



Professor Iosif Izrailevich Vorovich (1920 – 2004)

The Scientific Legacy of Academician Iosif Izrailevich Vorovich On the 90th Anniversary of his Birth (from: *Journal of Applied Mathematics and Mechanics* 74 (2010) 629–632)

Iosif Izrailevich Vorovich would have celebrated his 90th birthday on 21 June 2010. A member of the Russian Academy of Sciences, he was a leading Russian scientist who achieved much in the fields of mechanics and mathematics. Vorovich was a remarkable teacher, and he founded an authoritative school of mechanics both at home and abroad. A detailed description of his life was published in *Prikladnaya Matematika i Mekhanika* (V.A. Babeshko, A.V. Belokon', and V.I. Yudevich Iosif Izrailevich Vorovich. *Prikl. Mat. Mekh.*, 2000;64(3)) and was given in a book of reminiscences about him (*Reminiscences of Academician I.I. Vorovich*. Rostov-on-Don: Rostov State Communications University: 2004).

Vorovich completed the fourth year at Moscow State University just before Germany invaded Russia in 1941, and then, during the war, graduated from the N.Ye. Zhukovskii Airforce Engineering Academy. He then served in the armed forces, continued his academic studies at the Academy and defended his Candidate Dissertation in 1950. His subsequent research and teaching were closely associated with Rostov State University, to which he was appointed in the same year. A young Candidate of Sciences, strong and energetic and bursting with creative thoughts and ideas, he immediately became a centre of attraction among Rostov mathematicians. Junior member of the teaching staff, senior lecturer, deputy professor, reader, Head of the Faculty of Elasticity Theory, professor, Director of the Scientific Research Institute of Mechanics and Applied Mathematics of Rostov State University, Honorary Soros Professor, corresponding member (1970) and full member of the USSR Academy of Sciences (1990) – these are the main landmarks in his many-sided research and teaching activity.

His brilliant defense of Doctoral Dissertation in 1950 was the forerunner to the setting up of the Faculty of Elasticity Theory at Rostov State University. In the very early stages of the establishment of the new faculty, at the beginning of the 1960 s, under his management, a number of the most central lines of research in mechanics took shape in a period of intense development of military technology: the non-linear theory of shells and problems of the stability of thin-walled structures, the non-linear theory of hydrodynamic stability, mixed static and dynamic problems of elasticity theory, the mathematical theory of thick plates and shells, and the mathematical theory of initial-boundary-value problems of viscoelasticity.

After a brilliant education as both a mathematician and an engineer, he employed all his potential to investigate the most diverse problems of mechanics and applied mathematics. In his research, he not only used state-of-the-art mathematics, functional analysis methods, and operator beam and non-linear operator equation theory but also created a new body of mathematics and was drawn to tackling any problems encountered by mathematicians. This principle of his scientific life, based on the rigorous mathematical formulation of problems and a comprehensive analysis, made it possible to achieve important results in the most difficult areas of the mechanics of deformable solids. Rigor of formulation, the use of new methods for investigating solvability and uniqueness, the wide use of asymptotic methods in the investigation of specific problems – these are the hallmarks of the researchers at the Rostov School of Mechanics.

One of the first problems to whose solution Vorovich made a fundamental contribution was the investigation of the existence of a solution of problems of the statics and dynamics of non-linear thin-walled structures. He was the first to develop topological and variational approaches to the problem of the solvability of the principal boundary-value problems of non-linear shell theory, and to provide a rigorous proof of the applicability of a number of direct methods of the Bubnov–Galerkin type to the construction of an approximate solution of non-

