(from PRESSURE VESSELS AND PIPING: DESIGN TECHNOLOGY – 1982, A DECADE OF PROGRESS, S.Y. Zamrik and D. Dietrich, editors, published by ASME; Chapter 2.4 "Plastic Buckling" by David Bushnell, pp. 47-117)(This is an abridged version. See the full-length paper for more: bosor5.papers/1982.decade.pdf)

reprinted from
Pressure Vessels and Piping: Design Technology – 1982
A Decade of Progress
Edited by S.Y. Zamrik, and D. Dietrich
(Book No. G00213)

published by
THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS
345 East 47th Street, New York, N.Y. 10017
Printed in U.S.A.

BOSOR5

2.4 PLASTIC BUCKLING

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ABSTRACT

The phenomenon of plastic buckling is first illustrated by the behavior of a fairly thick cylindrical shell, which under axial compression deforms at first axisymmetrically and later nonaxisymmetrically. Thus, plastic buckling encompasses two modes of behavior, nonlinear collapse at the maximum point in in a load vs deflection curve and bifurcation buckling. Accurate prediction of critical loads corresponding to either mode in the plastic range requires a simultaneous accounting for moderately large deflections and nonlinear, irreversible, path-dependent material behavior. A survey is given of plastic buckling which spans three areas: asymptotic analysis of post-bifurcation behavior of perfect and imperfect simple structures, general nonlinear analysis of arbitrary structures, and nonlinear analysis for limit load collapse and bifurcation buckling of shells and bodies of revolution. A discussion is included of certain conceptual difficulties encountered in plastic buckling models, in particular those having to do with material loading rate at bifurcation and the apparent paradox that use of deformation theory often leads to better agreement with tests on structures with very simple prebuckling equilibrium states than does use of the more rigorous incremental flow theory. In the survey of general nonlinear structural analysis emphasis is given to formulation of the basic equations, various elastic-plastic material models, and strategies for solving the nonlinear equations incrementally. In the section on buckling of axisymmetric structures, numerous examples including comparisons of test and theory reveal that critical loads are not particularly sensitive to initial imperfections when the material is stressed beyond the proportional limit. A final summary includes suggestions for future work.

INTRODUCTION

WHAT IS PLASTIC BUCKLING?

To most engineers the word "buckling" evokes an image of failure of a structure which has been compressed in some way. Pictures and perhaps sounds come to mind of sudden, catastrophic collapse involving very large deformations. From a scientific and engineering point of view, the interesting phases of buckling phenomena generally occur before the deformations are very large, when to the unaided eye, the structure appears to be undeformed or only slightly deformed.

In static analysis of perfect structures, there are two phenomena loosely termed "buckling": collapse at the maximum point in a load vs deflection curve and bifurcation buckling. These are illustrated in Figs. 1 and 2. The axially compressed cylinder shown in Fig. 1 deforms approximately axisymmetrically along the path OA until a maximum or limit load λ_L is reached at point A. If the axial load λ is not sufficiently relieved by the reduction in axial stiffness, the perfect cylinder will fail at this limit load, following either the path ABC along which it continues to deform axisymmetrically, or some other path ABD along which it first deforms axisymmetrically from A to B and then nonaxisymmetrically from B to D. Limit point buckling, or "snap-through", occurs at point A and bifurcation buckling at point B. The equilibrium path OABC corresponding to the axisymmetrical mode of deformation is called the fundamental path and the post-bifurcation equilibrium path BD, corresponding to the nonaxisymmetrical mode of deformation, is called the secondary path. The significance of the word "plastic" in the title is that buckling of either type occurs at loads for which some or all of the structural material has

