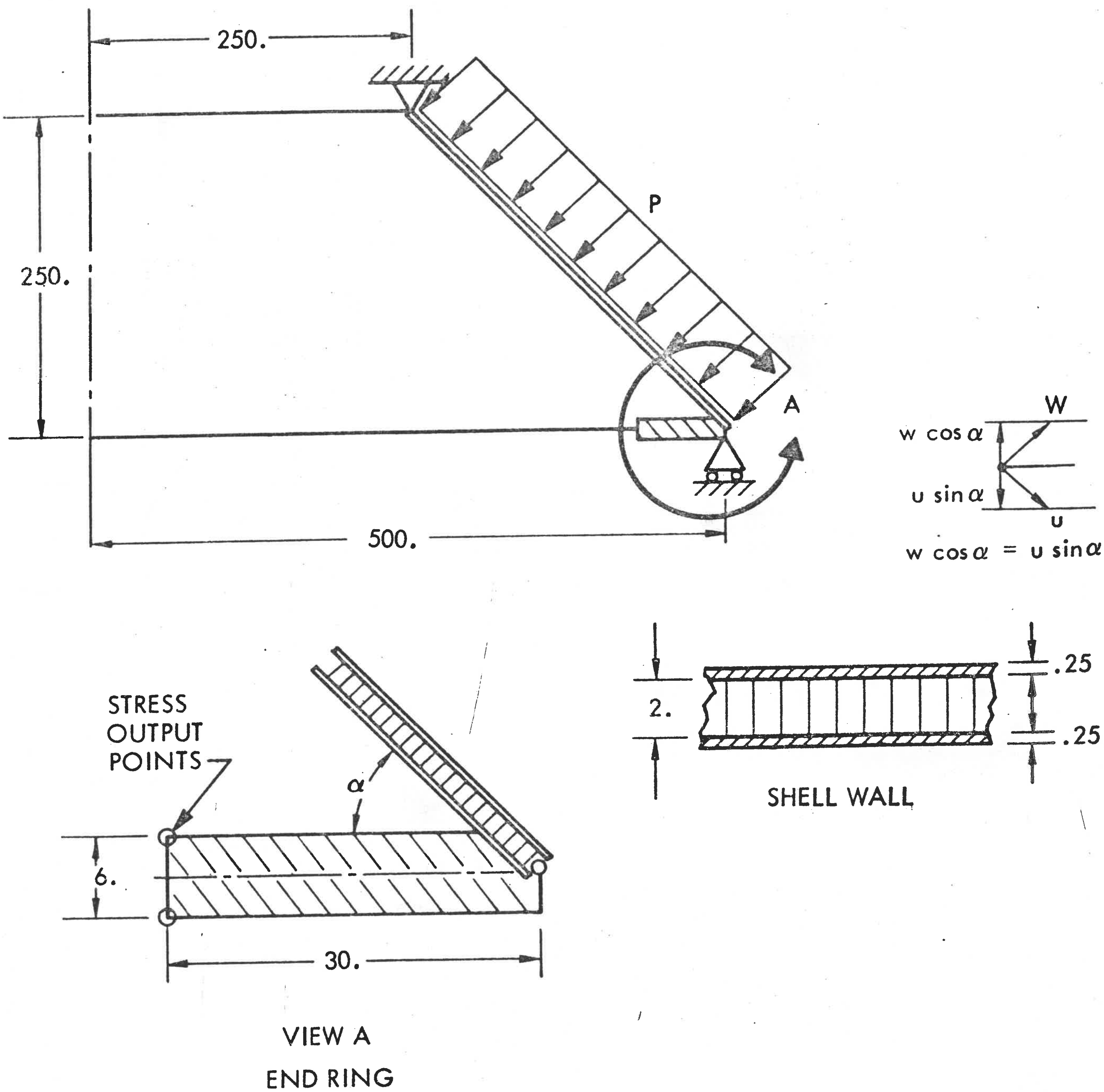

Fig. 2.6 Conical Shell

Material:

The shell is of aluminum sandwich construction. The middle layer carries only transverse shear. (Transverse shear deformation is neglected in the STAGS program).

Shell wall; inner and outer layers:

$E = 70\ 000$ MPa

$\nu = 0.3$

$G = 26\ 900$ MPa

Stiffener:

$E = 70\ 000$ MPa

$\nu = 0.3$

$G = 26\ 900$ MPa

Load:

A uniform external pressure of 0.1 MPa is applied.

Grid:

Since the load as well as the structure is axially symmetric a specialized program could have been used. Using STAGS we consider a narrow strip, say 5^o; two columns are defined. In the axial direction we define a grid that has a constant spacing within each of three segments as shown in Figure 2.7.

Mode of Analysis:

A linear stress analysis is requested. Displacements, stress resultants and stresses are printed at all points. Stress in the stiffener is computed for the three points marked in Figure 2.6.

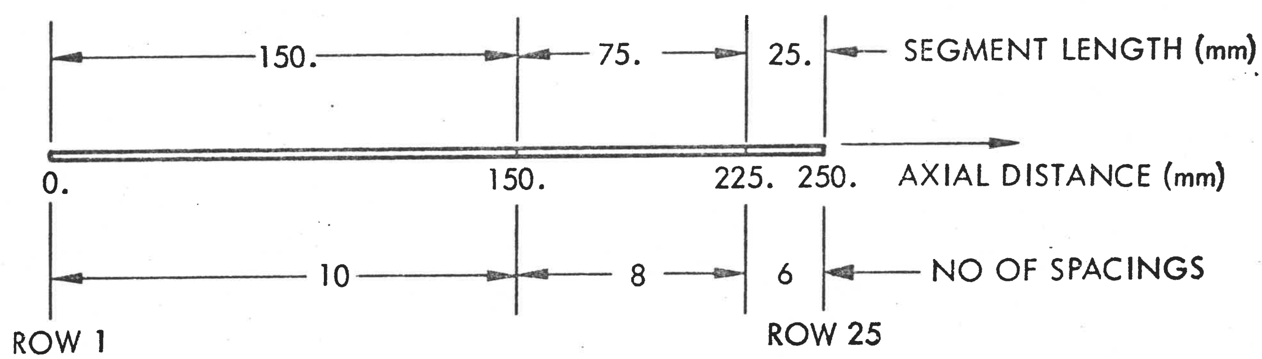

Fig. 2.7 Grid in Axial Direction

2.5.2 Input Preparation

Stiffener properties:

For a rectangular stiffener we can use the special input only if the angle α (Figure 2.6) equals 90^o. The general case is illustrated in Figure 2.8.

Example Case 4

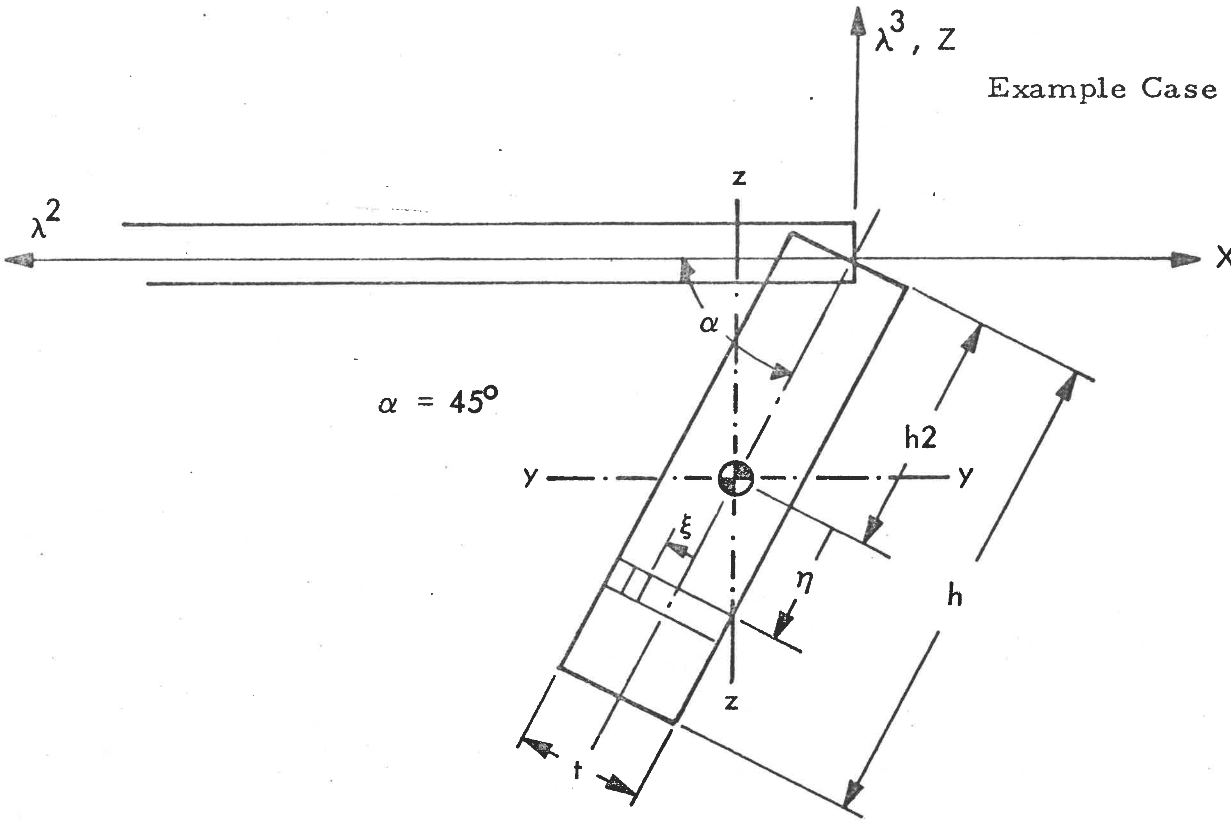

Figure 2.8 Stiffener Geometry

We notice that the λ^2-axis is positive in the opposite direction to X. This is necessary in order that λ^1, λ^2, λ^3 be a right-handed system. For notations see M-3 card (Volume 2)

$$SA = ht = 180 \text{ mm}^2$$

$$SIY = \int_{-h/2}^{h/2} \int_{-t/2}^{t/2} (\eta \cos\alpha \pm \xi \sin\alpha)^2 \, d\xi d\eta =$$

$$= \frac{th}{12} (h^2 \cos^2\alpha + t^2 \sin^2\alpha) = 7020 \text{ mm}^4$$

$$SIZ = \int_{-h/2}^{h/2} \int_{-t/2}^{t/2} (\eta \sin\alpha - \xi \cos\alpha)^2 \, d\xi d\eta =$$

$$= \frac{th}{12} (h^2 \sin^2\alpha + t^2 \cos^2\alpha) = 7020 \text{ mm}^4$$

$$SIYZ = -\int_{-h/2}^{h/2} \int_{-t/2}^{t/2} -(\eta \cos\alpha - \xi \sin\alpha)(\eta \sin\alpha + \xi \cos\alpha)\, d\xi d\eta =$$

$$= \frac{th}{12} \sin\alpha \cos\alpha \, (h^2 - t^2) = 6480 \text{ mm}^4$$

SEY = 10.61 mm

SEZ = -10.61 mm $\qquad$ $SJ = 30 \cdot 6^3/3 = 2160 \text{ mm}^4$

Stress output points:

U1(I) = 0.0 , $33 \sin 45^o$ = 23.3 , $27 \sin 45^o$ = 19.1

Z1(I) = 0.0 , - 19.1 , - 23.3

Boundary Conditions:

At the small end of the cone the shell is free to rotate but the three displacement components are constrained.