linear problems of the deformation of thin-walled structures. A feature of his approach to the generalized formulation of problems was its close tie with the mechanical content of the problem and with general variational principles of mechanics. The latest mathematical methods he used were based not only on formal mathematical transformations and estimations but also on a deep understanding of the mechanical essence of the problems under investigation. His approach always included an in-depth thematical analysis of the boundary-value problems of continuum mechanics, with a subsequent construction and substantiation of computing schemes. The results obtained using this approach are graphic and have a clear mathematical interpretation. The topological approach he developed, and based on the Schauder principle, goes far beyond non-linear shell theory and makes it possible, with practically no change to the mathematical methods used, to investigate the corresponding boundary-value problems, to examine and substantiate problems of the convergence of various methods in non-linear continuum mechanics. These ideas made it possible, for example, to prove, in combined studies with Yu.P. Krasovskii (Dokl. Akad. Nauk SSSR, 1959;126(4)), the convergence of the method of elastic solutions in plasticity theory, proposed earlier by A.A. Il'yushin.

One of the main lines of research that underwent rapid development in the 1960s and 1970s in mechanics, including the Rostov School of Mechanics, under his management, involved the development of shell and thick plate theory and the substantiation of the limit transition from three-dimensional problems of elasticity theory to two-dimensional problems. The numerous phenomenological models of shell theory that had been created by this time, which differed in allowing for particular factors, had to be compared on the basis of model solutions constructed using three-dimensional equations. Based on rigorous analysis, Vorovich was the first to provide a classification of the main approaches to constructing a plate and shell theory that was widely recognized among specialists.

Asymptotic methods played a huge role in the development and comparison of plate and shell theories. Vorovich emphasized on more than one occasion that plate and shell theory was by its very nature asymptotic. In the 1960 s, in work by Friedrichs, Dressler and Goldenveizer, the principles of the asymptotic method of integrating three-dimensional equations of elasticity theory for a homogeneous isotropic plate, based on two iteration processes, were set out. Vorovich and Aksentyan (Prikl. Mat. Mekh., 1963;27(6)) used another method for the asymptotic integration of three-dimensional equations of elasticity theory, based on Lurie's method of homogeneous solutions (1942). This approach was extremely constructive from the viewpoint of a qualitative analysis of the stress-strain state (SSS), and also in substantiating the process of a limit transition; such segmentation of the problem is particularly important when analyzing the SSS in the vicinity of concentrators – ribs and holes; based on this approach, the various types of SSS were classified, and a quantitative analysis was made of the effect of the geometric and physical parameters on the stress concentration.

The possibilities of the homogeneous solution method devised by Yu.A. Ustinov, who, with his students, extended the results of investigations of inhomogeneous plates with strong thickness inhomogeneity, were broadened considerably. In this case, an asymptotic analysis of the problem in the form of expansions in two parameters enabled the features of the SSS for multilayer plates and shells to be revealed and provided an estimate by various applied theories. Subsequently, this line of research was successfully developed for laminated and inhomogeneous structures by I.G. Kadomtsev, M.F. Mekhtiyev, and N.V. Boyev, and for electroelastic plates and inhomogeneous waveguides by I.P. Getman and V.Ye. Zhirov.

Vorovich obtained important results when investigating the stability of both thin shells and of three-dimensional problems of deformable media. He was one of the pioneers in the use of probabilistic methods in the problem of shell stability. At the start of the 1960s, he wrote several papers devoted to the influence of initial imperfections

and random loads on the stability of thin-walled structures and established important theorems on the number of solutions of non-linear boundary-value problems of the mechanics of structures and the stability of these solutions. The necessary and sufficient conditions for the stability of the natural state of a viscoelastic body that he formulated are of considerable practical importance, and also many other results in the area of the deformation and stability of structures of complex rheology. This line of research, based on rigorous tensor calculus, was developed by L.M. Zubov, L.P. Lebedev, and V.A. Yeremeyev. Stability was investigated for problems of thermoviscoelasticity, for Cosserat media and for thin-walled structures with complex properties. In recent years, many results have been extrapolated to models of media with an internal structure, to cases when there are dislocations and disclinations, and to shells of foam and porous materials.