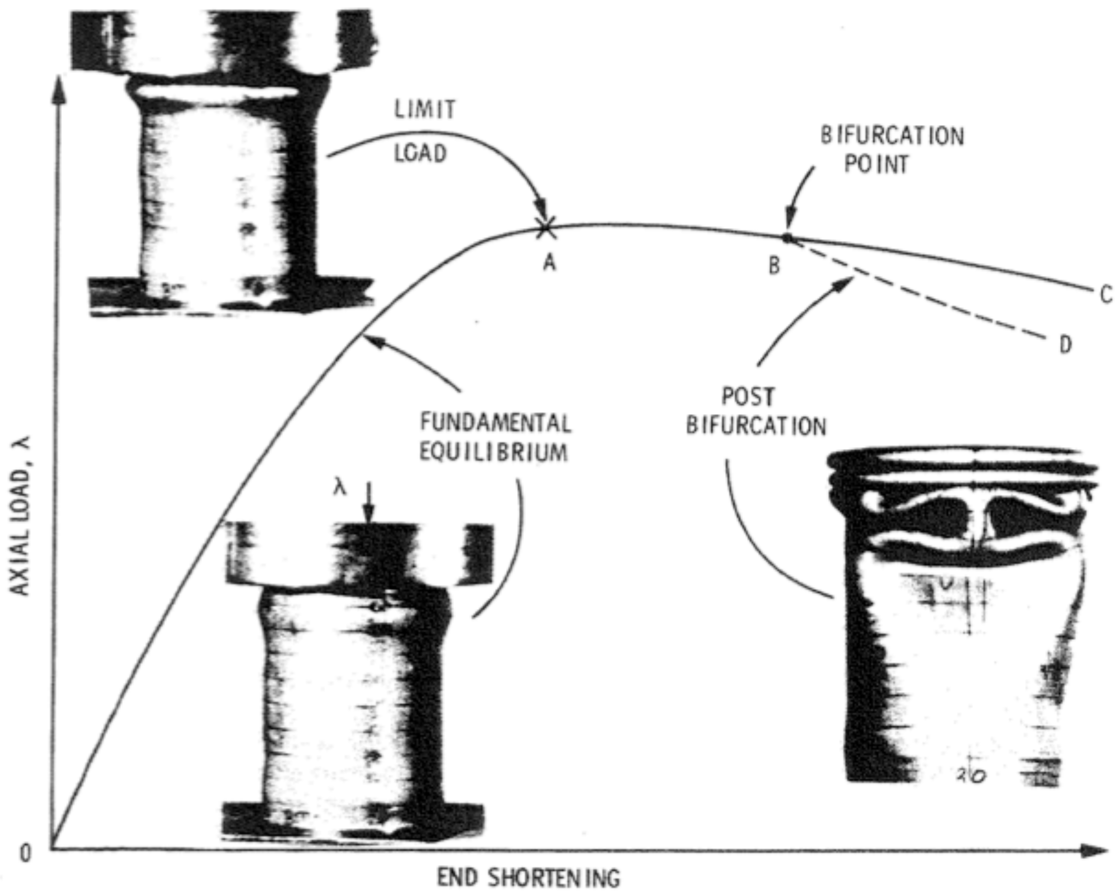

FIG. 1 LOAD-END SHORTENING CURVE WITH LIMIT POINT A, BIFURCATION POINT B, AND POST-BIFURCATION EQUILIBRIUM PATH, BD. (PHOTOGRAPHS COURTESY SOBEL AND NEWMAN [237]).

been stressed beyond its proportional limit. The example in Fig. 1 is somewhat unusual in that the bifurcation point B is shown (correctly) to occur after the collapse point has been reached. In this particular case, therefore, bifurcation buckling is of less engineering significance than axisymmetric collapse.

A more commonly occurring situation is illustrated in Fig. 2(a). The bifurcation point B is between O and A. If the fundamental path OAC corresponds to axisymmetrical deformation and BD to nonaxisymmetrical deformation, then initial failure of the structure would generally be characterized by rapidly growing nonaxisymmetrical deformations. In this case the collapse load of the perfect structure λ_L is of less engineering significance than the bifurcation point, λ_C.

In the case of real structures which contain unavoidable imperfections, there is no such thing as bifurcation buckling. The actual structure will follow a fundamental path OEF, with the failure corresponding to "snap-through" at point E at the collapse load λ_s. However, the bifurcation buckling analytical model is valid in that it is convenient and often leads to a good approximation of the actual failure load and mode. For more general background on buckling of perfect and imperfect structures, see Ref. [1].

CAPSULE OF PROGRESS IN THE 1970'S IN PLASTIC BUCKLING ANALYSIS

Recent progress in our capability to predict plastic buckling failure can be categorized into four main areas, three of them dealing primarily with structural modeling, and the fourth dealing primarily with material characterization.