At the large end of the cone the shell again is free to rotate. The circumferential displacement can be constrained on regular data cards. The displacement in the axial direction is constrained, i.e., $U \sin\alpha - W \cos\alpha = 0$. We leave both U and W free on the regular data cards and enforce the condition U - W = 0 ($\alpha = 45^o$) in a user written UCØNST. The condition is enforced at columns 1 and 2 and row 25.

The regular data cards and the user written subroutine are listed in Table 2.4.

Table 2.4a

Data Cards for Example Case 4

```
EXAMPLE CASE 4 , CONICAL SHELL WITH END RING
C THE FOLLOWING CARDS ARE INCLUDED : B-1,C-1,C-2,E-1,I-1,I-2,J-1,J-2,J-3,K-1,
C K-2,L-1,L-2,M-1,M-2,M-3,N-1,O-1,P-1,P-2A,P-2B(3 CARDS)
    0    1    1    0    0    2    0
   25    2
    1    1    0    0    1    0    3
  1.
    6    1    0    0    0    0
  0.         250.       0.         5.         250.       500.
    3    0    0
  150.       75.        25.
   10    8    6
    0    4    0    4
    0    0    0    1
    1    0    1    1
    1    0
 -1.              4    3    0    0
    1    0    3    0 70000.    26900.     0.         0.
  0.         0.        -23.3      19.1      -19.1       23.3
  180.       7020.      7020.      6480.      2160.      10.61      -10.61
    1   25    1    2
    0    1    1    1    0    1
    3    0
  0.              3    0    0    1
  .25        70000.     .3
  2.         0.         0.
  .25        70000.     .3
```

Table 2.4b

User Written Subroutine for Example Case 4

```
      SUBROUTINE UCONST
C     EXAMPLE CASE 4
C     BOUNDARY CONDITIONS ON LINE 3.
      DIMENSION IBRNCH(20),IX(20),IY(20),ID(20),CC(20)
      E=70000.
      IX(1)=25
      IX(2)=25
      ID(1)=1
      ID(2)=3
      CC(1)=E
      CC(2)=-E
      IBRNCH(1)=1
      IBRNCH(2)=1
      DO 10  I=1,2
      IY(1)=I
      IY(2)=I
   10 CALL CONSTR(2,IBRNCH,IX,IY,ID,CC)
      RETURN
      END
```

2.6 Example Case 5: Plates with Variable Thickness

2.6.1 Case Description

A rectangular wing consists of two plates with variable thickness connected to one another by two hinges with torsional springs. Geometric data are shown in Fig. 2.9.

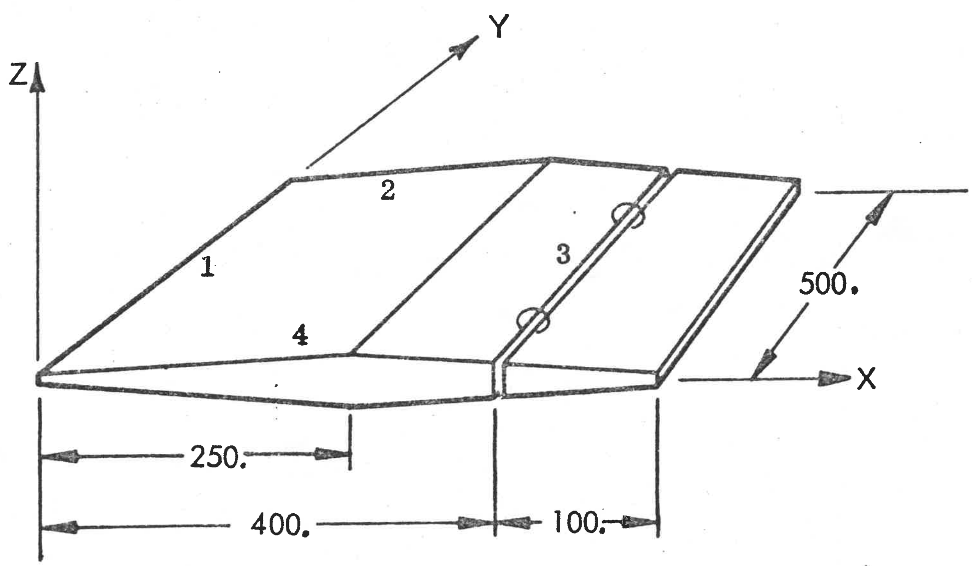

Fig.2.9 Plate Assembly

The two torsion springs are positioned 100 mm from the plate edges (Y = 100, Y = 400 mm). The thickness of the plate combination is

6. + 54. * X/250. if X < 250

and 60. - 54.* (X-250) / 250.if X > 250

On the larger plate the edge at Y = 500 is clamped. All other edges on both plates are free except for the hinges with torsional springs connection between the plates.

Each of the torsional springs have a stiffness of 10^7 N mm/radian.

Material:

The single layer plate wall is made of aluminum with

$$E = 70\,000 \text{ MPa}$$
$$\nu = 0.3$$

and the mass density

$$\rho = 0.26 \times 10^{-5} K_g/mm^3$$

Load:

There is no load on the plate assembly.

Mode of Analysis:

An analysis is requested which gives the four lowest modes of small vibration and corresponding frequencies.

Grid:

A finite difference grid with a 50 mm spacing in both direction is considered to be adequate.

2.6.2 Input Preparation

The two plates are defined as two separate shell branches. The hinges with torsional springs represent the only connection between the plates. The hinge connection is represented through user written subroutine UCONST, while the torsional springs are introduced as finite elements.

The plate thickness must be defined by a user written subroutine WALL. For the larger plate (BRANCH 1)

$$X \leq 250 \; ; \; T = 6. + 0.216\,X$$

$$X > 250 \; ; \; T = 114. - 0.216\,X$$

For the second plate we can set X = 400. on side 1 in which case we have:

$$T = 114. - 0.216\,X$$

Input data cards and the user written subroutine UCONST are shown in Table 2.5. Therefore there are no branch connection cards (NINT=0 on B-1 card).

Table 2. 5a.

Data Cards for Example Case 5

```
EXAMPLE CASE 5 , PLATE COMBINATION
C THE FOLLOWING GENRAL CONTROL CARDS ARE INCLUDED: B=1,C=1,F=2,F=3
    2    2    Ø    Ø    Ø    6    1 98Ø7.
    9   11    3   11
    1    2   3Ø    Ø
 Ø.        Ø.        Ø.        4
C THE FOLLOWING CARDS DEFINE FINITE ELEMENTS: H=1,H=3(4 CARDS),H=8(2 CARDS)
    Ø    Ø    Ø    4    Ø    Ø    2    Ø    Ø
    1    1    9    3
    2    1    9    9
    3    2    1    3
    4    2    1    9
    1    3 1     E+Ø8
    2    4 1     E+Ø8
C THE FOLLOWING CARDS ARE INCLUDED FOR BRANCH 1 : I=1,I=2,J=2,K=1,O=1,P=1
    2    Ø    Ø    Ø    Ø    Ø
 Ø.        4ØØ.      Ø.        5ØØ.
    Ø    Ø    Ø
    3    2    3    3
    1    1    1    1    Ø    Ø
    1
C THE FOLLOWING CARDS ARE INCLUDED FOR BRANCH 2 : I=1,I=2,J=2,K=1,O=1,P=1
    2    Ø    Ø    Ø    Ø    Ø
 4ØØ.      5ØØ.      Ø.        5ØØ.
    Ø    Ø    Ø
    3    3    3    3
    1    1    1    1    Ø    Ø
    1
```

Table 2.5b

User Written Subroutines Example Case 5

```
      SUBROUTINE WALL (IBRNCH,X,Y,Z,ICFB,ISTFF,IPRW,RHOA,CCC)
C     EXAMPLE CASE 5
      COMMON/LAYU1/TL(20),EX3(20),EY3(20),U21(20),G3(20),ZET3(20),
     1           RHO3(20),LAYS,LSTRS
      Z=0,
      ICFB=3
      ISTFF=1
      IPRW=1
      EX3(1)=70000.
      EY3(1)=EX3(1)
      U21(1)=.3
      G3(1)=EX3(1)/(2.*(1.+U21(1)))
      ZET3(1)=0.
      RHO3(1)=.0000026
      LAYS=1
      LSTRS=0
      TL(1)=6.+54.*X/250.
      IF(IBRNCH.EQ.1) RETURN
      TL(1)=60.-54.*(X-250.)/250.
      RETURN
      END

      SUBROUTINE UCONST
C     EXAMPLE CASE 5
C     HINGE CONNECTIONS
      DIMENSION IBRNCH(20),IX(20),IY(20),ID(20),CC(20)
      E=70000.
      IBRNCH(1)=1
      IBRNCH(2)=2
      IX(1)=9
      IX(2)=1
      CC(1)=E
      CC(2)=-E
      DO 10 I=1,3
      CALL CONSTR(2,IBRNCH,IX,3,I,CC)
   10 CALL CONSTR(2,IBRNCH,IX,9,1,CC)
      RETURN
      END
```

2.7 Example Case 6. Spherical Cap with Gravity Load

2.7.1 Case Description

Geometry:

A spherical cap as shown in Fig. 2.10 is supported at four points around the circumference.

