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STRESS, STABILITY AND VIBRATION OF COMPLEX, BRANCHED SHELLS OF REVOLUTION

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(This is an abridged version. See the full-length paper for more: bigbosor4.papers/1974.basic.pdf)

ABSTRACT

A comprehensive computer program, designated BOSOR4 (2011 NOTE: now superseded by BIGBOSOR4), for analysis of the stress, stability and vibration of segmented, ring-stiffened, branched shells of revolution and prismatic shells and panels is described. The program performs large deflection axisymmetric stress analysis, small-deflecton nonsymmetric stress analysis, modal vibration analysis with axisymmetric nonlinear prestress included, and buckling analysis with axisymmetric or nonsymmetric prestress. One of the main advantages of the code is the provision for realistic engineering details such as eccentric load paths, internal supports, arbitrary branching conditions, and a library of wall constructions. The program is based on the finite difference energy method which is very rapidly convergent with increasing numbers of mesh points. The organization of the program is briefly described with flow of calculations charted for each of the types of analysis. Overlay charts and core storage requirements are given for the CDC 6600, IBM 370, and UNIVAC 1108 versions of BOSOR4. A large number of cases is included to demonstrate the scope and practicality of the program. (2011 NOTE: BIGBOSOR4 now runs on LINUX. BIGBOSOR4 will handle many more shell segments than BOSOR4.)

INTRODUCTION

The purpose of this paper is to describe a computerized method of analysis for composite, branched shells of revolution. The main advantage of the analysis method and computer program is its direct and efficient applicability to practical engineering design problems involving very complex shells of revolution or prismatic shell structures such as corrugated panels or noncircular cylinders. Emphasis is placed on analytical results for a variety of 'real-world' engineering problems. Details of the analysis method are reported in Ref. [1]. (For the list of references see the BOSOR4 paper.)

Extensive literature exists on analysis and computer programs for shells and solids of revolution. Figure 1 contains the names of many computer programs and names of originators of other computer programs that cover this field. The names are given in a 'coordinate system' arranged such that increasingly 'general purpose' computer codes lie increasing distances from both axes. Other codes, existing just outside of the region depicted, apply to structures that are 'almost' shells of revolution, such as shells with cutouts, shells with material properties that vary around the circumference, or panels of shells of revolution. The region shown in Fig. 1 is divided by a heavy line into two fields: Programs lying within the heavy line are based on numerical methods that are essentially one-dimensional, that is, the dependent variables are separable and only one spatial variable need be discretized; programs lying outside the heavy line are based on numerical methods in which two spatial variables are discretized.

It is generally true that analysis methods and programs lying outside the heavy line require perhaps an order of magnitude more computer time for a given case with given nodal point density than do those lying inside the line. This distinction arises because the bandwidths and ranks of equation systems in two-dimensional numerical analyses are greater than those in one-dimensional numerical analyses. Certain of the areas in Fig. I are blank. Those near the origin correspond in general to cases for which closed-form solutions exist and for which slightly more general programs are clearly applicable. The blank areas lying near the outer boundaries of the chart are *for* the most part covered by more general programs such as NASTRAN, SNAP, REXBAT, STAGS, STRUDL, and ASKA [20].

INCREASING GENERALITY OF STRUCTURE →	Axisymmetric Linear Stress	Axisymmetric Large-Deflection Stress	Axisymmetric Elastic-Plastic Stress	Axisymmetric Large-Deflection Elastic-Plastic Stress	Nonsymmetric Linear Stress	Eigenvalue Vibration	Eigenvalue Buckling	Dynamic Response, 1-D Nonlinear	Dynamic Response, 2-D Linear	Static 2-D Nonlinear	Dynamic Response, 2-D Nonlinear
Thick-Walled Shells or Solids	MARCAL WILSON WASP	EPIC(5)	SAAS-II (4) EPIC(5)		FARSS (6)				GHOSH(7)		
Branched, Segmented Shells	BOSOR4(1)	BOSOR4(1) COHEN (2)			BOSOR4(1) COHEN(2) SABOR5 CAPELLI(3) LESTINGI(18) RADKOWSKI	BOSOR4(1) COHEN(2)	BOSOR4(1) COHEN(2)		SABOR5-DRASTIC5		
Segmented Shells	SADISTIC IV SABOR I WASP	BOSOR3 SADAOS SALORS STARS II	SADAOS	GERDEEN(19)	BOSOR 3 KALNINS SABOR 3 STARS II SALORS	BOSOR3 KALNINS BALOR	BOSOR3 KALNINS BALOR STARS II	SADAOS	SABOR5-DRASTIC		
Thin-Walled Simple Shells		BOSOR2 WILSON-JONES(8)	KHOJASTEH-BAKHT(9) SHARIFI(10)	MARCAL LEVINE(11) EPSOR YAGMAI(12)	SCHAEFFER MARGOLIAS(13)	BOSOR2 ADELMAN(14) MARGOLIAS(13)	BOSOR2 HSEUH(15) MARGOLIAS(13)	GIRLS II	SABOR5-DRASTIC	BALL(17) SNASOR(24) GREENBAUM MARTIN	BALL(17) SHORE(16) DYNASOR II (24)
Stiffened Cylinders							SCAR				
Monocoque Cylinders										NULACS	CYLINDER GRIVET SMERSH SPECTRE
Rings or Infinite Cylinders								UNIVALVE II GIRLS I PUFF			