The three areas dealing with structural modeling are:

1. Development of asymptotic postbuckling theories and applications of these theories to specific classes of structures, such as simple plates, shells and panels (Refs. [2-18]).

2. Development of general-purpose computer programs for calculation of static and dynamic behavior of structures includ-

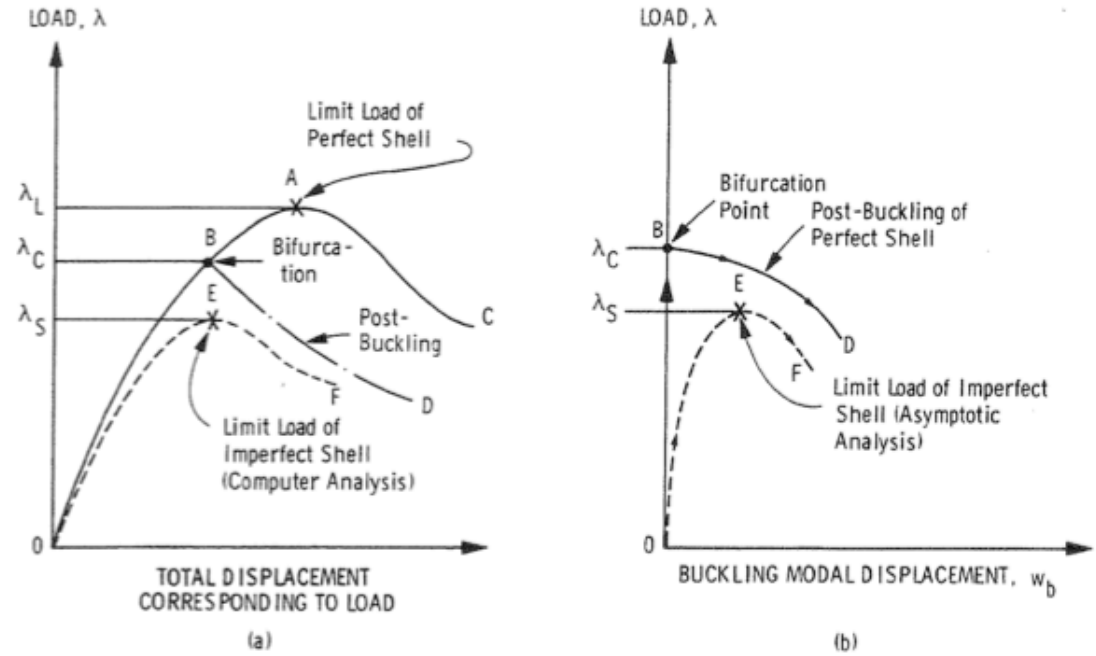

FIG. 2 LOAD-DEFLECTION CURVES SHOWING LIMIT AND BIFURCATION POINTS. (A) GENERAL NONLINEAR ANALYSIS. (B) ASYMPTOTIC ANALYSIS.

ing large deflections, large strains, and nonlinear material effects (Refs. [19-21]).

3. Development of special purpose computer programs for limit point axisymmetrical buckling and non-axisymmetrical bifurcation buckling of axisymmetric structures (Refs. [22-23]).

Asymptotic Analysis

The elastic-plastic bifurcation and asymptotic post-buckling analyses [2-18] rest on the theoretical foundations established by Hill [24-27], whose formulation of the bifurcation problem applies to solids with smooth or piecewise smooth yield surfaces and small or large strains, and Koiter [28-29], whose general elastic post-bifurcation theory leads to an expansion for the load parameter λ in terms of the buckling modal amplitude w_b which is valid in the neighborhood of the critical bifurcation point in (λ, w_b) space. Hill proved that in order for bifurcation of equilibrium paths to occur at stresses beyond the proportional limit, the initial post-buckling path must in general have a positive slope; plastic bifurcation buckling occurs under increasing load. Therefore, it is crucial to determine the post-bifurcation path until it reaches a limt load. The primary aims of the asymptotic analyses of Refs. [2-18] are to calculate limit loads for perfect and imperfect structures. These analyses have contributed vital physical insights into the plastic buckling process and the effect of structural or loading imperfections on this process.