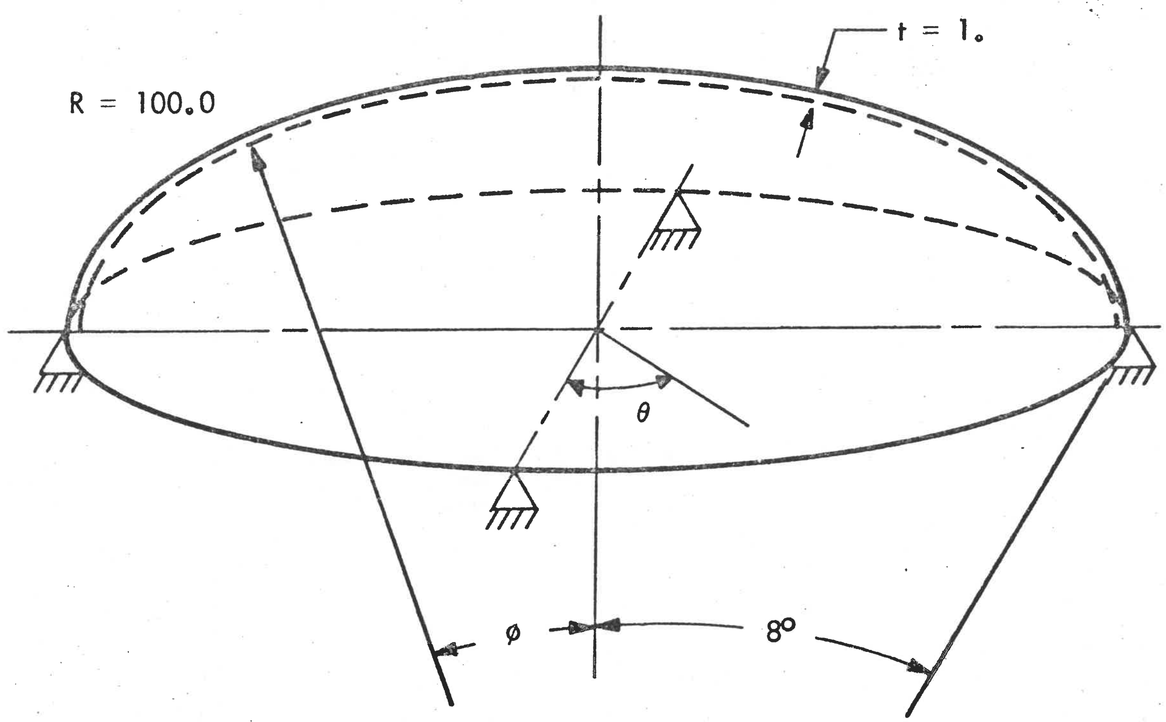

Fig. 2.10 Spherical Cap on Point Support

Material:

The single layer shell wall as well as the stiffeners are made of aluminum with

$$E = 70\,000 \text{ MPa}$$

$$\nu = 0.3$$

Load:

The shell is subjected to gravity load directed along the axis. The size of the load is 1.5 MPa.

Mode of Analysis:

A linear stress analysis is requested. Due to symmetry conditions only a 45° section of the cap needs to be considered.

Grid:

A grid with 9 gridlines in each direction is suggested. The grid is made somewhat closer in both directions in the neighborhood of the point support.

2.7.2 Input Preparation

The cap will be modeled as a shell branch extending from $\phi = 1°$ to the edge at $\phi = 8°$. At the apex we will cover the one degree hole by use of 8 triangular finite elements. The finite difference grid has rows at ϕ = 1, 2.5, 4, 5, 6, 6.5, 7, 7.5 and 8° and columns at θ = 0., 2, 5, 9, 15, 21, 29, 37, 45°.

Since the load has components both in the normal and tangential (along meridian) directions and these vary with the shell coordinate the easiest way to define the load is by use of a user written subroutine USRLD.

In the normal direction, we have $P_W = -1.5 \cos \phi$ and in tangential direction $P_U = 1.5 \sin \phi$ MPa.

On the finite elements we apply one third of the load on the element at each corner. The two loads (on each element) at the circumference are most easily applied as a line load on <u>the shell segment.</u> The load at the apex is one third of the load on the 1.0^o cap, that is on a circle with a radius of

$$r = R \quad \sin \ 1.0^o = 1.75.$$

On the finite element node at the apex we apply in the x-direction (global system)

$$P_x = \frac{1}{3} * \frac{45}{360} * 1.5 * \pi * 1.75^2 = 0.6 \ N$$

The line loads on the shell segments are

$$P_w = -2 * 0.6 * \cos (1.0^o) / (1.75 * \pi * 45/180)$$

$$= -0.873 \ \cos (1.0^o) \ N/mm$$

and

$$P_u = 0.873 \sin (1.0^o) \text{ N/mm.}$$

Boundary Conditions:

Symmetry conditions prevail along the meridional edges (sides 2 and 4) while the sides 1 and 3 are left free. The support at $\theta = 0$, $\phi = 8$ is introduced by use of load cards (zero displacement).

Input data cards and user written subroutine USRLD are shown in Table 2.6.

Table 2.6a

Data Cards for Example Case 6

```
EXAMPLE CASE 6 , SPHERICAL CAP WITH GRAVITY LOAD
C THE FOLLOWING GENERAL CONTROL CARDS ARE INCLUDED : B=1,C=1,E=1
    0    1    0    0    0    0    1
    9    9
 1.
C THE FOLLOWING CARDS DEFINE FINITE ELEMENTS : H=1,H=2,H=3(9 CARDS),H=4,H=5,
C H=6,H=10(8 CARDS)
    0    0    1    9    1    0    0    0    8
         1 -100.     0.        0.             11111
    2    1    1    1
    3    1    1    2
    4    1    1    3
    5    1    1    4
    6    1    1    5
    7    1    1    6
    8    1    1    7
    9    1    1    8
   10    1    1    9
    1
.6            1    1    1
70000.     .3        26900.    0.        0.
    1    2    3 1.      0.
    1    3    4 1.      0.
    1    4    5 1.      0.
    1    5    6 1.      0.
    1    6    7 1.      0.
    1    7    8 1.      0.
    1    8    9 1.      0.
    1    9   10 1.      0.
```

Table 2.6a

Continued

```
C J=7(2 CARDS),K=1,L=1,L=2(3 CARDS),O=1,P=1,P=2A,P=2B
C THE FOLLOWING CARDS DEFINE THE SHELL BRANCH : I-1,I-2,J-1,J-4(2 CARDS),
     7    1    0    0    0    0
 1.        8.        0.        45.       100.
   -1    -1
 1.        2.5       4.        5.        6.        6.5       7.        7.5
 8.
 0.        2.        5.        9.        15.       21.       29.       37.
 45.
     3    4    3    4
     3    1
 0.             -1    1    9    1
 0.             -1    2    9    1
 0.             -1    3    9    1
     1    1    1    1    0    1
     3
 0.              1
 1.        70000.     .3
```

Table 2.6b

User Written Subroutine for Example Case 6

```
      SUBROUTINE USRLD (IBRNCH,X,Y,NROW,NCOL,K)
C     EXAMPLE CASE 6
C     SURFACE TRACTIONS
C
      DO 10  N=1,NROW
      DO 10  M=1,NCOL
      TH=3.14159*X(N)/180.
      P1=-1.5*COS(TH)
      P2=1.5*SIN(TH)
      CALL FORCE (P1,4,3,N,M)
   10 CALL FORCE (P2,4,1,N,M)
C
C     LINE LOADS
C
      DO 20 M=1,NCOL
      P1=-.873*.9998
      P2=.873*.0175
      CALL FORCE (P1,2,3,1,M)
   20 CALL FORCE (P2,2,1,1,M)
      RETURN
      END
```

2.8 Example Case 7 Quadrilateral Plate with Discrete Stiffeners

2.8.1 Case Description

Geometry:

A quadrilateral plate with five discrete stiffeners is considered. The geometry of plate and stiffeners is shown in Fig. 2.11

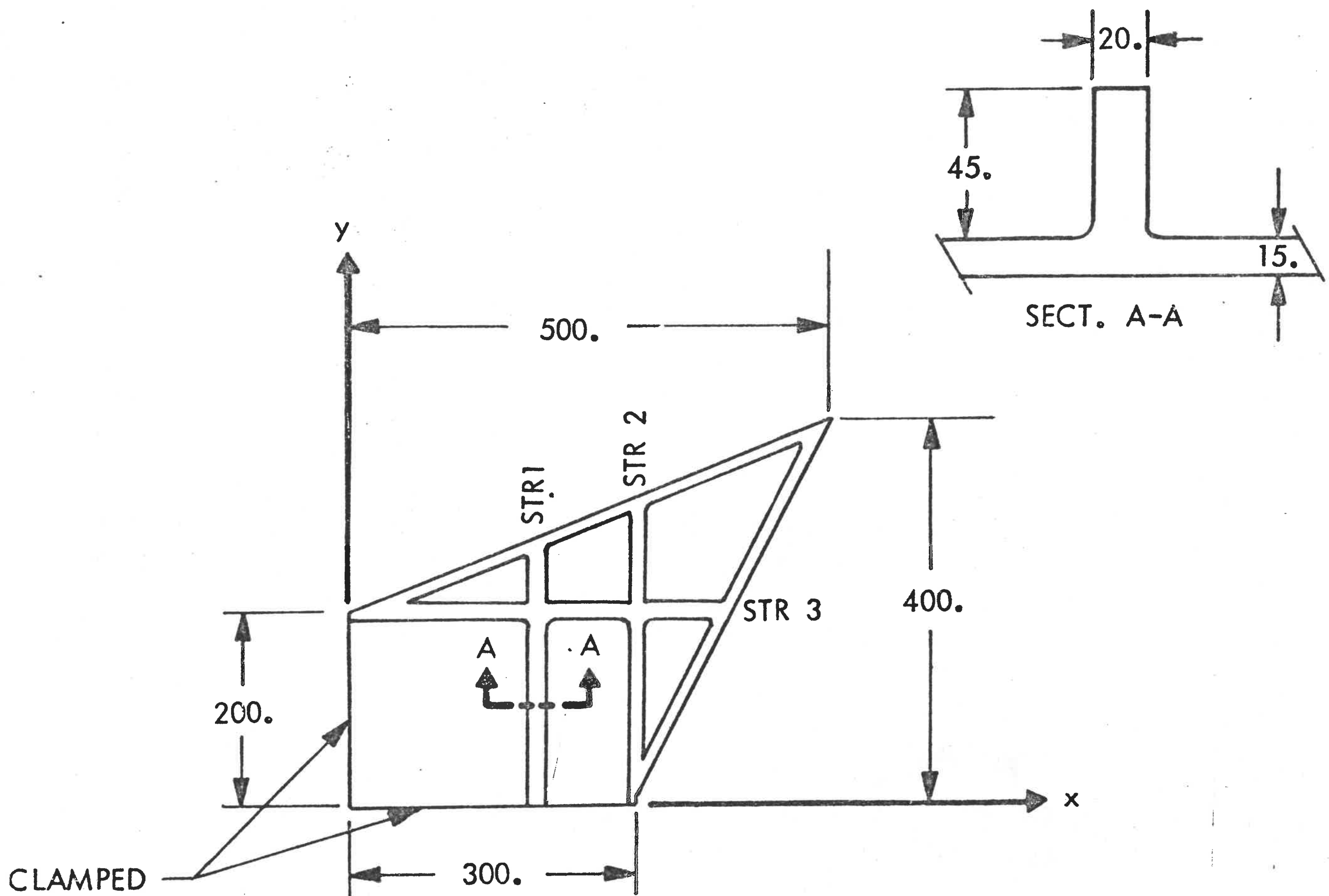

Fig. 2.11 Quadrilateral Plate with Stiffeners

The plate thickness is uniform and all the stiffeners have the same cross-section. Two of the plate edges are clamped as shown in the figure and the other two are free.

