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STRESS, BUCKLING, AND VIBRATION OF PRISMATIC SHELLS

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(This is an abridged version. See the full-length paper for more: bigbosor4.papers/1971.prismatic.pdf)

ABSTRACT

A computer code for the general treatment of complex shells of revolution is applied to the analysis of prismatic shells such as oval cylinders, corrugated sheets, and longitudinally stiffened cylinders with stringer discreteness retained in the model. Cylinders of circular and noncircular cross section are treated as portions of very slender toroids. Corrugated and beaded panels are treated as portions of cylinders with very large radii. Longitudinally stiffened cylinders are treated as portions of very slender toroids with discrete ring stiffeners. The technique also permits analysis of buckling of cylinders with nonsymmetric pressure or thermal loading. The analysis is based on the finite-difference method which is used in conjunction with energy minimization. Several numerical examples are given. Included are convergence studies proving the validity of the technique and buckling calculations for nonuniformly loaded cylinders, externally pressurized and axially compressed elliptic cylinders, and axially compressed corrugated and beaded panels. Vibration modes and frequencies are calculated for an eccentrically stiffened cylinder in which the stringers are treated as discrete.

INTRODUCTION

The motivation behind much of the research activity in shell analysis is to reduce computer time and core storage required to solve complex problems. It is advantageous whenever possible to reduce the number of degrees of freedom required by separation of variables and to optimize computer efficiency by setting up stiffness matrices with as narrow bands as possible. Currently, problems in complex shell analysis can be classified into two groups: that which involves two-dimensional discretization and that which involves one-dimensional discretization. The two-dimensional numerical analysis generally requires one to several orders of magnitude more computer time to solve than does the one-dimensional problem. The computer time increases quadratically with the bandwidth of the stiffness matrix and linearly with the number of degrees of freedom. Matrix bandwidths for two-dimensional problems are much wider than those for one-dimensional problems and the number of degrees of freedom required for convergence to a given accuracy is greater.

This research was motivated by the need for economical computer solutions to problems traditionally associated with two-dimensional numerical analyses but amenable by means of an exchange of independent variables to solution by separation of variables with consequent reduction to one-dimensional numerical treatment. In this class are included stress, buckling and vibration problems for simply-supported prismatic shells. Stress analysis can be performed for prismatic shells with loads that vary in the two coordinate directions. Buckling and vibration analyses are restricted to systems in which both the loads and the geometry are prismatic-constant in the axial direction.

Figure 1 gives examples of prismatic shells: Fig. la shows an oval cylinder which may be subjected to combinations of pressure and axial loading; Fig. lb is a cylinder with a pressure or thermal load which varies in the circumferential direction; Figs. 1c and 1d represent typical advanced structural panels being considered for hypersonic cruise vehicles, lightweight space systems shrouds, and space shuttles; and Fig. le shows a general prismatic shell with stringers which can be treated as discrete elastic structures. The oval cylinder under axial compression has been investigated by Kempner and Chen [1], Hutchinson [2], and Almroth, Brogan, and Marlowe [3]. Elliptic cylinders under external pressure have been treated by Yao and Jenkins [4]. Liaw [5] gives a survey of papers published before April 1969 on the stability of cylindrical and conical shells of noncircular cross section. (See the paper for a list of the references.)