General Nonlinear Analysis

The general-purpose computer programs in widespread use since the early 1970's and presently being written are based on principles of continuum mechanics established for the most part by the late 1950's and set forth in several texts [30-35]. The structural continuum is discretized into finite elements as described in the texts [36-40] and various strategies are employed to solve the resulting nonlinear problem [41-71]. The nonlinearity is due to moderately large or very large deflections and nonlinear material behavior. Various plasticity models are described in texts [72-76], conference proceedings [77-78], and survey articles [79-82]. Additional papers on the formulation, discretization, and solution of nonlinear structural problems appear in the symposia proceedings [83-87]. The primary aim of this vast body of work, most of which was done in the 1970's, has been to produce reliable analysis methods and computer programs for use by engineers and designers. Thus, the emphasis in the literature just cited is not on acquiring new physical insight into buckling and post-bifurcation phenomena, but on creating tools that can determine the equilibrium path OEF in Fig. 2 for an arbitrary structure and on proving that these tools work by use of demonstration problems, the solution of which is known. In most cases no formal distinction is made between prebifurcation and post-bifurcation regimes; in fact, simple structures are modeled with imperfections so that potential bifurcation points are converted into limit points. The plastic buckling problem loses its special qualities as illuminated so skillfully in Refs. [2-18] and becomes just another nonlinear analysis, requiring perhaps special physical insight on the part of the computer program user because of potential numerical traps such as bifurcation points and limit points usually (and sometimes spuriously!) revealed by changes in the sign of the determinant of a stiffness matrix.

Figures 2(a) and (b) illustrate the two very different approaches to the plastic buckling problem described in the last two paragraphs. In the general nonlinear approach the computations involve essentially a "prebuckling" analysis, or a determination of the unique equilibrium states along the fundamental path OEF in Fig. 2(a). In the asymptotic approach (Fig. 2b) the prebuckling state is usually statically determinate. The secondary path BD of the perfect structure and (in the elastic case) the limit point E on the fundamental path of the imperfect structure are determined by expansion of the solution in a power series of the bifurcation modal amplitude which is asymptotically exact at the bifurcation point B.

Axisymmetric Structures

The third approach to the plastic buckling problem, development of special-purpose programs for the analysis of axisymmetric structures, forms a sort of middle ground between the asymptotic analysis and the general-purpose nonlinear analysis. The approach is similar to the asymptotic treatment because in applications it is restricted to a special class of structures and the distinction between prebuckling equilibrium and bifurcation buckling is retained. It is similar to the general nonlinear approach in that the continuum is discretized and the nonlinear prebuckling equilibrium problem is solved "by brute force." The emphasis is on the calculation of the prebuckling fundamental path, OB in Fig. 2(a), and determination of the bifurcation point B and its associated buckling mode, not on calculation of post-bifurcation behavior BD or of the load-deflection path of the imperfect structure. The goals of this third approach are to create an analysis tool for use by engineers and designers and to use this tool in extensive comparisons with tests both to varify it and to obtain physical insight into the plastic buckling process.

SUMMARY OF THIS SURVEY ON PLASTIC BUCKLING

Certain difficulties associated with plastic bifurcation buckling theory are discussed in the next section. Following this, the asymptotic methods of Hutchinson, Tvergaard, and Needleman [2-18] are described and examples shown. The general nonlinear approach is then summarized with emphasis on the Total Lagrangian vs Updated Lagrangian formulations, strategies for solving nonlinear equations, and characterization of elastic-plastic material behavior. Next, a strategy for axisymmetric nonlinear snap-through and nonaxisymmetric bifurcation buckling of elastic-plastic axisymmetric shells is described, followed by numerous examples in which experimental and theoretical results are compared. The chapter closes with some recommendations for future work.