Material:

The plate is made of a composite material consisting of three fiber-reinforced layers, each of .05 mm thickness and with the fiber orientations 0^o, 90^o, and 0^o with respect to the x-axis.

Each of the layers have the following material properties

E_1	=	290 000	MPa
E_2	=	6 100	MPa
G	=	4 200	MPa
ν_{12}	=	.0065	

The stiffeners are made of the same material with the fibers running in the direction of the stiffener.

Load:

The plate is subjected to a uniform lateral pressure of 0.5 MPa.

Mode of Analysis:

Linear stress analysis.

Grid:

A reasonably accurate analysis should be obtained by use of a grid with 9 points on the side along the x-axis and 7 points on the side along the y-axis. A uniform spacing can be used.

2.2.2 Input Preparation

A standard geometry routine is available for the quadrilateral plate. For this case the inplane displacement components are not orthogonal. The angle ζ (see Fig. 3.10 in Volume 2) between the fiber direction in the layers and the shell coordinate X′ varies with the shell coordinate Y′, Fig.2.12 Consequently a user written subroutine WALL will be needed for definition of the shell wall properties.

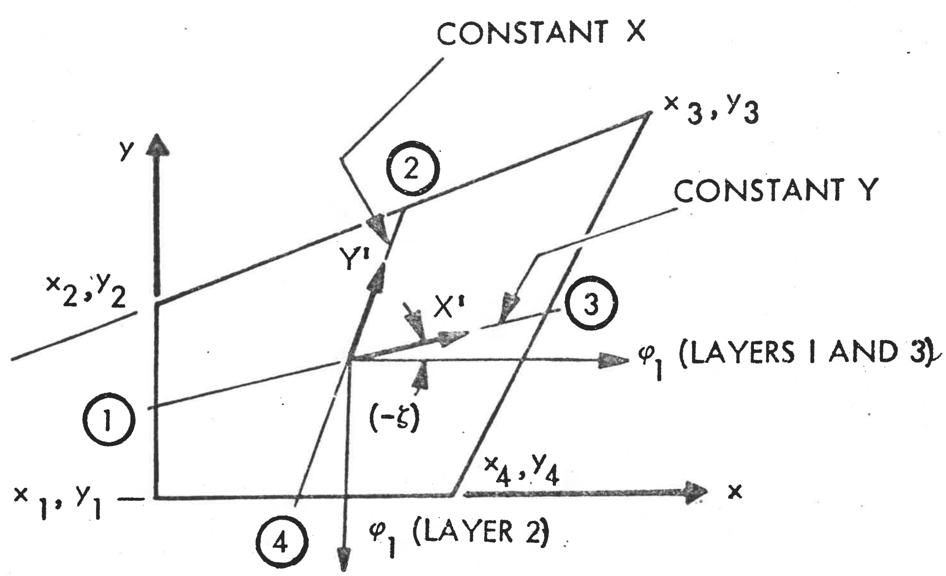

Fig. 2.12 Orientation of Material Coordinates

The angle ζ (in degrees) for layers 1 and 3 is given by

$$\zeta_1 = -(180\ \pi)\arctan[(\partial y/\partial X)\ /\ (\partial x/\partial X)]$$

The angle ζ for the second layer is given by

$$\zeta_2 = \zeta_1 - 90^{\circ}$$

In the standard geometry routine

$$x = (x_4 - x_1)\ X + (x_2 - x_1)\ Y + (x_1 - x_2 + x_3 - x_4)\ XY + x_1$$

$$y = (y_4 - y_1)\ X + (y_2 - y_1)\ Y + (y_1 - y_2 + y_3 - y_4)\ XY + y_1$$

Here X is the distance along the side $x_1y_1 \rightarrow x_4y_4$ (side 4) normalized with respect to the length of that side. X is zero at x_1y_1 and x_2, y_2, it is equal to 1.0 at x_3, y_3 and x_4, y_4). Y is the distance along the side $x_1y_1 \rightarrow x_2y_2$ normalized with the respect to the length of that side (side 1).

Substituting actual values for the corner coordinates we find

$$x = 300\ X + 200\ XY \qquad \text{(a)}$$

$$y = 200\ Y + 200\ XY \qquad \text{(b)}$$

or

$$\partial y/\partial X = 200\ Y$$

$$\partial x/\partial X = 300 + 200\ Y$$

The tangent for the slope of a gridline at constant Y is

$$\frac{dy}{dx} = \frac{200\ Y}{300 + 200\ Y} = Y/(1.5 + Y)$$

We have three stiffeners off gridlines, and the location of these are most easily defined by use of a user written subroutine GSLAG. Two of the stiffeners run on a constant value of the Cartesian coordinate x, say x_o. With

$$300\ X + 200\ XY = x_o$$

$$YC = Y = (x_o - 300X)/\ 200X$$

$$XC = X = x_o\ /\ (300 + 200Y)$$

$$F = \partial(XC)\ /\ \partial Y = -\frac{200\ x_o}{(300 + 200Y)^2}$$

For a stiffener on constant value of the cartesian coordinate y, we have

$$200\ Y + 200\ XY = y_o$$

$$XC = (y_o - 200Y)\ /\ (200\ Y)$$

$$YC = y_o\ /\ [200\ (1 + X)]$$

$$G = \partial\ (YC)\ /\ \partial X = -\frac{y_o}{200\ (1 + X)^2}$$

Stringer no. 1 starts at x, y = 200, 0 and ends at x, y = 200, 280. That is (from equations (a) and (b)):

XS, YS = 0.667, 0.0

XE, YE = 0.4 , 1.0

Stringer no. 2 starts at x, y = 300, 0 and ends at x, y = 300, 320

XS, YS = 1.0 , 0.0

XE, YE = 0.6 , 1.0

Stringer no. 3 starts at x, y = 0, 200 and ends at x, y = 400, 200

XS, YS = 0.0 , 1.0

XE, YE = 1.0 , 0.5

Data cards and user written subroutine WALL and GSLAG are shown in Table 2.7.

Table 2.7a

Data Cards for Example Case 7

```
EXAMPLE CASE 7, QUADRILATERAL PLATE
C THE FOLLOWING CARDS ARE INCLUDED : B-1,C-1,C-2,E-1,I-1,I-2,J-1,K-1,L-1,L-2,
C M-1,M-2,M-4,O-1,P-1
    0    1    1    0    0    0    0
    9    7
    1    1    1    3    5    0    2
 1.
    3    1
 0.         0.         0.         200.       500.       400.       300.       0.
    0    0    0
    2    3    3    2
    1    0
 .5             4    3    0    0
    1    1    2    0 290000.   4200.      0.         0.
 7.5        0.        52.5       0.
 20.        45.        0.        30.
    0    1    1    1    0    1
    1    0
```

Table 2.7b

User Written Subroutines for Example Case 7

```
      SUBROUTINE WALL (IBRNCH,X,Y,Z,ICFH,ISTFF,IPRW,RHOA,CCC)
C     EXAMPLE CASE 7
      COMMON/LAYO1/TL(20),EX3(20),EY3(20),U21(20),G3(20),ZET3(20),RHO3
     1(20),LAYS,LSTRS
      Z=0.
      ICFH=3
      ISTFF=0
      IPRW=1
      DO 10 I=1,3
      TL(I)=5.
      EX3(I)=290000.
      EY3(I)=6100.
      U21(I)=.0065
   10 G3(I)=4200.
      LAYS=3
      LSTRS=1
      DYX=Y*(1.5+Y)
      ZET3(1)=-(3.14159/180.)*ATAN(DYX)
      ZET3(2)=ZET3(1)-90.
      ZET3(3)=ZET3(1)
      RETURN
      END
```

Table 2.7b

Continued

```
      SUBROUTINE GSLAG(IBRNCH,K,X,Y,XC,YC,F,G,XS,YS,XE,YE)
C     EXAMPLE CASE 7
      IF (K.EQ.3) GO TO 10
      X0=200.
      IF (K.EQ.2) X0=300.
      XC=X0/(300.+200.*Y)
      YC=(X0-300.*X)/(200.*X)
      YS=0.
      YE=1.
      F=-200.*X0/(300.+200.*Y)**2
      IF (K.EQ.2) GO TO 20
      XS=.667
      XE=.4
      GO TO 30
   20 XS=1.
      XE=.6
      GO TO 30
   10 CONTINUE
      Y0=200.
      XC=(Y0-200.*Y)/(200.*Y)
      YC=Y0/(200.*(1.+X))
      XS=0.
      YS=1.
      XE=1.
      YE=.5
      G=-Y0/(200.*(1.+X)**2)
   30 RETURN
      END
```

2.9 Example Case 8: Cylindrical Shell with Rectangular Cutout

2.9.1 Case Description

Geometry:

A circular cylinder with a rectangular cutout is considered. Dimensions are shown in Figure 2.13

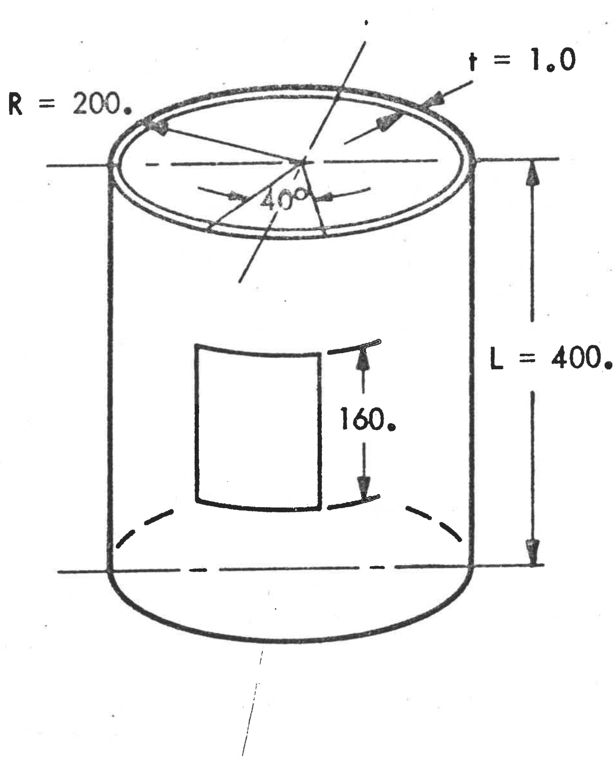

Figure 2.13 Cylinder with Cutout

The two ends of the cylinder are rigidly attached to rigid end plates.

