



Professor Fred W. Williams, FEng (?-2019)

Obituaries: (1) by David Kennedy and (2) by Ranjan Banerjee:

(1) by David Kennedy:

Fred Williams was the son of Sir Frederic Calland Williams, creator of the first electronic stored-program digital computer. After obtaining a First Class degree at Cambridge Fred studied for a PhD at Bristol supervised by Sir Alfred Pugsley, whose old school, hands-off approach aided Fred's development as an independent researcher.

Fred then spent 3 years teaching at a university in Nigeria, where he and his new wife Anne were terrifyingly caught up in some civil unrest. Safely back in the UK he joined Professor Bill Wittrick at Birmingham, applying his civil engineering expertise to the buckling and vibration of aircraft structures. In Fred's words, he "quickly experienced a sense of privilege, respect and even awe... Bill's courtesy and consideration were unflinching. An abiding memory is the way he would identify and remove one's areas of ignorance ... in a way which was direct but never crushing. .. Bill, like the best of master craftsmen, passed on by example and training the very highest standards of scholarly integrity, enthusiasm and thoroughness." Words that perfectly describe Fred himself.

Fred recalls an occasion when "Wittrick and I worked intensively together all day, including lunch, to meet the need of one of his research students. An algorithm grew out of the discussion, to the surprise of both of us, as the result of numerous thoughts (some of which were helpful and some of which proved to be misleading). A key thought occurred over lunch but almost faded before it could be fully developed in the afternoon. Without

doubt, this algorithm was the most comprehensive joint discovery of which I have first hand experience and memories of that day are still clear about twenty years later.”

In 1975 Fred became the youngest professor in Cardiff, where he remained for the rest of his career. But not before he had written a computer program VIPASA with the Wittrick-Williams algorithm at its heart.

At a conference he met Dr Mel Anderson from NASA Langley Research Center, who went on to incorporate VIPASA into a design program which became widely used in the US aerospace industry. Mel came to the UK with his family for a year to work with Fred on a new analysis code VICON. They also developed 3D frame software which was used in the design of the space shuttle. With funding from British Aerospace and NASA, Fred’s team at Cardiff further consolidated the software into a design code VICONOPT which is still used in the aerospace industry.

In 1988 Fred met Professor Zhong Wanxie from Dalian University of Technology. So began a passion that continued throughout his career. He visited China every year, sometimes for several months, and also established the Cardiff Advanced Chinese Engineering Centre, hosting scores of visitors including senior professors with whom we have collaborated ever since. Fred was intrigued by the Chinese version of a well-known proverb, and posted a notice on his office door saying “Failure is killing only two birds with one stone”. Sadly he was asked to remove it on the occasion of a royal visit.

As head of Cardiff’s Division of Structural Engineering, Fred’s recipe for success was “Quality, Income, Papers”. He led by example, publishing over 400 papers and carrying his research into his teaching, particularly his final year module on plate theory.

Fred is remembered with respect and affection by generations of research students, many of whom have advanced to successful careers in academia or industry. Always firm but fair, explaining and encouraging. One student recalls enthusiastically trying to explain something but making a fundamental mistake, upon which Fred called out “STOP!” If your meeting with Fred was in the late afternoon you knew it would have to continue until the problem was solved. I would often return home late having been “Fredded”.

But Fred could never be just a colleague. To all of us and our families he became a friend, generous in his hospitality, keen to share his love of travelling and the great outdoors. He contributed enthusiastically to the success of ISVCS and organised the 2003 event at his beloved Keswick.

In 2000 Fred took up a prestigious 3 year appointment at City University of Hong Kong, also continuing our collaboration with universities in mainland China. I remember arriving in Beijing on an overnight flight and finding Fred waiting for me in the hotel lobby with equations to solve. The Wittrick-Williams algorithm featured in a textbook by Tsinghua’s Professor Yuan Si and became widely known in China.

Returning to Cardiff Fred worked part-time, then was appointed Emeritus Professor, winding down to enjoy a well earned retirement. His last visit to China was in 2013 when Dalian appointed him a Guest Professor.

Fred leaves a legacy of expertise in structural mechanics, and his work will surely continue through those of us who have had the privilege of learning from him. I have received condolences from over 70 of his colleagues and students in some 20 different countries.

In January Fred wrote to some of his closest friends, “I thank you from the bottom of my heart for all your friendship has meant to me and wish you well for the future.” We thank Fred for teaching and inspiring us and for letting us share in his wonderful life.

David Kennedy

(2) by Ranjan Banerjee

Fred Williams: A friend, mentor and an indomitable spirit in my life

In November 1978 I had just finished my PhD Viva at Cranfield. The same evening, I had a phone call from Professor Fred Williams which was a turning point in my career and life. I was thinking of packing my luggage and leaving the United Kingdom for good. I remember the telephone conversation vividly even to this day. Fred asked me on the phone very directly, “Will you be interested in a post-doctoral research in Cardiff?” I knew very little about Fred at that time, but I was told that he was one of the youngest and brightest professors in the country. My telephone conversation with Fred was amazing. I thought that this professor, whom I had never met, could really be my mentor and guru to fulfil my ambitions in life. How can a person be so precise when he talks? He was open, honest, clear and importantly very straightforward and exactly to the point. He said that if I was interested, he would invite me to come to Cardiff for an interview for the post-doc job.