INCREASING GENERALITY OF ANALYSIS OR PHENOMENON →

FIG. 1. Computer programs for shells and solids in revolution.

The numerals next to the program name or to the originator names correspond to references at the end of the text. Names shown with no numerals are referenced in one or more of the survey papers, Refs. [20-23]. These papers describe the various numerical procedures used, and in one case [21] hint as to the availability of some of the codes.

(2011 NOTE: Regard the above as interesting history, perhaps a bit quaint, from the days when core storage on the computer was severely limited and much of a developer's time was spent devising ways to shoehorn his or her computer program into the machine.)

This paper will focus on a description of the general branched shell-of-revolution analyzer called BOSOR4. The goals of the research leading to the BOSOR4 code have been to provide as general as possible an engineering tool within the restriction of one-dimensional discretization; to include as much capability as possible for analysis of practical engineering structures, which include meridional discontinuities, weld mismatches, composite materials, discrete rings, sliding constraints, etc. to make the computer program easy to use by means of logical arrangement of input data, internal diagnostics, plots, and a complete user's manual; to make the code as efficient as possible; and to maximize its availability by converting it and checking it out on three major systems-the Univac 1108, the CDC 6600, and the IBM 370/165. Program tapes and manuals are available through the author or through the COSMIC system. (2011 NOTE: BIGBOSOR4 runs on LINUX and can be downloaded from the web site.) Described in this paper are the scope of BOSOR4, the analysis method, and the program organization; in addition, several cases involving nonlinear stress analysis, buckling, and vibration of segmented, branched, ring-stiffened shells of revolution with various wall constructions are discussed.

SCOPE OF THE BOSOR4 COMPUTER PROGRAM

The BOSOR4 code performs stress, stability, and vibration analyses of segmented, branched, ring-stiffened, elastic shells of revolution with various wall constructions. Figure 2 shows some examples of branched structures which can be handled by BOSOR4. Figure 2a represents part of a multiple-stage rocket treated as a shell of seven segments; Fig. 2b represents part of a ring-stiffened cylinder in which the ring is treated as two shell segments branching from the cylinder; Fig. 2c shows the same ring-stiffened cylinder, but with the ring treated as 'discrete', that is the ring cross section can rotate and translate but not deform, as it can in the model shown in Fig. 2b. Figures 2d-f represent branched prismatic shell structures, which can be treated as shells of revolution with very large mean circumferential radii of curvature, as described in Ref. [25] and later in this paper.

The program is very general with respect to geometry of meridian, shell-wall design, edge conditions, and loading. It has been thoroughly checked out by comparisons with other known solutions and tests. The BOSOR4 capability is summarized in Table 1. The code represents three distinct analyses:

(1) A nonlinear stress analysis for axisymmetric behavior of axisymmetric shell systems (large deflections, elastic)

(2) A linear stress analysis for axisymmetric and nonsymmetric behavior of axisymmetric shell systems submitted to axisymmetric and nonsymmetric loads

(3) An eigenvalue analysis in which the eigenvalues represent buckling loads or vibration frequencies of axisymmetric shell systems submitted to axisymmetric loads. (Eigenvectors may correspond to axisymmetric or nonsymmetric modes.)

BOSOR4 has an additional branch corresponding to buckling of nonsymmetrically loaded shells of revolution. However, this branch is really a combination of the second and third analyses just listed.