Material:

The shell has a single layer aluminum wall.

$$E = 70\ 000 \quad \text{MPa}$$

$$\nu = 0.3$$

Load:

The shell is subjected to axial compression through the application of two opposite forces at the center of the end plates. The end plates are free to rotate.

Mode of Analysis:

A bifurcation buckling analysis is requested. The buckling mode is assumed to be symmetric about the midplane and an symmetric with respect to a plane through the cylinder axis and the center of the cutout.

Grid:

Due to symmetry in geometry and loading, we consider a segment covering 180° circumferentially and half the cylinder length. It is most efficient to use a grid with points concentrated in the area where the buckling occurs. A grid pattern of this type is suggested in Fig. 2.14 and used in the example case input.

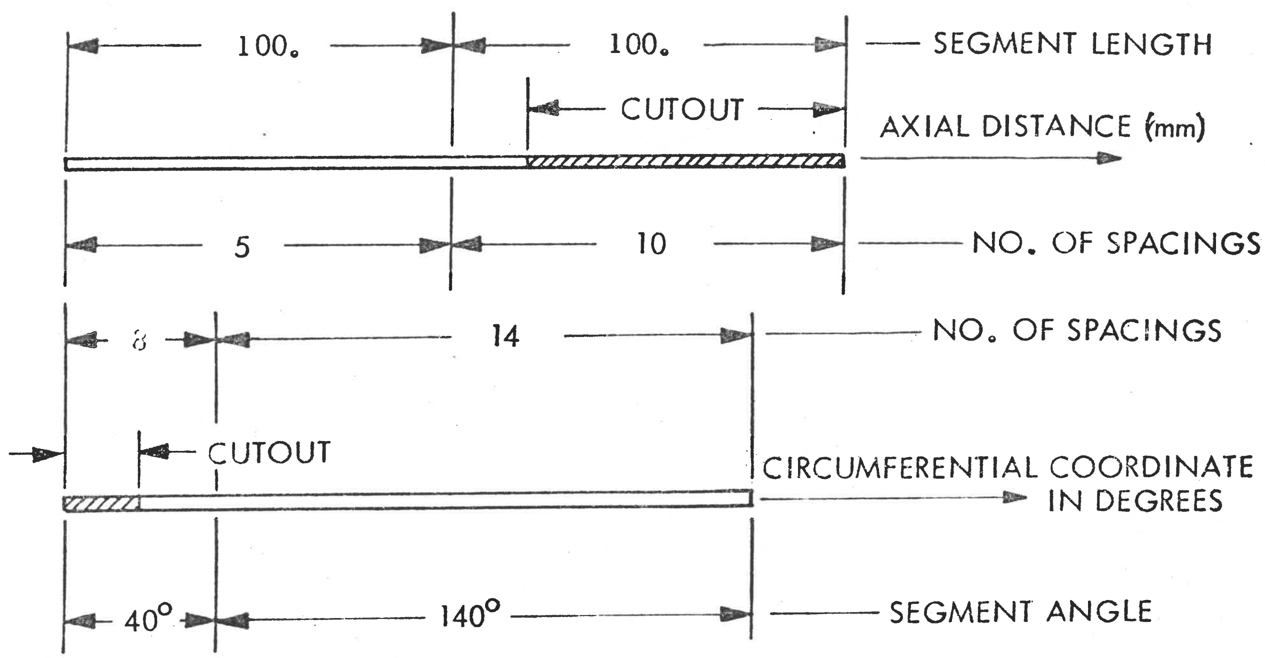

Fig. 2.14 Grid Spacings for Cylinder with Cutout

2.3.2 Input Preparation

The description of the geometry offers no difficulties in this case. The load can be applied either by definition of a force or a displacement. The computed buckling load is the same in those cases. (For a nonlinear collapse analysis, it would be better to increment the axial displacement.)

The only difficulty lies in the definition of boundary conditions. The end plates will tend to rotate on application of the load as shown in Figure 2.15.

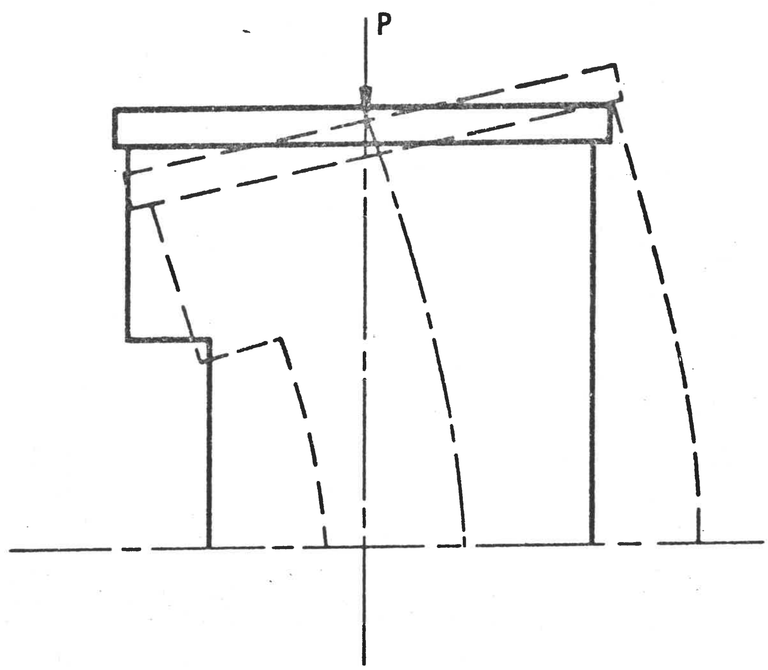

Fig. 2.15 Deformed Cylinder

One way to define the case is to add a fictitious very stiff ring at the cylinder end and to apply an axial line load along the edge. However, if the ring is too stiff the equation system may become ill conditioned; and if it is not stiff enough the chosen ring properties will affect the deformation of the cylinder. Another way to define the case is to use a user written subroutine, UCØNST, to constrain the points on the cylinder edge to remain on a circle, that is free to translate and rotate. Neglecting non-linear terms in the boundary conditions we have v = w = o. That is we can define simple support conditions. The end plate rotates through an angle Ω and translates through the distance u_o as shown in Fig. 2.16.

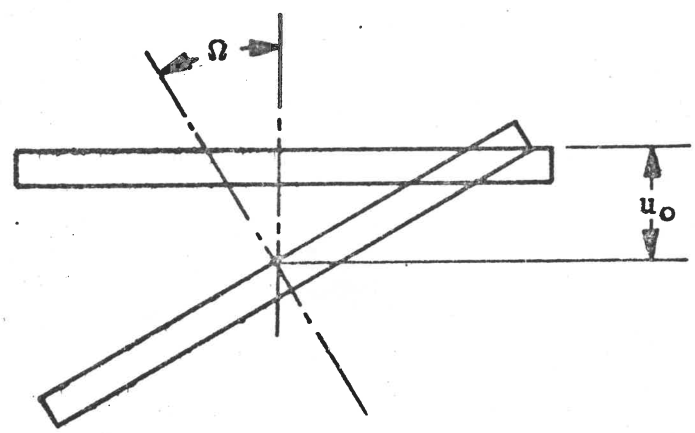

Fig. 2.16 Displacement of Cylinder Edge

The rotation of the end plate can be expressed in terms of any two values of the axial displacement.

With $\theta = 0$ at the center of the cutout the column no 14 will be at $\theta = 90^o$. Hence we can set:

$$tg\,\Omega = (U(1) - U(14))/R$$

The points on the cylinder edge will remain in the same plane if

$$U = U(14) + R \cos \theta \, tn\Omega$$

Since θ (or the surface coordinate Y) is not available in UCONST we define:

For $\begin{cases} J \leq 9, \theta_J = (J-1)\, 5\pi/180 \\ J > 9, \theta_J = 40\pi/180 + (J-9)\, 10\pi/180 \end{cases}$

(J = column number)

and the condition

$$U(J) = U(14) + (U(1) - U(14)) \cos \theta_J$$

This condition is enforced for all values of J except J=1 and J=14. U(14) is the load parameter (determined by data cards L-1, L-2). U(1) is unrestrained to give the plate the freedom to rotate.

Finally the rotation of the edge is given by the condition that the shell wall remains normal to the end plate. This requires that β_1 (rotational freedom) is given by

$$\beta_1 = -tg\, \Omega \cos \theta \text{ or}$$

$$\beta_1 (J) = - (U(1) - U(14)) \cos \theta_J / R$$

for J=1, 23

The total number of constraints is

NCONST = 21 + 23 = 44

Data cards and user written subroutine UCONST are shown in Table 2.8.

Table 2.8a

Data Cards for Example Case 8

```
EXAMPLE CASE 8 , CYLINDER WITH RECTANGULAR CUTOUT
C      THE FOLLOWING CARDS ARE INCLUDED :B=1,C=1,E=1,F=2,F=3,I=1,I=2,J=1,J=2,
C      J=3,J=5,J=6,J=8,K=1,L=1,L=2,O=1,P=1,P=2A,P=2B
    1    1    0    0    0   44    0
   16   23
 1.
    1    2  120    0
 0.        0.        0.             2
    5    1    0    0    0    0
 0.        200.      0.        180.      200.
    2    2    1
 100.      100.
    5   10
 40.       140.
    8   14
    8   16    5    0    0
    1    4    4    4
    1    0
 .001         -1    1    1   14
    1    1    1    1    0    1
    3    0
 0.           1
 1.        70000.    .3
```

Table 2.8b

User Written Subroutine for Example Case 8

```
      SUBROUTINE UCONST
C     EXAMPLE CASE 8
      DIMENSION IBRNCH(20),IX(20),IY(20),ID(20),CC(20)
      DIMENSION TH(23),ATH(23),BTH(23)
      H=3.14159/180.
      E=70000.
C   ATH(J)=E*COS(TH(J)), BTH(J)=E*COS(TH(J))
C   ATH(J)*U(1)*BTH(J)*U(14)-E*U(J)=0 FOR J=2,23 EXCEPT J=14
C   ATH(J)*U(1)-ATH(J)*U(14)+E*R*BETA1(J)=0 FOR  J=1,23
      IY(1)=1
      IY(2)=14
      DO 10 K=1,3
      IBRNCH(K)=1
      ID(K)=1
   10 IX(K)=1
      DO 40 J=1,23
      AJ=J
      IF(J.GT.9) GO TO 20
      TH(J)=(AJ-1.)*5.*H
      GO TO 30
   20 TH(J)=40.*H+(AJ-9.)*10.*H
   30 ATH(J)=E*COS(TH(J))
      BTH(J)=E-ATH(J)
      IY(3)=J
      ID(3)=4
      CC(1)=ATH(J)
      CC(2)=-ATH(J)
      CC(3)=E*200.
      CALL CONSTR (3,IBRNCH,IX,IY,ID,CC)
      IF(J.EQ.1.OR.J.EQ.14) GO TO 40
      ID(3)=1
      CC(2)=BTH(J)
      CC(3)=-E
      CALL CONSTR (3,IBRNCH,IX,IY,ID,CC)
   40 CONTINUE
      RETURN
      END
```