WHERE PLASTIC BUCKLING FITS INTO THE BIG PICTURE

Since most plastic buckling analyses are probably performed with use of general-purpose computer programs, and since

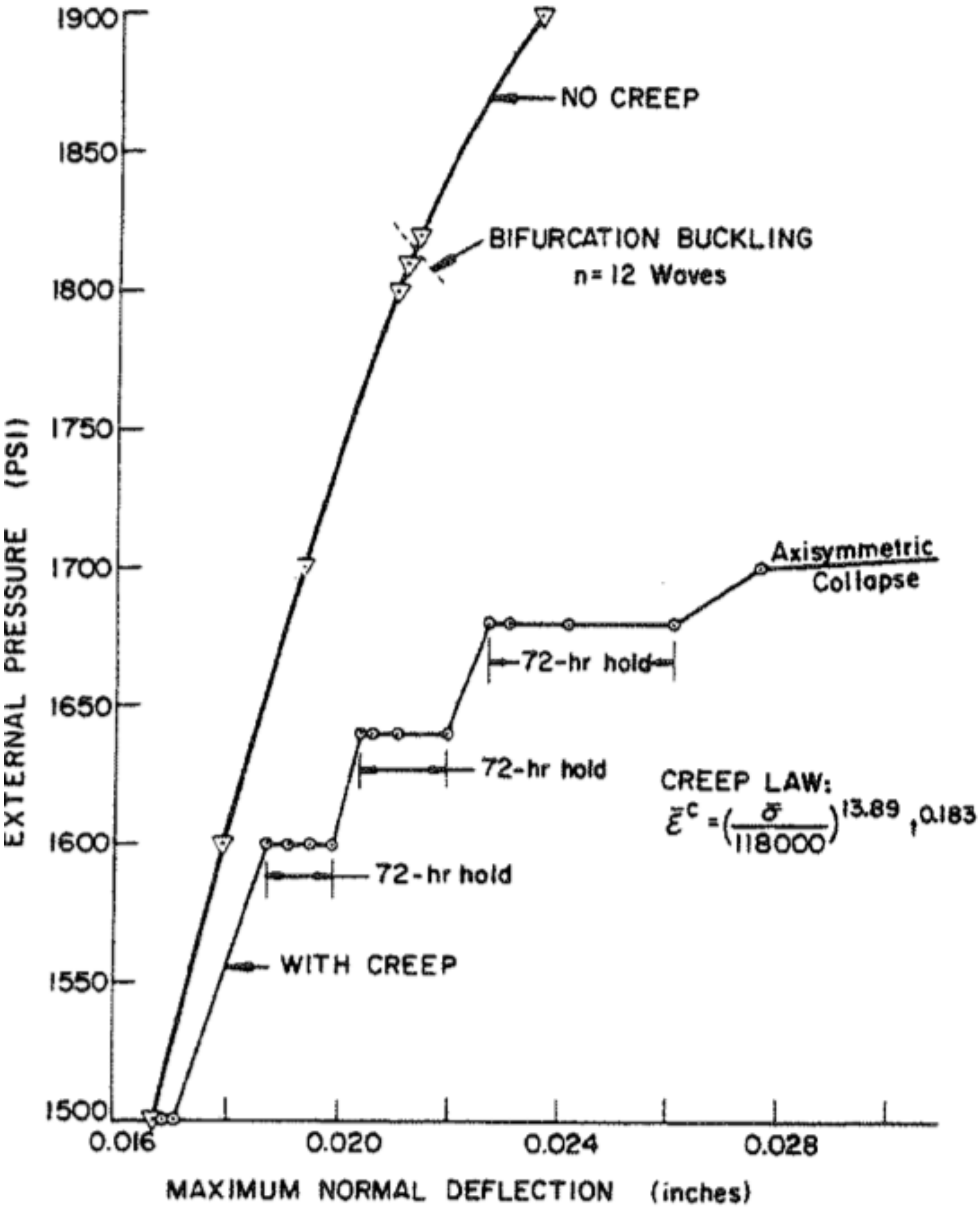

Fig. 50 Load-deflection curves for externally pressurized, ring-stiffened, titanium cylindrical shell with and without primary creep included in the analysis. (from PRESSURE VESSELS AND PIPING: DESIGN TECHNOLOGY – 1982, A DECADE OF PROGRESS, S.Y. Zamrik and D. Dietrich, editors, published by ASME; Chapter 2.4 “Plastic Buckling” by David Bushnell, pp. 47-117)