The development of the classical theory of contact problems, originating from the work of Hertz, gained new impetus in the middle of the twentieth century in the investigation of contact interaction both under static loading and under dynamic conditions. Vorovich was the initiator of the development of a rigorous mathematical theory of mixed continuum mechanics problems, including mixed elasticity theory problems. He was the first to provide a rigorous mathematical formulation of static and dynamic contact problems for layer-type regions, thereby paving the way for the investigation of problems with mixed boundary conditions using an asymptotic analysis of the integral equations with an irregular kernel that arise (I.I. Vorovich and Yu.A. Ustinov. *Prikl. Mat. Mekh.*, 1959;23(3)). Consequently, asymptotic methods for solving static contact problems (large - methods) were developed by V.M. Aleksandrov and his students and were brought to a level where they could be used in engineering applications in the form of formulae and tables.

In a monograph (I.I. Vorovich, V.M. Aleksandrov and V.A. Babeshko, *Non-classical Mixed Problems of Elasticity Theory*. Moscow: Nauka; 1974), mathematical models for solving static contact problems using different asymptotic approaches were presented and rigorously substantiated. This line of research was continued in the Rostov School of Mechanics, involving the development of analytical methods for solving static contact problems for bodies with complex physicomaterial properties, by V.M. Aleksandrov, M.I. Chebakov and S.M. Aizikovich.

Allowance for the dynamic nature of applied loads led to the launch in the Rostov School of Mechanics of a new line of research associated with the development of methods for solving dynamic contact problems. Here, to investigate new classes of mixed problems of elasticity theory, new methods were required. In the field of dynamic contact problems, Vorovich provided a rigorous mathematical formulation of mixed problems, established the presence both of a continuous and of a discrete spectrum of the operators investigated, and indicated algorithms for computing them. In joint monographs with V.A. Babeshko and O.D. Pryakhina, mathematical methods were given and rigorously proved for solving dynamic contact problems for semi-bounded domains. V.A. Babeshko developed apparatus for factorizing functions and matrix functions, by means of which it proved possible to reduce the initial boundary-value problems to operator equations of the second kind with completely continuous operators, to design effective computing schemes for constructing solutions, and to uncover the fine structure of the formation of wave fields. Subsequent development of factorization methods and the creation by V.A. Babeshko of the method of fictive absorption for solving a broad class of integral equations and systems made it possible to investigate new classes of problems of continuum mechanics for laminated structures (problems for arbitrary areas of contact in the presence of stratification – Ye.V. Glushkov and N.V. Glushkova; anisotropic and electroelastic media – A.O. Vatul'yan and O.D. Pryakhina; prestressed media – V.V. Kalinchuk). Work by Vorovich helped promote significantly the important development of mathematical methods for problems of geophysics and seismology, which was reflected by the large number of subsequent studies and monographs by V.A. Babeshko, where new approaches were

formulated to solving a broad class of boundary-value problems of mathematical physics based on differential and integral method of factorization, new ideas were presented that made it possible to create new computing technologies, and, in particular, the theory of block structures and block elements was developed.

Important results based on an analysis of Timoshenko-type non-linear integral equations in the theory of elastoplastic collision of bodies were obtained by I.G. Kadomtsev; they allowed the goodness of fit of different applied theories in this area to be assessed.

The development of the superimposition method enabled M.G. Seleznev to analyse wave fields in mixed problems of vibration of semi-bounded bodies with cavities and inclusions, to establish the ranges of variation of the parameters, where the proposed schemes for constructing approximate solutions are fairly effective, and to develop a line of research with applications in problems of the designing foundations and road surfaces, and in building.

Developing methods for solving different practical problems concerning the contact interaction of bounded elastic bodies, Vorovich devised approaches that later spawned one of the modern computing technologies – the boundary integral equation method and the boundary element method based on it. Investigations in this area using the superimposition method enabled A.V. Belokon' to reduce mixed problems for finite bodies of canonical shape to systems of boundary integral equations, to analyze the structure of solutions, and to propose effective approximate methods for constructing and investigating them.