2.10 Example Case 9: Curved Tube in Bending

2.10.1 Case Description

Geometry:

A curved tube consists of a cylindrical shell attached to a segment of a toroidal shell. Dimensions are shown in Fig. 2.17

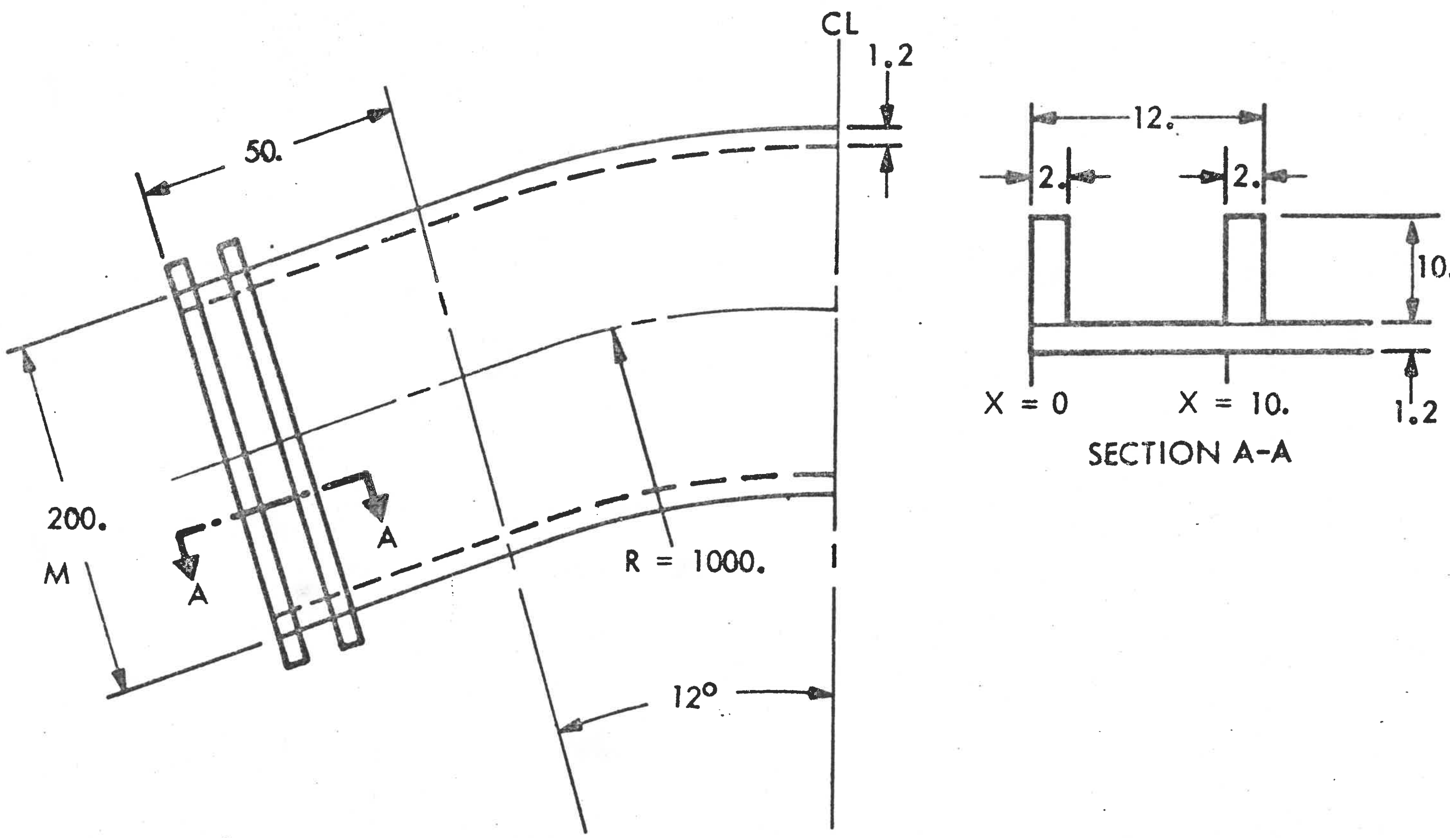

Fig. 2.17 Curved Tube

Material:

The single layer shell wall as well as the two ring stiffeners are made of aluminum with

$$E = 70\,000.\ \text{MPa}$$

$$\nu = 0.3$$

Load:

The tube is subjected to a bending moment M at each of the free edges. The applied line load is proportional to the distance to the axis, i.e., proportionally to cos θ, where θ is the angular coordinate. The total applied moment is $2*10^6$ Nmm.

Mode of Analysis:

A geometrically nonlinear static analysis is requested.

Grid:

A reasonably accurate result should be obtained with 13 gridlines in the circumferential direction. In the "axial direction" we use 5 gridlines on the cylindrical part and 9 gridlines on the toroidal part.

2.10.2 Input Preparation

Cartesian coordinate systems and the numbers corresponding to the boundary lines on each of the two branches are shown in Fig.2.18. The directions of the surface coordinates are also indicated. We notice that the Y-coordinate

The bending moment can be introduced as a number of point forces on regular data cards or through a user written subroutine USRLD. A USRLD-routine is defined here.

The line load on the free edge of the cylinder varies as

$$N_x = N_o \cos (Y)$$

$$\text{Where } N_o = M/(2R^2) = 2*10^6/(2*10^4) = 100$$

In order to prevent rigid body translation of the shell we enforce the condition V = 0 at column 7 (90^o) at the symmetry plane. W = 0 at column 1 (0^o) would also prevent rigid body translation, but this condition may cause excessive radial displacement since loads are applied in a "weak" shell direction. Load cards (L-1, L-2) are used for this purpose.

Data cards and user written subroutine USRLD are shown in Table 2.9.

on the cylinder, branch 1 side 3, is positive in the opposite direction to the X coordinate on the torus, branch 2 side 4. Therefore, on the D-1 card NBOUND must be negative.

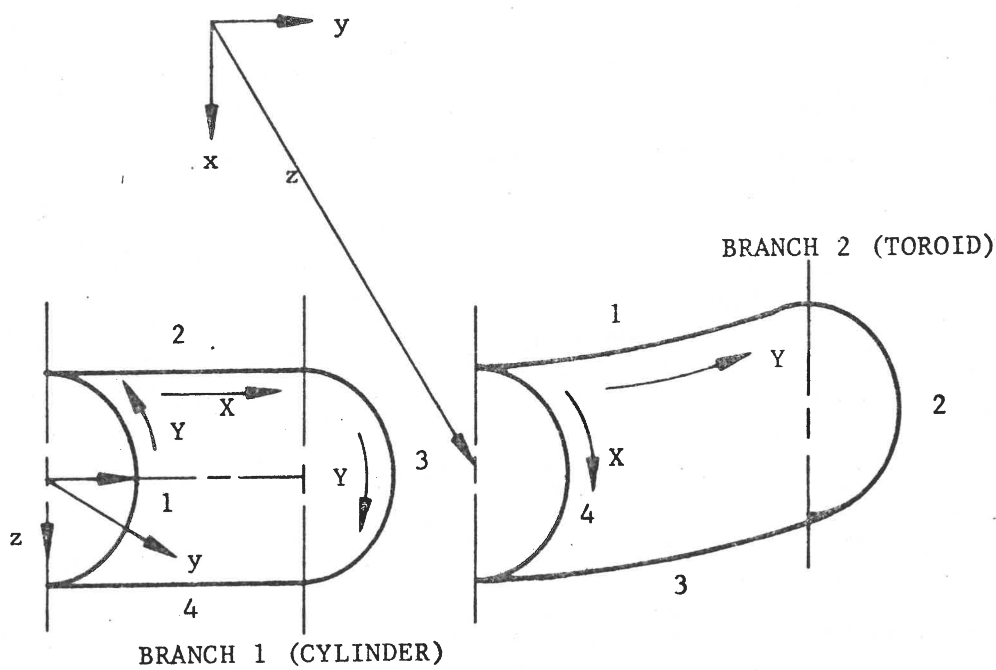

Fig. 2.18 Coordinate Systems for Torus-Cylinder Combination

Table 2.9a

Data Cards for Example Case 9

```
EXAMPLE CASE 9 , CURVED TUBE IN BENDING
C  THE FOLLOWING GENERAL CONTROL CARDS ARE INCLUDED : B=1,C=1,C=2,D=1,E=1,F=1
    3    2    1    1    0    0    0
    5   13    9   13
    1    2    0    0    2    0    0
    1    3    2   -4
 .1        .1        1.
    0  120    2    5    1
C  THE FOLLOWING CARDS ARE INCLUDED FOR BRANCH 1 : I=1,I=2,J=1,K=1,L=1,M=1,M=4,
C  N=1(2 CARDS),O=1,P=1,P=2A,P=2B
    5    1    0    0    0    0
 0.        50.       0.        180.      100.
    0    0    0
    3    4    6    4
    0    1
    1    1    0    0 70000.   26900.    0.        0.
 2.        10.       1.        5.6
    1    0    0    0 0.      0.        180.
    1    0    0    0 10.     0.        180.
    0    3    0    3    0    1
    3    0
 0.             1
 1.2       70000.    .3
```

Table 2.9a

Continued

```
C  THE FOLLOWING CARDS ARE INCLUDED FOR BRANCH 2 : I-1,I-2,I-3A,I-4,J-1,
C  K-1,L-1,L-2,O-1,O-2(2 CARDS),P-1,P-2A,P-2B
     8     1     2     0     0     0
 0.         180.        0.          12.         1000.       100.
 50.         0.        -100.
-90.         0.        -90.
     0     0     0
     4     4     4     6
     1     0
 0.              -1     2     9     7
     0     3     0     3     2     1
     3     9     1
     3     9    13
     3     0
 0.               1
 1.2        70000.     .3
```

Table 2.9b

User Written Subroutines for Example Case 9

```
      SUBROUTINE USRLD (IBRNCH,X,Y,NROW,NCOL,K)
C  EXAMPLE CASE 9
      DIMENSION X(NROW),Y(NCOL)
      IF (K.EQ.2) GO TO 20
      DO 10 I=1,NCOL
      T=Y(I)*3.14159/180.
      P=100.*COS(T)
   10 CALL FORCE (P,2,1,1,I)
   20 CONTINUE
      RETURN
      END
```

2.11 Example Case 10. Spherical Shell Roof with Square Plan Form

2.11.1 Case Description

Geometry:

A spherical shell roof with square plan form is considered. The geometry is shown in Fig. 2.19.