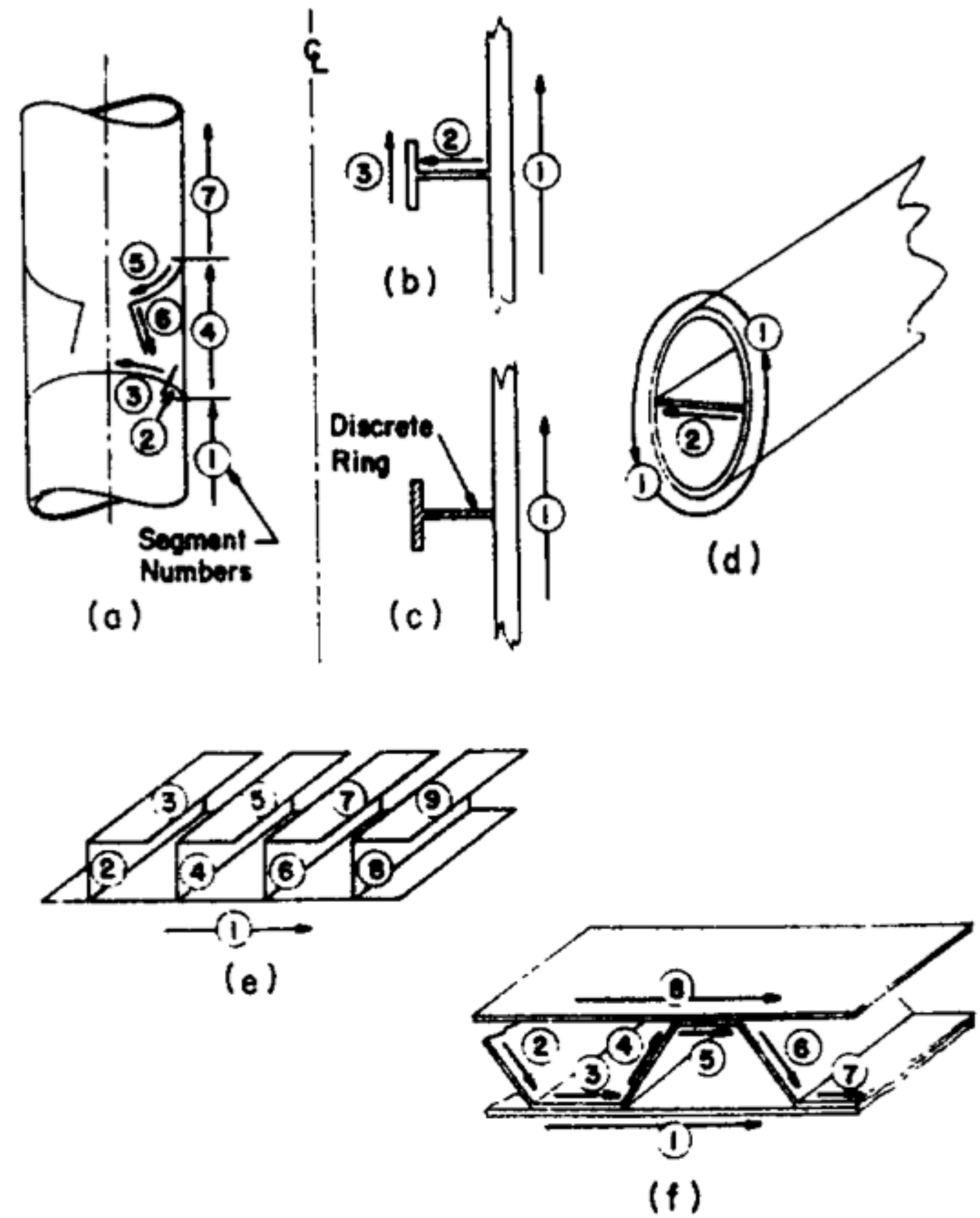

Fig. 2 Examples of branched structures that can be analyzed with BOSOR4/BIGBOSOR4. (a) multi-segment, branched shell of revolution, (b) part of a cylindrical shell with an internal T-shaped ring in which the ring web and ring outstanding flange are modeled as two little flexible shell segments, (c) part of a cylindrical shell with an internal T-shaped ring in which the ring is modeled as a "discrete ring", the cross section of which can translate and rotate but which does not deform, (d – f) three different prismatic thin-walled structures. (from Computers & Structures, Vol. 4, pp 399 – 435, 1974)

2011 NOTE: the program, BIGBOSOR4, supersedes BOSOR4. BIGBOSOR4 will handle shells of revolution and prismatic shells with many more segments than will BOSOR4. BIGBOSOR4 will accommodate models with up to 295 shell segments.
(see genopt.papers/2000.wavycyl.pdf
and genopt.papers/2011.trusscomp.pdf.)