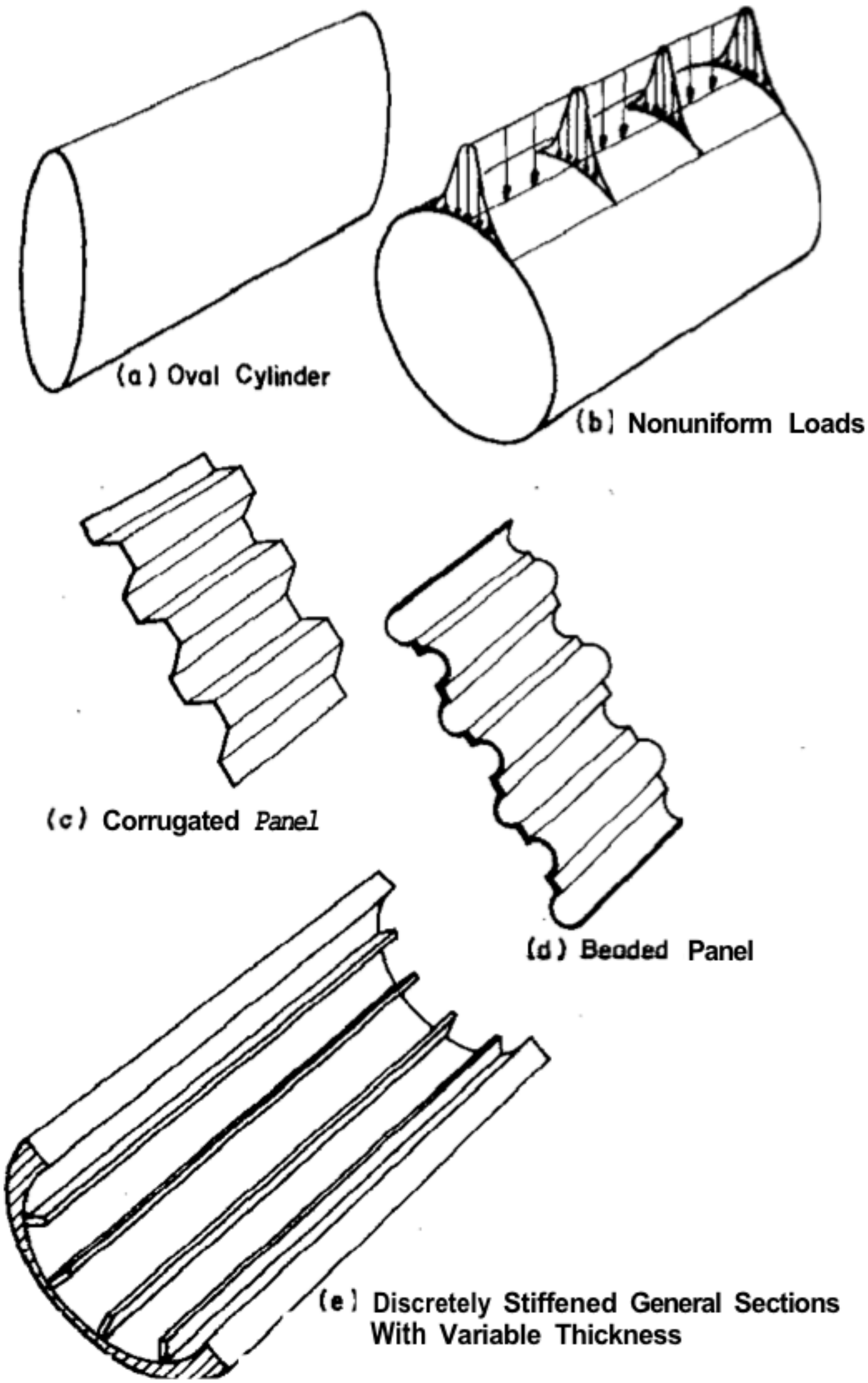

Fig. 1 Some typical prismatic shell structures. (from AIAA Journal, Vol. 9, No. 10, pp 2004 – 2010, 1974)