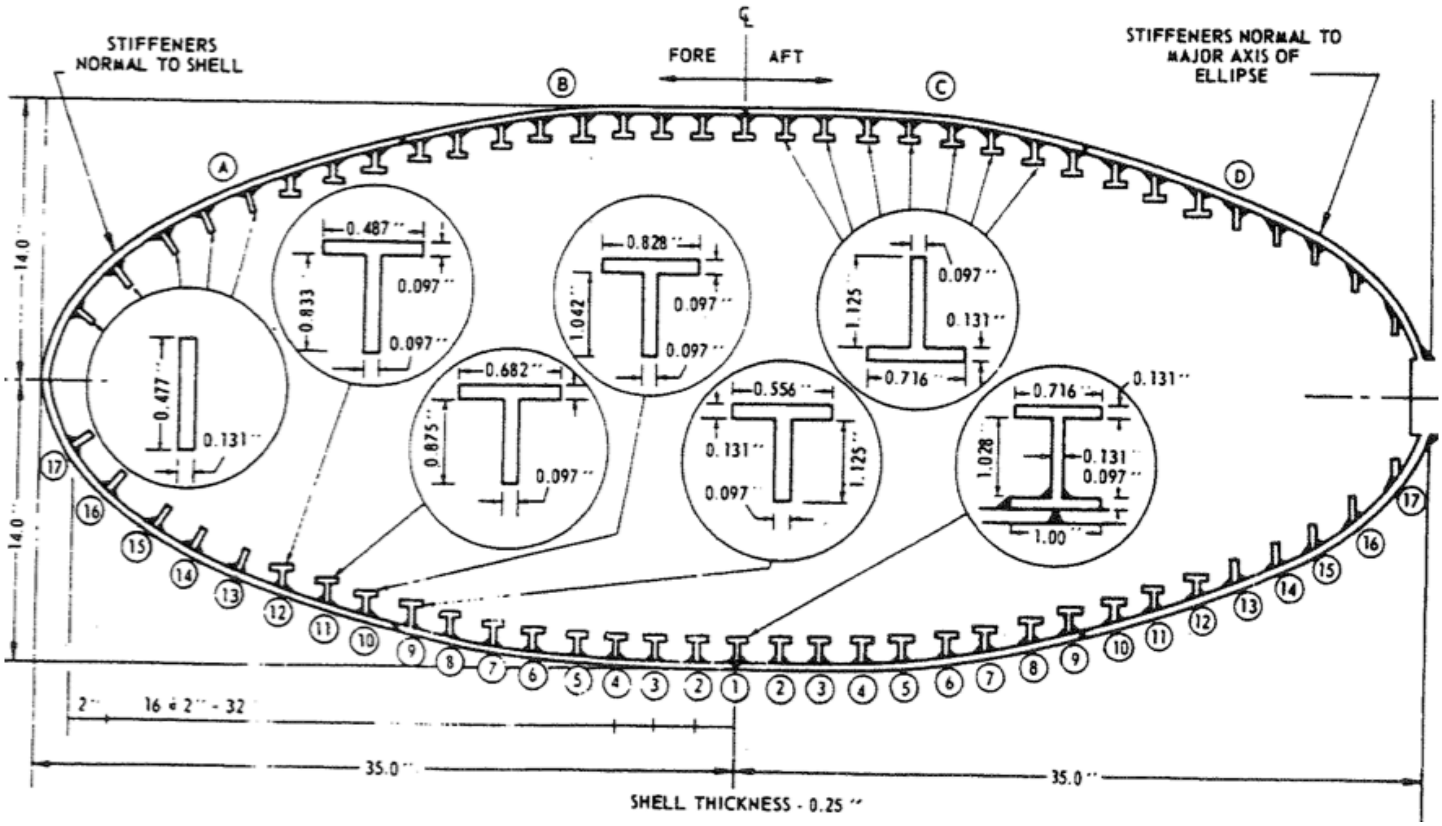

Fig. 74 Externally pressurized steel ellipsoidal shell with internal rings welded to it. (from PRESSURE VESSELS AND PIPING: DESIGN TECHNOLOGY – 1982, A DECADE OF PROGRESS, S.Y. Zamrik and D. Dietrich, editors, published by ASME; Chapter 2.4 "Plastic Buckling" by David Bushnell, pp. 47-117)