Table 1 BOSOR4 capability summary. (from Computers & Structures, Vol. 4, pp 399 – 435, 1974)

Type of analysis	Shell geometry	Wall construction	Loading
Nonlinear axisymmetric stress	Multiple-segment shells, each segment with its own wall construction, geometry, and loading	Monocoque, variable or constant thickness	Axisymmetric or nonsymmetric thermal and/or mechanical line loads and moments
Linear symmetric or nonsymmetric stress	Cylinder, cone, spherical, ogival, toroidal, ellipsoidal, etc.	Skew-stiffened shells	Axisymmetric or nonsymmetric thermal and/or mechanical distributed loads
Stability with linear symmetric or nonsymmetric prestress or with nonlinear symmetric prestress	General meridional shape; point-by-point input	Fiber-wound shells	Proportional loading
Vibration with nonlinear prestress analysis	Axial and radial discontinuities in shell meridian	Layered orthotropic shells	Non-proportional loading
Variable mesh point spacing within each segment	Arbitrary choice of reference surface	Corrugated, with or without skin	
	General edge conditions	Layered orthotropic with temperature-dependent material properties	
	Branched shells	Any of above wall types reinforced by stringers and/or rings treated as "smeared out"	
	Prismatic shells and composite built-up panels	Any of above wall types further reinforced by rings treated as discrete	
		Wall properties variable along meridian	

In the BOSOR4 code, the user chooses the type of analysis to be performed by means of a control integer INDIC:

INDIC= -2 Stability determinant calculated for given circumferential wave number *N* for increasing loads until it changes sign. Nonlinear prebuckling effects are included. INDIC is then changed automatically to -1 and calculations proceed as if INDIC has always been -1.

INDIC= -1 Buckling load and corresponding wave number *N* are determined, including nonlinear prebuckling effects. *N* corresponding to a *local* minimum critical load $L_{c},(N)$ is automatically sought.

INDIC = 0 Axisymmetric stresses and displacements calculated for a sequence of stepwise increasing loads from some starting value to some maximum value, including nonlinear effects. Axisymmetric collapse loads can be calculated.

INDIC = 1Buckling loads are calculated with nonlinear bending theory for a *fixed* load. Buckling loads are calculated for a range of circumferential wave numbers. Several buckling loads for each wave number can be calculated.

INDIC = 2 Vibration frequencies and mode shapes are calculated, including the effects of prestress obtained from axisymmetric nonlinear analysis. Several frequencies and modes can be calculated for each circumferential wave number.

INDIC = 3 Nonsymmetric or symmetric stresses and displacements are calculated for a range of circumferential wave numbers. Linear theory is used. Results for each harmonic are automatically superposed. Fourier series for nonsymmetric loads are automatically computed or may be provided by the user.

INDIC = 4 Buckling loads are calculated for nonsymmetrically loaded shells. The prebuckled state is obtained from linear theory (INDIC=3) or read in from cards. The 'worst' meridional prestress distribution (such as the distribution involving the maximum negative meridional or hoop prestress resultant) is chosen by the user, and this particular distribution is assumed to be axisymmetric in the stability analysis, which is the same as that for the branch INDIC= 1.

The variety of buckling analyses (INDIC= -2, -1, 1, and 4) is provided to permit the user to approach a given buckling problem in a number of different ways. There are cases for which an INDIC= -1 analysis, for example, will not work. The user can then resort to an INDIC= -2 analysis, which requires more computer time, but which is generally more reliable. Buckling of a shallow spherical cap under external pressure is an example. In an INDIC= -1 analysis of the cap, the program generates a sequence of loads that ordinarily should converge to the lowest buckling load, with nonlinear prebuckling effects included. Depending on the cap geometry and the user-provided initial pressure, however, one of the loads in the sequence may exceed the axisymmetric collapse pressure of the cap. This phenomenon can occur if the bifurcation buckling loads are just slightly smaller than the axisymmetric collapse loads. The user can obtain a solution with use of INDIC= - 2, in which the bifurcation load is approached from below in a 'gradual' manner. The BOSOR4 manual [1] contains an example of this case. A somewhat different illustration is provided in the section on analytical results.