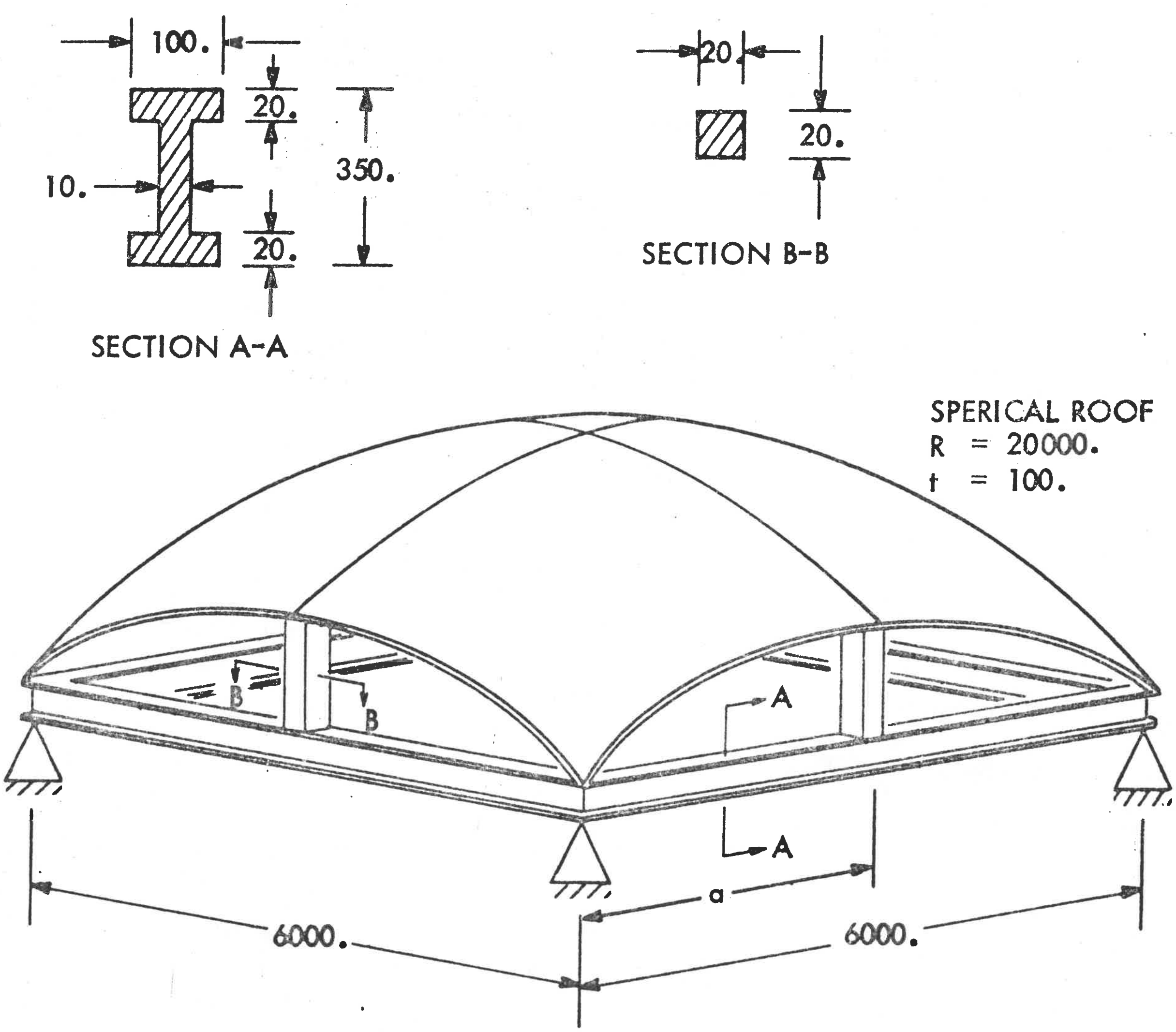

Fig. 2.19 Spherical Shell Roof

The roof is rigidly attached to the beams at each of the four corners. The verticals carry axial loads only.

Material:

The roof is made of concrete with

$$E = 30\ 000.\ \text{MPa}$$

$$G = 13\ 000.\ \text{MPa}$$

$$\nu = .15$$

$$\rho = 3.4 \quad 10^{-6}\ \text{kg/mm}^3$$

The beams and the vertical bars are made of steel with

$$E = 210\ 000\ \text{MPa}$$

$$\nu = 0.3$$

Load:

The shell is subjected to a uniform vertical load (gravity). The load intensity is 0.01 N/mm^2.

Mode of Analysis:

A linear stress analysis is requested. Due to symmetry conditions only one quarter of the roof needs to be considered if triangular shell elements are used it would be possible to do the problem with one eighth of the shell, due to the diagonal symmetry.

Grid:

We will use a grid that is square in the projection on the plane. Seven rows and seven columns should give reasonably accurate results.

2.11.2 Preparation of Input

The coordinate system used in the standard geometry routine is not suitable with this type of boundaries. Therefore, we will derive a user-written geometry routine, LAME. The Cartesian coordinates are shown in Fig. 2.20.

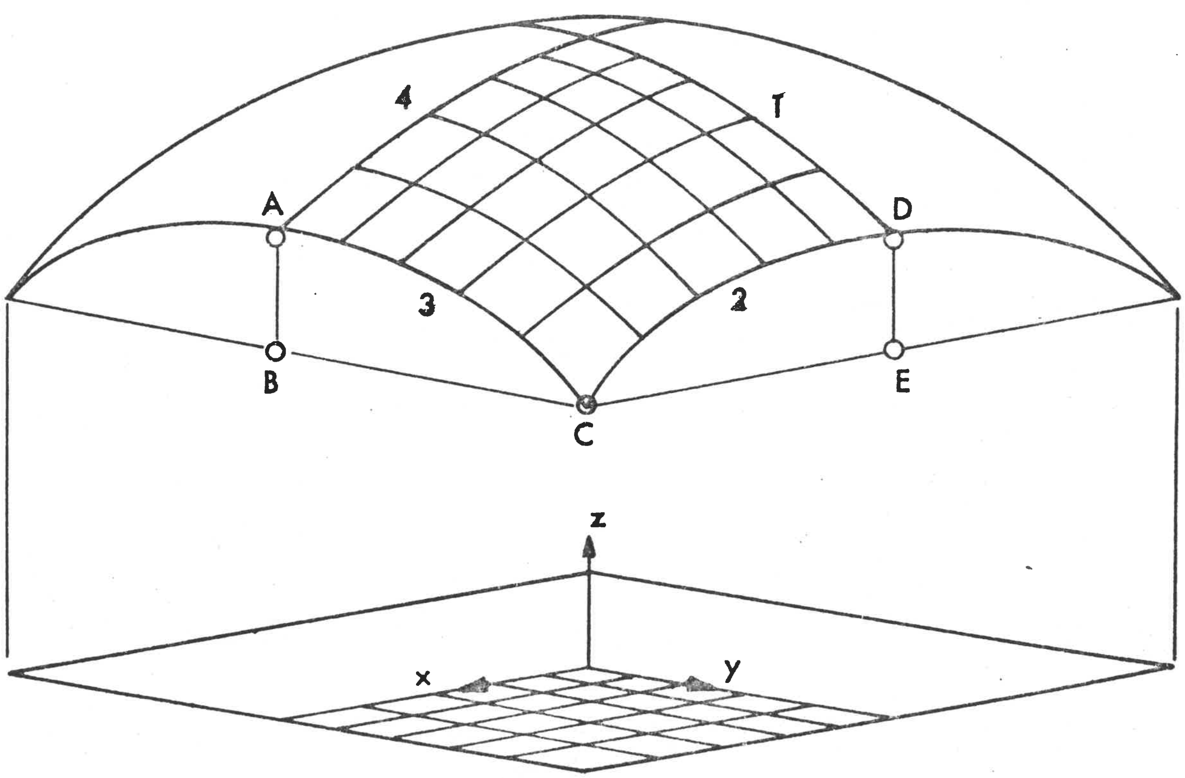

Fig. 2.20 Geometry

The two surface coordinates are defined by

$$X = x/R$$

$$Y = y/R$$

Lines of constant X and Y values form a grid that is rectangular in its projection on the plane. Consequently we have

$$x = RX$$

$$y = RY$$

$$z = R\sqrt{1 - X^2 - Y^2}$$

In order to obtain somewhat simpler expressions in the derivation of the geometric constants, we can substitute

$$X = \cos\alpha$$

$$Y = \cos\beta$$

$$\sqrt{1 - X^2 - Y^2} = \cos\gamma$$

The derivation of geometric constant is somewhat less tedious if this substitution is used.

We have then, for example:

$$\frac{\partial(\cos\gamma)}{\partial X} = \frac{1}{2}(1 - X^2 - Y^2)^{-\frac{1}{2}}(-2X) = -\cos\alpha/\cos\gamma$$

We will find use for the following derivatives

$$\partial/\partial X(\cos\alpha,\ \sin\alpha,\ \cos\gamma,\ (\cos\gamma)^{-1},\ (\cos\gamma)^{-2}) =$$

$$= (1,\ -\cot\alpha,\ -\cos\alpha/\cos\gamma,\ \cos\alpha/\cos^3\gamma,\ 2\cos\alpha/\cos^4\gamma)$$

and

$$\partial/\partial Y(\cos\beta,\ \sin\beta,\ \cos\gamma,\ (\cos\gamma)^{-1},\ (\cos\gamma)^{-2}) =$$

$$= (1, -\cot\beta,\ -\cos\beta/\cos\gamma,\ \cos\beta/\cos^3\gamma,\ 2\cos\beta/\cos^4\gamma)$$

$x,_X = R$ $\qquad$ $x,_Y = 0$

$y,_X = 0$ $\qquad$ $y,_Y = R$

$z,_X = -R\cos\alpha/\cos\gamma$ $\qquad$ $z,_Y = -R\cos\beta/\cos\gamma$

The coefficients of the first fundamental form are:

$$A = R\sqrt{1+\cos^2\alpha/\cos^2\gamma} = R\sin\beta/\cos\gamma$$

$$B = R\sin\alpha/\cos\gamma$$

$$C = R^2\cos\alpha\cos\beta/\cos^2\gamma$$

$$\sqrt{A^2B^2-C^2} = \frac{R^2}{\cos^2\gamma}\sqrt{\sin^2\alpha\sin^2\beta-\cos^2\alpha\cos^2\beta}$$

$$= R^2/\cos\gamma$$

The normal to the surface has the components

$$n_x = (R^2\cos\alpha/\cos\gamma)/(R^2/\cos\gamma) = \cos\alpha$$

$$n_y = (R^2\cos\beta/\cos\gamma)/(R^2/\cos\gamma) = \cos\beta$$

$$n_z = R^2/(R^2\cos\gamma) = \cos\gamma$$

The length of the normal should equal unity:

$$\sqrt{\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma} = 1.$$

$x_{,XX} = 0$

$y_{,XX} = 0$

$$z_{,XX} = -\frac{R}{\cos \gamma} - R \cos \alpha \frac{\cos \alpha}{\cos^2 \gamma} = -R \sin^2 \beta / \cos^3 \gamma$$