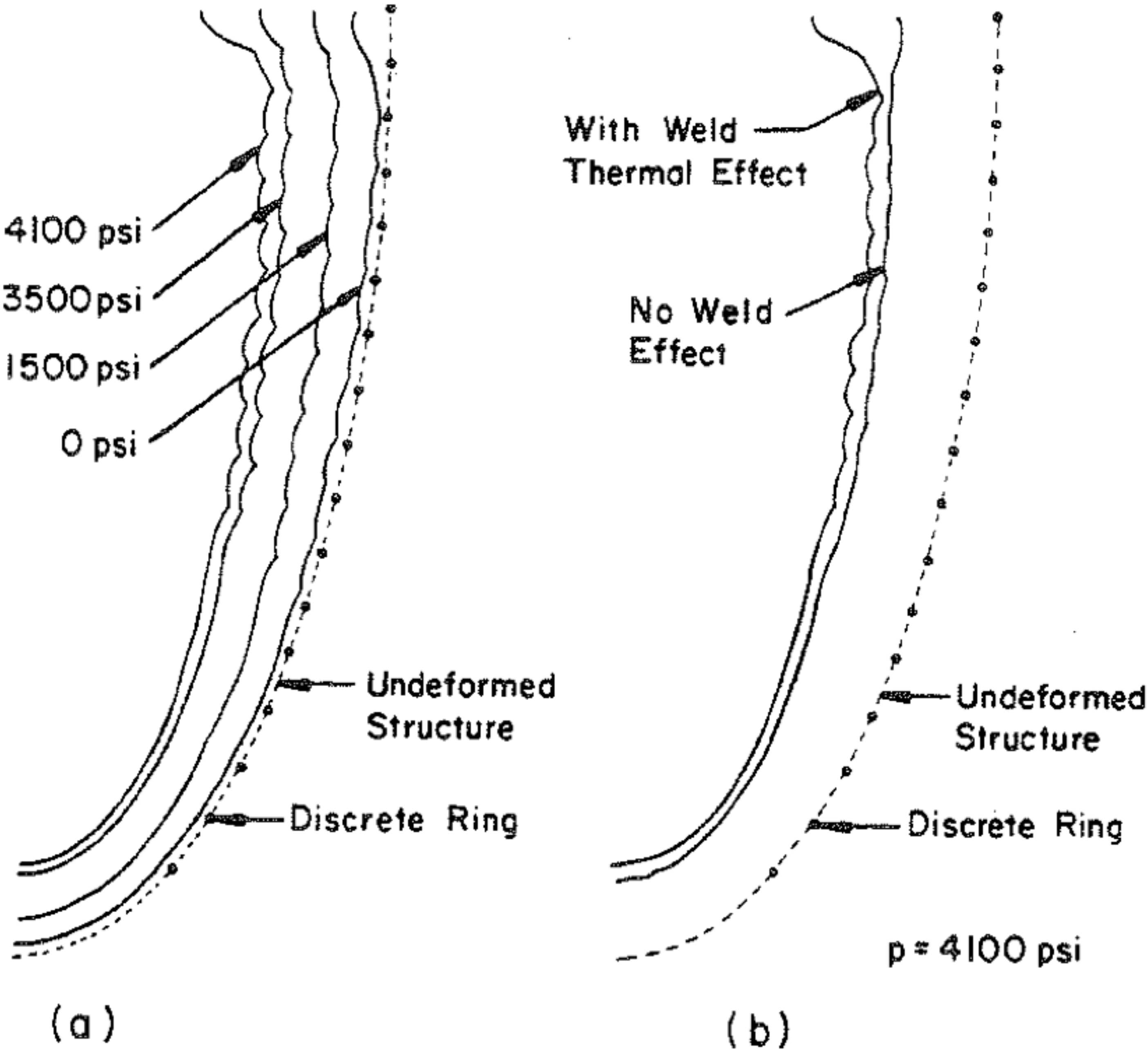

Fig. 77 Pre-buckling deflections with increasing external pressure and comparison with and without the weld cool-down effect. (from PRESSURE VESSELS AND PIPING: DESIGN TECHNOLOGY – 1982, A DECADE OF PROGRESS, S.Y. Zamrik and D. Dietrich, editors, published by ASME; Chapter 2.4 “Plastic Buckling” by David Bushnell, pp. 47-117)