The branch INDIC= 1 is provided because it is sometimes desirable to know several buckling eigenvalues for each circumferential wave number, *N,* and because there may exist more than one minimum in the critical load vs N-space.This is especially true for complex shell structures with many segments and load types. Such a structure can buckle in many different ways. The designer may have to eliminate several possible failure modes, not just the one corresponding to the lowest pressure, for example. The INDIC=4 branch is provided for two reasons: The user can calculate buckling under nonsymmetric loads without having to make two separate runs, an INDIC=3 run and an INDIC= 1 run. In addition, this branch permits the user to bypass the prebuckling analysis and read prebuckling stress distributions and rotations directly from cards. This second feature is very useful for the treatment of composite branched panels under uniaxial or biaxial compression.

The BOSOR4 program, although applicable to shells of revolution, can be used for the buckling analysis of composite, branched prismatic panels by means of a 'trick' described in detail in [25]. This 'trick' permits the analysis of any prismatic shell structure that is simply-supported at particular stations along its length. Any boundary conditions can be used along generators. In [25] many examples are given, including nonuniformly loaded cylinders, noncircular cylinders, corrugated panels, and cylinders with stringers treated as discrete.

This and the following pages show additional figures from the 1974 paper on BOSOR4.

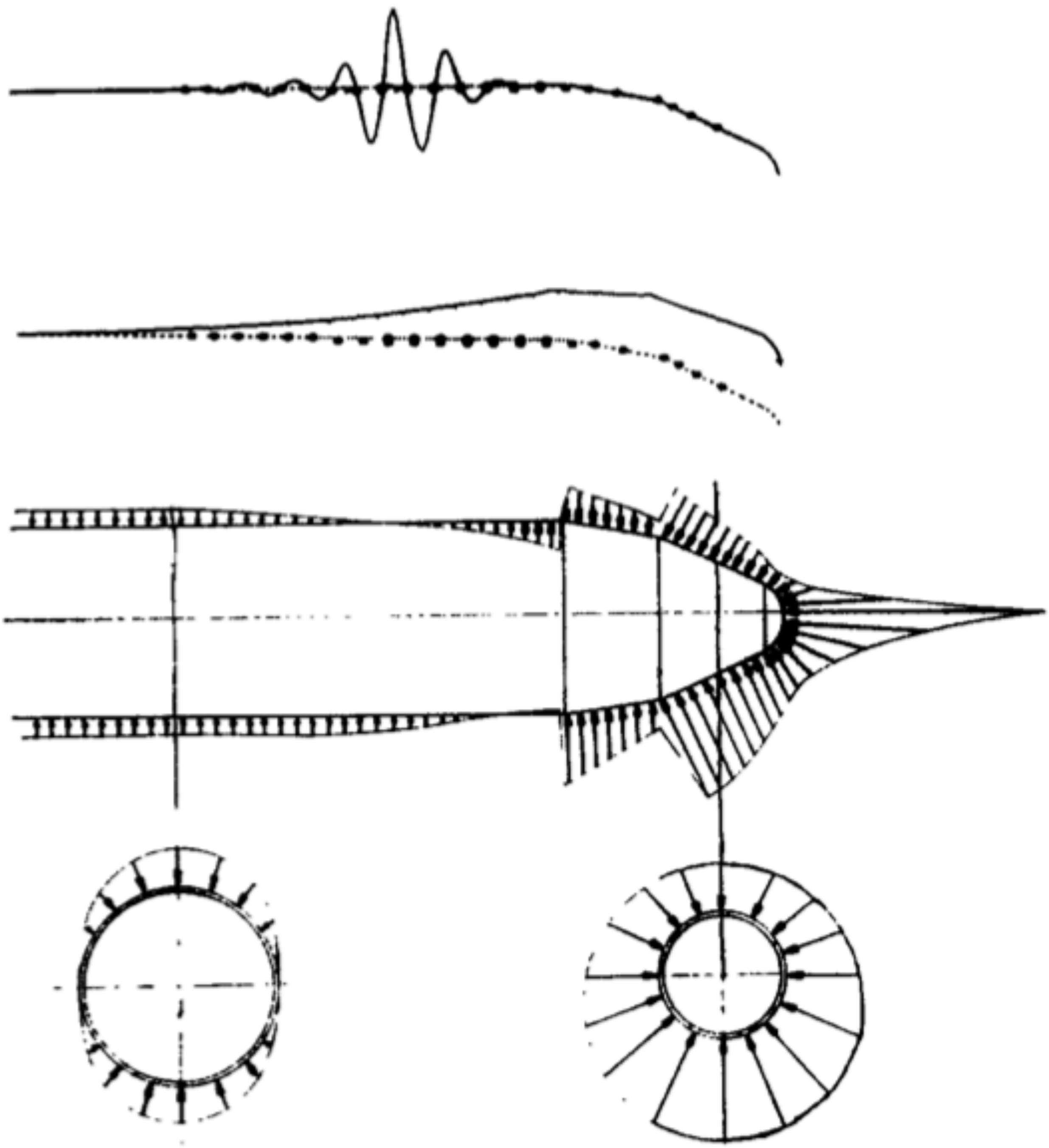

Fig. 23 Pressure distribution on the shroud, prebuckling deflection, and buckling mode (top). (from Computers & Structures, Vol. 4, pp 399 – 435, 1974)

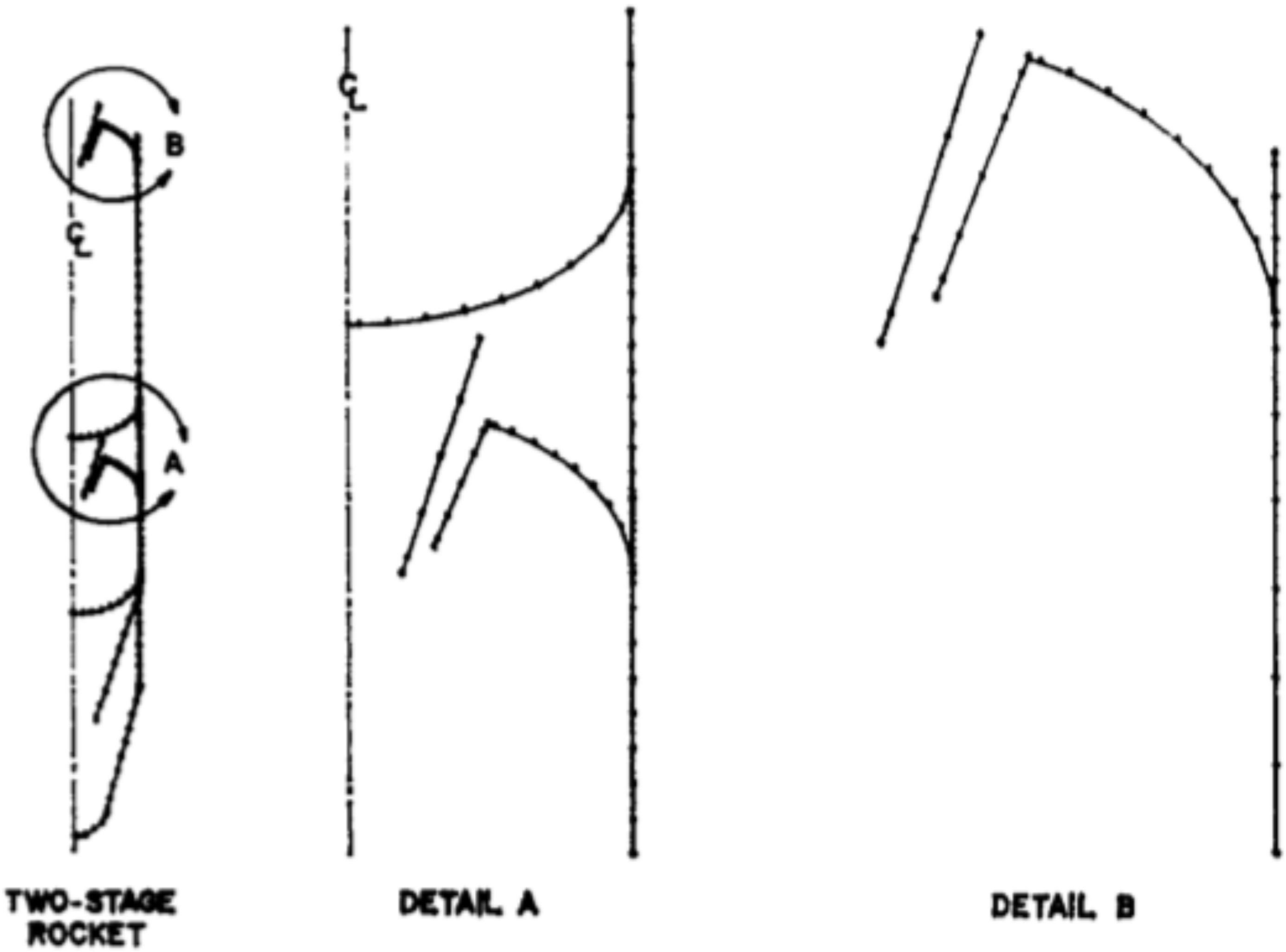

Fig. 24 Two-stage rocket: segmented, discretized model for BOSOR4/BIGBOSOR4. (from Computers & Structures, Vol. 4, pp 399 – 435, 1974)