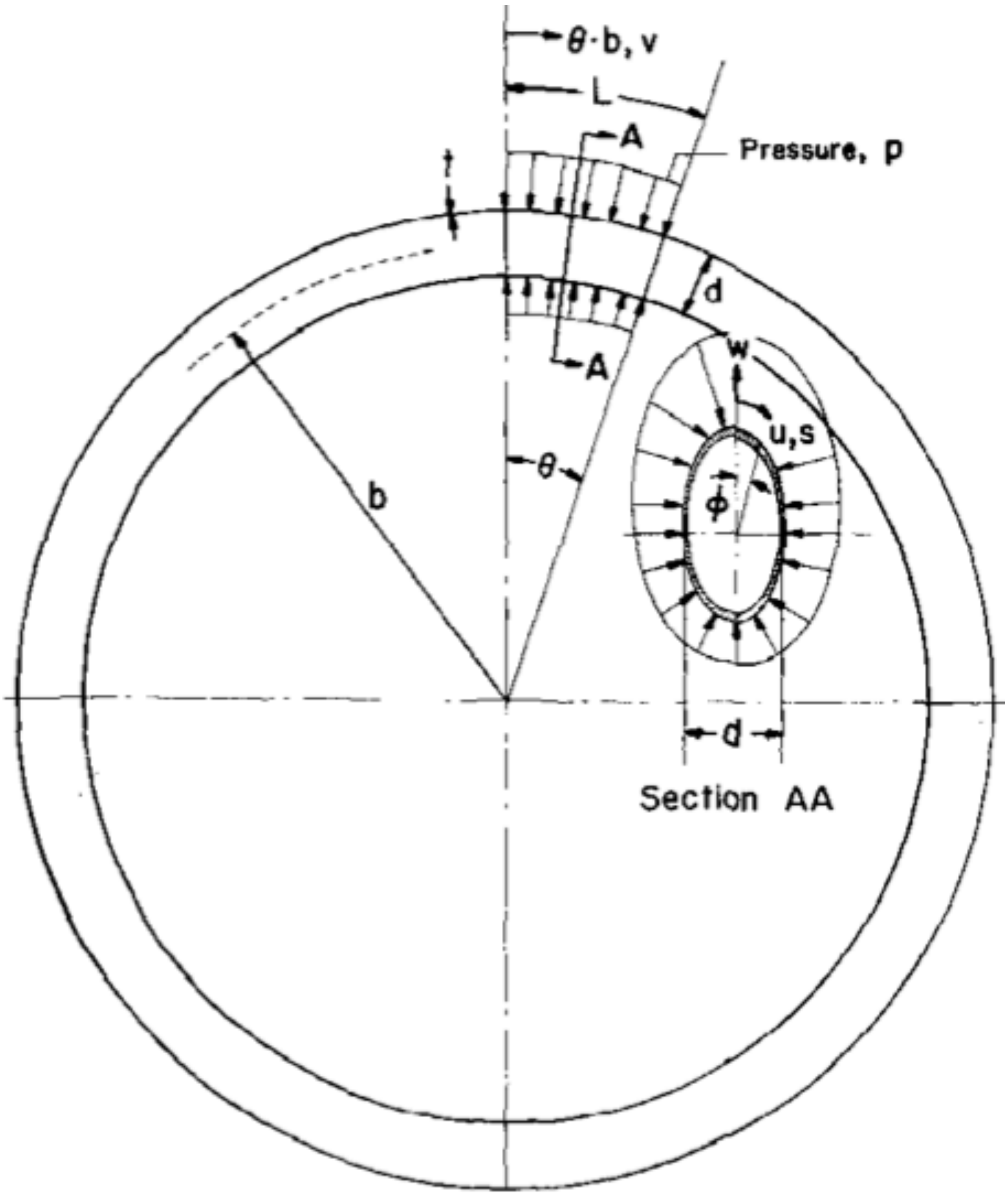

Fig. 2 Noncircular cylindrical shell treated as a portion of a torus with a large radius b and length L = theta x b. (from AIAA Journal, Vol. 9, No. 10, pp 2004 – 2010, 1974)