$x_{,XY} = 0$

$y_{,XY} = 0$

$$z_{,XY} = -R \cos \alpha \frac{\cos \beta}{\cos^3 \gamma} = -R \cos \alpha \cos \beta / \cos^3 \gamma$$

$x_{,YY} = 0$

$x_{,YY} = 0$

$$z_{,YY} = -\frac{R}{\cos \gamma} - R \cos \beta \frac{\cos \beta}{\cos^3 \gamma} = -R \sin^2 \alpha / \cos^3 \gamma$$

It follows that the coefficients of the second fundamental form are:

$$D = n_Z z_{,XX} = -\cos \gamma \cdot R \frac{\sin^2 \beta}{\cos^3 \gamma} = -R \sin^2 \beta / \cos^2 \gamma$$

$$E = n_Z z_{,XY} = -\cos \gamma \cdot R \cos \alpha \cos \beta / \cos^3 \gamma = -R \cos \alpha \cos \beta / \cos^2 \gamma$$

$$F = n_Z z_{,YY} = -\cos \gamma \cdot R \sin^2 \alpha / \cos^3 \gamma = -R \sin^2 \alpha / \cos^2 \gamma$$

The derivatives of the geometric coefficients are:

$$\frac{\partial A}{\partial X} = R \sin \beta \cos \alpha / \cos^3 \gamma = A \cos \alpha / \cos^2 \gamma$$

$$\frac{\partial A}{\partial Y} = R \left| \sin\beta \; \frac{\cos\beta}{\cos^3\gamma} - \frac{1}{\cos\gamma} \cdot \frac{\cos\beta}{\sin\beta} \right| = R \cot\beta \; \cos^2\alpha \; / \; \cos^3\gamma$$

$$\frac{\partial B}{\partial X} = R \cot\alpha \; \cos^2\beta \; / \; \cos^3\gamma$$

$$\frac{\partial B}{\partial Y} = B \cos\beta \; / \; \cos^2\gamma$$

$$\frac{\partial C}{\partial X} = R^2 \cos\beta \; (\cos\alpha * 2 \cos\alpha/\cos^4\gamma + 1/\cos^2\gamma)$$

$$= R^2 \cos\beta \; (\sin^2\beta + \cos^2\alpha) \; / \; \cos^4\gamma$$

$$\frac{\partial C}{\partial Y} = R^2 \cos\alpha \; (\cos^2\beta + \sin^2\alpha) \; / \; \cos^4\gamma$$

$$\frac{\partial D}{\partial X} = -\, 2R \sin^2\beta / \cos^4\gamma$$

$$\frac{\partial D}{\partial Y} = -\, 2R \cos\beta \; \cos^2\alpha \; / \; \cos^4\gamma$$

$$\frac{\partial E}{\partial X} = -\, R \cos\beta \; (\sin^2\beta + \cos^2\alpha) \; / \; \cos^4\gamma$$

$$\frac{\partial E}{\partial Y} = -\, R \cos\alpha \; (\sin^2\alpha + \cos^2\beta) \; / \; \cos^4\gamma$$

$$\frac{\partial F}{\partial X} = -\, 2R \cos\alpha \;\; \cos^2\beta / \cos^4\gamma$$

$$\frac{\partial F}{\partial Y} = -2R \sin^2\alpha \; \cos\beta / \cos^4\gamma$$

Another user written subroutine is needed to define the loading. The distributed pressure is most easily defined by its components in the Cartesian system

$$PP = (0., \ 0., \ -0.01)$$

A transformation matrix was defined in LAME which transforms a vector from the X′, Y′, Z′ - system to the x, y, z system. The inverse of this matrix would transform the load vector P into a vector with its components in the X′, Y′, Z′ directions. A subroutine INVER3 residing in the STAGS program can be called for inversion. In USRLD we define the vector PP and the same transformation matrix as in LAME. After that we call INVER3 to obtain the transformation matrix TI. The vector PN gives the intensity of the three surface tractions and is obtained by multiplication of PP and TI.

The points ABCDE in Fig. 2.20 are element nodes. B and E are auxiliary points, whose position are determined by definition of the cartesian coordinates. The plane through points B and E is located at

$$z = 20\ 000 \sqrt{1 - (3/20)^2} = 19,744$$

Hence the coordinates for B are

3 000, 0, 19,774

and for E

0, 3 000, 19,774

Due to symmetry conditions point B is constrained from motion in the y direction and from rotation about x- and z-axes. Point E is constrained from motion in the x-direction and rotation around the y- and z-axes.

Cross-section properties for beam (H9a card)

$$A = 310 * 10 + 2 * 100 * 20 = 7100 \text{ mm}^2$$

$$AX \sim 330 * 10 = 3300 \text{ mm}^2$$

$$AZ = 0.8 * 2 * 100 * 20 = 3600 \text{ mm}^2$$

$$XI = 133.8 * 10^6 \text{ mm}^4$$

$$ZI = 3.33 * 10^6 \text{ mm}^4$$

Torsional constant:

$$AJ = (310 * 10^3 + 2 * 100 * 20^3) / 3 = 0.62 * 10^6 \text{ mm}^4$$

By setting $\zeta \equiv 0.0$ we will obtain stresses in a system with the φ_1 coordinate coinciding with the x' axis, the φ_2 coordinate is always normal to φ_1.

Table 2.10a

Data Cards for Example Case 10

```
EXAMPLE CASE 10A,SPHERICAL SHELL ROOF
C  ONE QUARTER MODEL
C  THE FOLLOING GENERAL CONTROL CARDS ARE INCLUDED : B=1,C=1,E=1
    0    1    0    0    0    0    1
    7    7
 1.
C  THE FOLLOING CARDS DEFINE FINITE ELEMENTS : H=1,H=2(2 CARDS),H=3(3 CARDS),
C  H=4,H=6,H=7(2 CARDS),H=9A,H=9B,H=9A,H=9B
    0    0    2    3    1    2    0    2    0
    2 3000.      0.        19774.            101010
    5 0.         3000.     19774.            011100
    1    1    7    1
    3    1    7    7
    4    1    1    7
    0
 210000.   .3        80700.
    1    2    1 400.
    4    5    1 400.
    2    3    1    1 7100.     3300.     3600.     133.8E+06 3.33E+06
 .62E+06   0.          0.        0.        0.        0.        175.
    3    4    5    1 7100.     3300.     3600.     133.8E+06 3.33E+06
 .62E+06   0.          0.        0.        0.        0.        175.
```

Table 2.10a

Continued

```
C   THE FOLLOWING CARDS DEFINE THE SHELL BRANCH
     1       0       0       0       0       0
 0.         3000.       0.          3000.      20000.
     0       0       0
     4       3       3       4
     3       1
 0.                 -1       1       7       7
 0.                 -1       2       7       7
 0.                 -1       3       7       7
     1       1       1       1       0       1
     3       0
 0.                  1
 100.       30000.     .15        0.          0.          0.          .34E-05
```

Table 2.10b

User Written Subroutine for Example Case 11

```
      SUBROUTINE USRLD (IBRNCH,X,Y,NROW,NCOL,K)
C     EXAMPLE CASE 10A
      DIMENSION X(NROW),Y(NCOL)
      DIMENSION T(3,3),TI(3,3),PP(3),PN(3)
C   A VECTOR PP(I) IS DEFINED ,REPRESENTING THE LOAD INTENSITY IN THE
C   DIRECTIONS DEFINED BY THE GLOBAL COORDINATE SYSTEM
      PP(1)=0.
      PP(2)=0.
      PP(3)=-.01
      DO 20 L=1,NROW
      DO 20 M=1,NCOL
      SA=SQRT(1.-X**2)
      SB=SQRT(1.-Y**2)
      Z=SQRT(1.-Y**2-X**2)
      T(1,1)=Z/SB
      T(1,2)=0.
      T(1,3)=-X/SB
      T(2,1)=0.
      T(2,2)=Z/SA
      T(2,3)=-Y/SA
      T(3,1)=X
      T(3,2)=Y
      T(3,3)=Z
      CALL INVER3 (T,TI)
      DO 10  I=1,3
      S=0.
      DO 30  J=1,3
```

Table 2.10b

Continued

```
30 S=S+TI(I,J)*PP(J)
10 PN(I)=S
   CALL FORCE (PN(1),4,1,0,0)
   CALL FORCE (PN(2),4,2,0,0)
   CALL FORCE (PN(3),4,3,0,0)
20 CONTINUE
   RETURN
   END

      SUBROUTINE LAME (IBRNCH,PROP,X,Y)
C     EXAMPLE CASE 10A
      DIMENSION PROP(8)
      COMMON/FQA/NSHELL,A,B,AX,AY,BX,BY,XG,YG,ZG
      COMMON/FQB/C,CX,CY,D,DX,DY,E,EX,EY,F,FX,FY
C     ON DATA CARDS WE READ
C     PROP(1)=0.
C     PROP(2)=3000. (HALF LENGTH OF SIDE)
C     PROP(3)=0.
C     PROP(4)=3000.
C     PROP(5)=20000. (RADIUS OF SPHERE)
      R=PROP(5)
C     X=COS(ALPHA)
C     Y=COS(BETA)
      SA=SQRT(1.-X**2)
      SB=SQRT(1.-Y**2)
      Z=SQRT(1.-X**2-Y**2)
      XG=R*X
      YG=R*Y
      ZG=R*Z
      A=R*SB/Z
      B=R*SA/Z
      Z2=Z**2
      C=R**2*X*Y/Z2
      D=-R*SB**2/Z2
      E=-R*X*Y/Z2
      F=-R*SA**2/Z2
```

Table 2.10b

Continued

```
AX=A*X/Z2
AY=R*Y*X**2/(SB*Z*Z2)
BX=R*X*Y**2/(SA*Z*Z2)
BY=B*Y/Z2
CX=R**2*Y*(SB**2+X**2)/Z2**2
CY=R**2*X*(SA**2+Y**2)/Z2**2
DX=-2.*R*X*SB**2/Z2**2
DY=-2.*R*Y*X**2/Z2**2
EX=-R*Y*(SB**2+X**2)/Z2**2
EY=-R*X*(SA**2+Y**2)/Z2**2
FX=-2.*R*X*Y**2/Z2**2
FY=-2.*R*Y*SA**2/Z2**2
RETURN
END
```

Example cases 11 to 14 are in preparation.