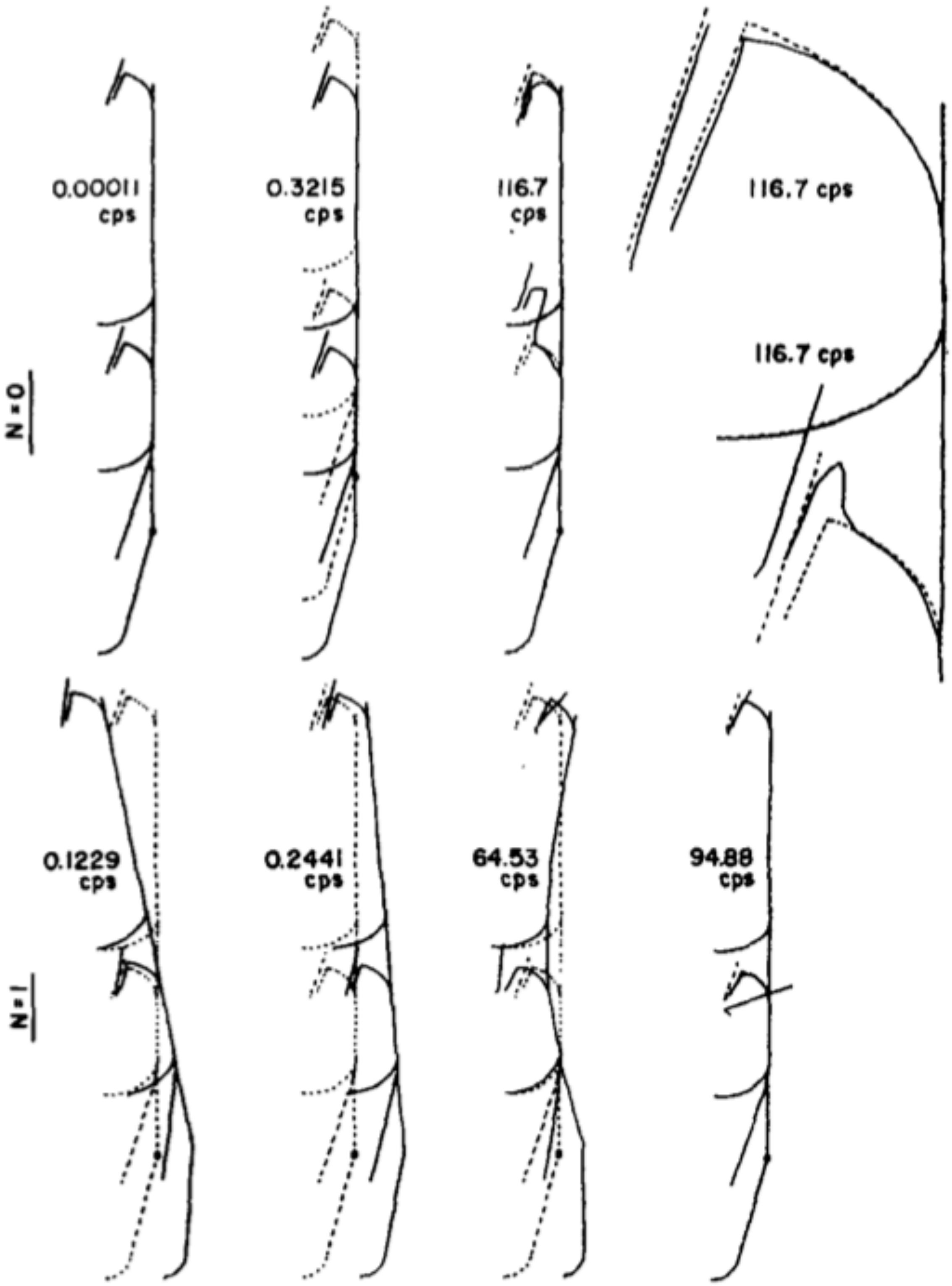

Fig. 25 Two-stage rocket: Vibration modes for N = 0 and N = 1 circumferential waves. (from Computers & Structures, Vol. 4, pp 399 – 435, 1974)