In joint work with A.V. Belokon', Vorovich studied problems of the correctness of the formulations of static and dynamic problems of coupled electroelasticity, investigated the spectral properties of piezoelectric bodies of limited dimensions, and substantiated the applicability of the Bubnov–Galerkin method. These investigations then formed the basis of the finite element modelling of coupled fields, which was developed in work by A.V. Belokon', A.V. Nasedkin and A.N. Solov'yev, which enabled the state-of-the-art finite element package ACELAN to be created.

Vorovich showed intuition in determining priority lines of research, and also had the know-how to combine a thoroughness in solving theoretical problems with the vision as to how the theory might be applied. This applies, in particular, to financial contract work on the investigation of the strength of wheels of complex design (the monograph: I.I. Vorovich, Yu.V. Safronov and Yu.A. Ustinov, *The Strength of Wheels of Complex Design: Investigation and Calculation*. Moscow: Mashinostroyeniye; 1967), and also to the development of an engineering procedure for calculating temperature regimes of polymer bearings (1967). From 1977, he supervised important developments of methods and software packages in the area of the vibroacoustics of complex shell structures, which were used in shipbuilding, aviation and other sectors; particularly urgent was the analysis of the vibrational characteristics of structures, allowing for contact with a liquid, and also the problem of the radiation and scattering (diffraction) of sound by elastic shells, which was especially important for analyzing the vibroacoustic characteristics of submarine objects, the levels of sound radiation of which are related to their principal properties. Despite the considerable attention devoted to this problem and the wealth of publications on relatively simple objects, in the period when this work was developed, problems of designing shell models taking adequate account of the geometry and principal design elements of objects, and also the parameters of the external medium, remained unresolved. Significant results in this line of research were obtained under the leadership of I.I. Vorovich, A.S. Yudin, A.Ya. Tsionskii, V.G. Safronenko, et al.

In the middle of the 1990s, Vorovich, foreseeing the expansion of areas of future applications, drew the

attention of his students and colleagues of the faculty and institute to a number of new areas of mechanics in which the solution of specific problems could not be obtained numerically, for example, using the finite element method. One of these new lines of research in mechanics and modern mathematical physics was the development of the theory of inverse problems. These problems were generally non-linear and irregular, and interested him both as a mechanician and as a mathematician. The simplest problems of this type arise in identifying of polymer materials within the framework of linear differential models; at the Scientific Research Institute of Mechanics and Applied Mathematics, not only were theoretical investigations along these lines successfully developed, but units were also set up that implemented this laborious procedure automatically. Problems in identifying cavities from a directional diagram – so-called geometric inverse problems, which have important applications in defectoscopy – were developed in a cycle of joint work with N.V. Boyev and M.A. Sumbatyan. Problems of load reconstruction were the focus of joint work with A.O. Vatul'yan and A.N. Solov'yev. Later, this research was developed along the lines of investigating problems of identifying cracks in solids and determining the variable characteristics of solids. These problems involve inverting compact operators and require regularization in a particular form. Subsequent development of methods for solving inverse problems of determining the coefficients of differential operators describing the vibrations of inhomogeneous elastic and electroelastic bodies for boundary fields of displacements was carried out by A.O. Vatul'yan and A.N. Solov'yev, who described methods for formulating iteration processes, and also economic computing schemes for constructing solutions were developed and scale computer experiments were carried out.