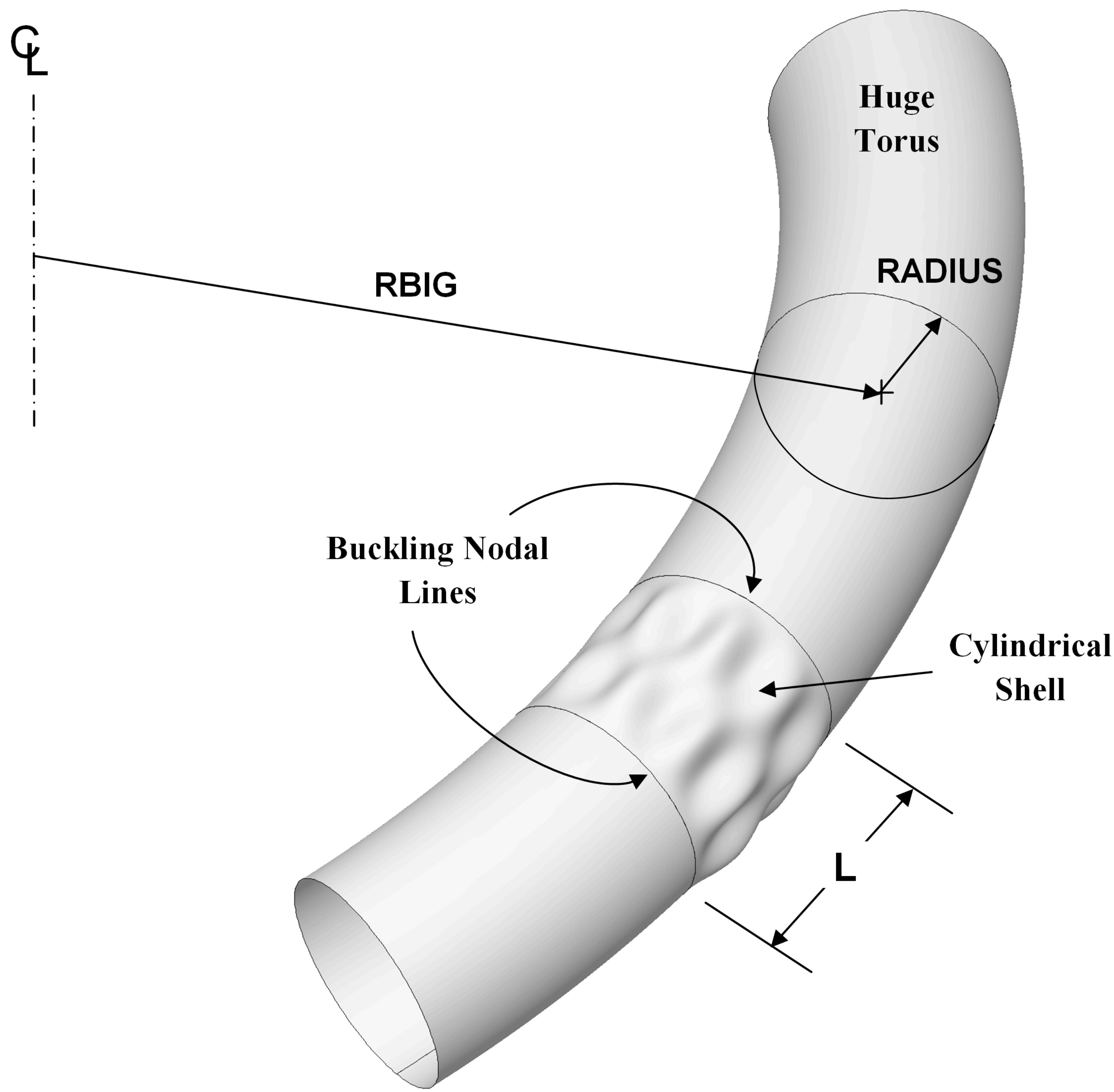

Fig. 3 Modeling a cylindrical shell as part of a huge torus (Figure by Robert P. Thornburgh, from AIAA 51st Structures, Structural Dynamics, and Materials Conference, AIAA Paper 2010-2927, 2010)