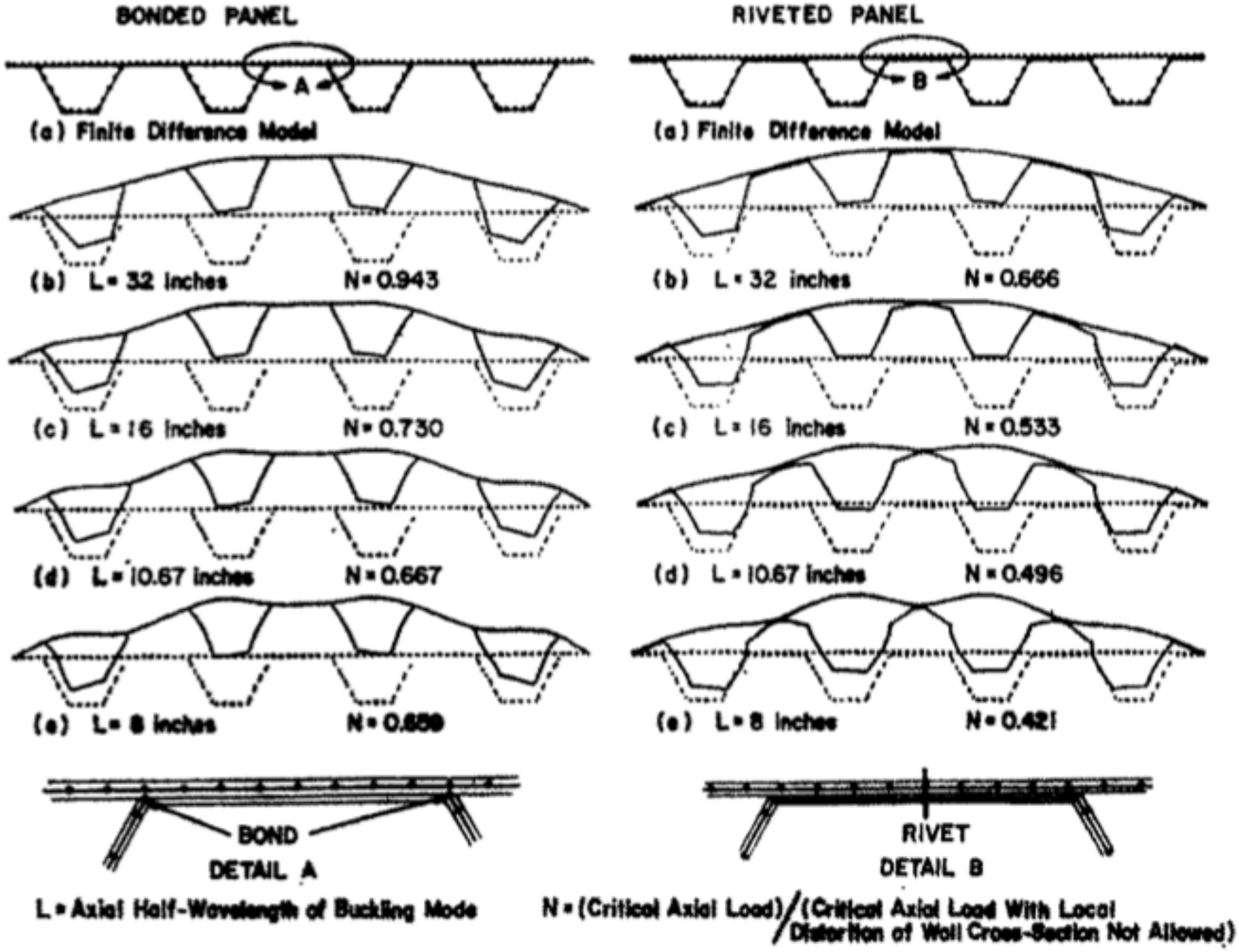

Fig. 34 Buckling modes of axially compressed, semi-sandwich, bonded and riveted corrugated panels modeled with BOSOR4 or BIGBOSOR4. . (from Computers & Structures, Vol. 4, pp 399 – 435, 1974)