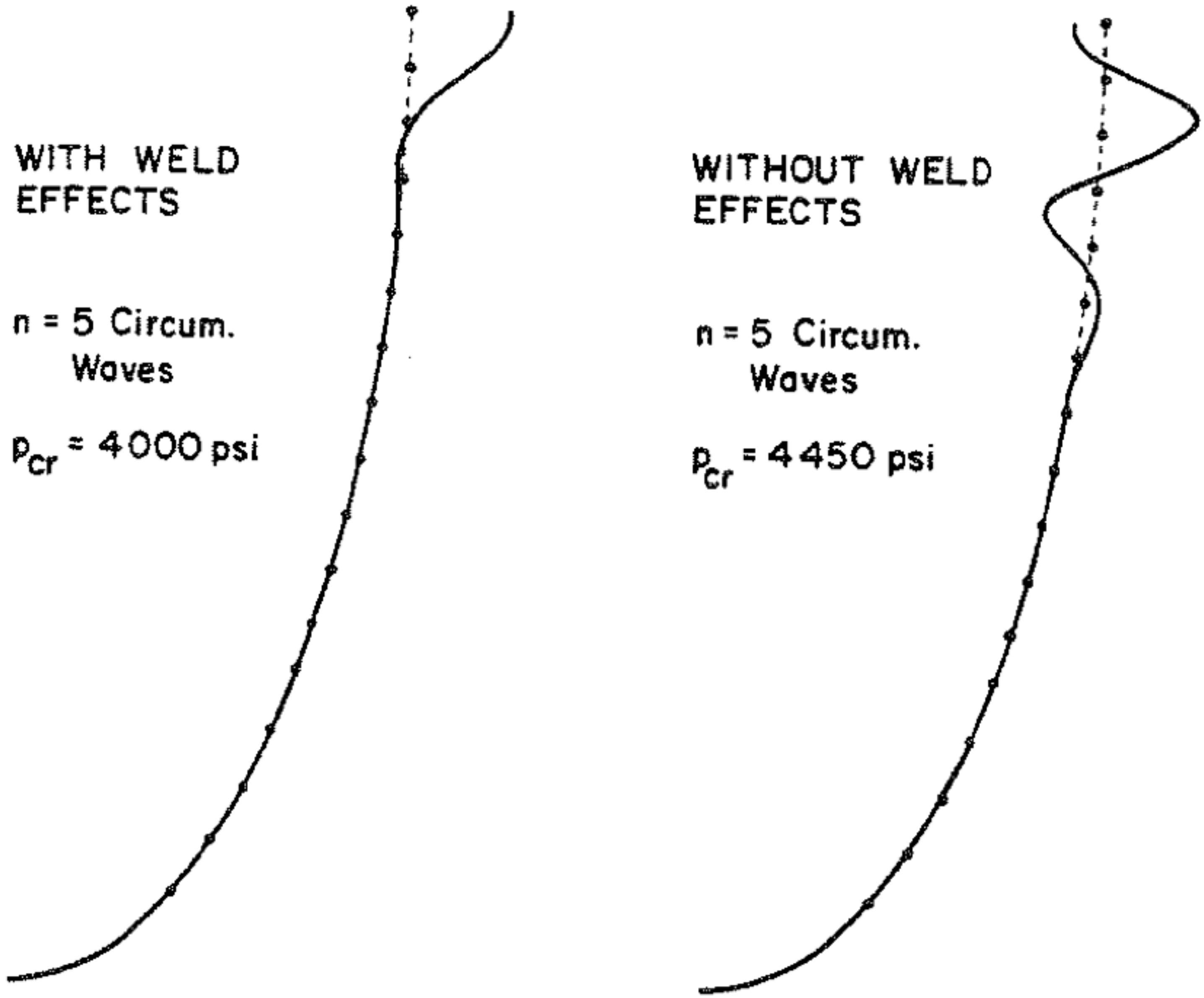

Fig. 78 Predicted bifurcation buckling modes and pressures with and without the weld cool-down effect included in the analysis. (from PRESSURE VESSELS AND PIPING: DESIGN TECHNOLOGY – 1982, A DECADE OF PROGRESS, S.Y. Zamrik and D. Dietrich, editors, published by ASME; Chapter 2.4 "Plastic Buckling" by David Bushnell, pp. 47-117)