Vorovich's entire scientific activity was aimed at solving central problems in new areas of mechanics and at establishing new laws existing in nature. Thus, in 1979, he published two papers, devoted to the analysis of the spectral properties of an elastic strip, in which he formulated the conditions for the occurrence of unrestricted resonances in semi-bounded media, which made it possible subsequently to register the discovery made jointly with academicians N.F. Obraztsov and V.A. Babeshko, of "The phenomenon of high-frequency resonance in semi-bounded media with inhomogeneities". This enabled work to be developed along these lines in a number of investigations. Thus, O.D. Pryakhina, together with Ye.I. Vorovich and O.D. Tukodova, investigated in detail the conditions and characteristics of the occurrence of resonance effects in an elastic strip and in elastic multilayer structures interacting with massive objects. V.V. Kalinchuk and T.I. Belyankova studied in detail the conditions for the occurrence of resonance situations in the contact interaction of elastic bodies with semi-bounded prestressed objects, and also in the case of electroelastic media with discrete electric systems with lumped parameters.

From the instant of the setting up in 1971 of the Scientific Research Institute of Mechanics and Applied Mathematics, Vorovich began to develop a new line of research in applied mathematics for the Rostov scientific school – the mathematical modeling of ecological and economic systems. The rich experience of mechanics in the modeling of systems with complex properties enabled him to apply his knowledge to the modeling of the ecological system of the Sea of Azov. In 1983, the principal result of this research, which was continued by a team of scientists, was awarded the State Prize of the USSR in the field of science. The mathematical modeling of ecological-economic systems was supplemented in 1996 with state-of-the-art tools – geoinformation technologies, and in 2008 with a new line of research – space monitoring; corresponding work is being conducted under the leadership of F.A. Surkov.

Vorovich was present at the very inception of the Rostov School of Mathematical Fluid Mechanics directed for many years by his student V.I. Yudovich, who was the first to use topological approaches to investigate various problems of classical fluid dynamics. In his studies, a consistent theory of the stability of motions of a viscous

fluid was developed. Extensive research was carried out on the linear and non-linear stability of different classes of motion of a viscous fluid – translational flows in channels, rotational flows between cylinders, and also problems, similar mathematically, concerning the onset of gravitational convection. Problems of the existence and uniqueness of non-viscous fluid flows, the problem of the solvability of the basic problems of fluid dynamics, the analysis of linear and non-linear stability of different classes of viscous fluid motion, the development of asymptotic models of convection, the creation of new physical models of a fluid, in particular, models of an incompressible fluid with a density independent of the pressure, models of a micropolar fluid, models of liquid crystals – this is far from a comprehensive list of the lines of research undertaken by V.I. Yudovich, M.Yu. Zhukov, and many of their students, the shaping of which was influenced to some extent by Vorovich.

Vorovich published about 300 scientific papers, including 16 monographs on various mechanics problems. He devoted the final years of his life to work that was dearest to him – the writing of a comprehensive mechanics textbook. Unfortunately, a large part of the book planned by the author remained unfinished. After his death, in 2004, through the efforts of his students (V.I. Yudovich and E.N. Potetyunko), the first part of this work was published – Lectures on Newtonian Dynamics. A Modern View of Newtonian Mechanics and its Development, consisting of 23 lectures. In subsequent work by the team of the Faculty of Elasticity Theory, and with the support of the Russian Foundation for Basic Research, the second part was prepared for publication in 2010, consisting of 26 lectures. The undoubted merits of the book include the breadth and depth of coverage of the material, layout accessibility, and the attempt to give practical applications. It contains, along with classical sections of theoretical and analytical mechanics, historical digressions and deep analytical investigations in various areas of mechanics and mathematics (contact problems, friction, potential theory, the ballistics of near-Earth space, elements of the theory of relativity and celestial mechanics, non-linear vibrations, the theory of stability, and the qualitative theory of differential equations). Furthermore, there are biographical details on outstanding mechanicians and a vast amount of reference material. The third part of this monumental work is in preparation.

Vorovich's students are left with memories of his charming smile and his remarkable stories and jokes; today we remember with gratitude the pleasure of the company of this simple and accessible man who made such a significant contribution to various areas of mechanics.

Translated by P.S.C.