That interview was an unforgettable experience. The most transformative period of my life began to emerge. It took me only a few seconds to realise that Fred was an incredibly intelligent man. His towering and charismatic personality impressed me enormously. He structured my interview in an extremely clever and well-thought-out manner. During the conversation, prior to the formal interview, he was in fact X-raying me. The formal interview took only fifteen minutes. I was offered the job of Research Associate and I was Fred's very first post-doc. My instinct told me that my future would be totally secure in his hands. I had an overwhelming feeling within me that I had to respect this man.

I joined Fred in Cardiff on 1 February 1979 and worked with him for six years on large (1km size) space structures, in close collaboration with NASA Langley Research Center. The Ranjan Banerjee that you see today was in the making. The magic of Fred worked wonders. His planning and thinking were immaculate. He knew how to inspire a person and how to get the best out of a person. He planned, he thought, and he always had a contingency plan in case something went wrong. He was such a structured person, always analytical in his approach, always precise. According to Fred, a researcher should be as objective as possible. In his presence I was determined to succeed.

I have many special memories of Fred which I have no space to include here. Fred was a highly principled person and he always stuck to his principles. From a personal perspective, the screw of my academic life turned because of Fred. He had so much influence on me and on my thinking. There is no drama, no poetry, no melodrama in this statement. He rekindled my hopes and aspiration even after I left him to join City University in 1985. He had the right ideas and a true sense of values. When I met him first, he weighed me up in just five minutes. He was a real and genuine visionary and certainly a meticulous man. He monitored and observed my progress all along and my success was uninterrupted because of him. I am still consolidating the knowledge that I acquired from him. I achieved more than I could have ever imagined. This has been possible because of Fred and I am very proud to have worked with him.

When I met Fred a couple of months before his death, he was in a sparkling mood, as sharp as ever. I treasure those moments. He has given me everything I asked for, more than I bargained for. I am very sad to see him depart and it truly breaks my heart. We will carry forward his ideas.

In summary, Fred has been to me a memory of the past, running into the present and flowing on into the ocean of the future.

Ranjan Banerjee

<http://65.54.113.26/Author/18394335>

Cardiff School of Engineering, UK
Chairman of Cardiff Advanced Chinese Engineering Centre
Institute of Modeling and Simulation in Mechanics and Materials

Experience:

Fred Williams obtained a First Class Honours degree at the University of Cambridge in 1961, his PhD in the area of Space Frame Domes from the University of Bristol in 1964 and his Doctor of Science degree from the University of Cambridge in 1985.

Prior to coming to Cardiff in 1975, he had a brief period at Freeman Fox and Partners before becoming a lecturer in Civil Engineering, first at Ahmadu Bello University in Nigeria (1964- 67) and then at the University of Birmingham (1967-75). While at Birmingham he made one of the rare breakthroughs in classical mechanics with Professor Wittrick. This breakthrough has since been named by others the Wittrick-Williams algorithm. He is currently involved in major new work on extending this algorithm and its areas of application.

At Cardiff, he has pioneered the use of the Wittrick-Williams algorithm principally in the areas of frameworks and prismatic plate structures of Aerospace proportions. Both of these areas have been funded by NASA under a co-operative agreement which has existed since 1981. The resulting space frame program BUNVIS-RG was released by NASA to US users in 1986/87 and successive versions of the plate code VICONOPT have also been released by NASA, starting in 1990/91. Areas of work additional to those listed above include: effects of

periodicity; stayed columns; “back of the envelope” calculation methods for gaining structural insight; design and optimisation methods; deployable structures; parallel computing; seismic response; wave propagation; postbuckling behaviour, and rapid equation solving.

Links with China are exceptionally close, with 20 Chinese full professors being co-authors of Journal papers and Professor Williams being a Guest Professor of Shanghai Jiao Tong University and of the University of Science and Technology of China. He is currently on a two year secondment as one of eight internationally leading research professors recruited by City University of Hong Kong.

Collaboration with industry is very close. VICONOPT is a 50,000 line FORTRAN code based on novel theory. It is used by nearly all the American Aerospace Companies and by British Aerospace (Military and Civil) who co-funded it with NASA for many years.

He is author of 276 published or accepted papers of which approximately 70% are in a wide range (over 46) of refereed Journals of international standing.

Selected Publications:

Williams, F. W. and Wittrick, W. H. Computational procedures for a matrix analysis of the stability and vibration of thin flat-walled structures in compression. *Int. J. Mech. Sci.*, 1969, 11(12), 979-998.

Wittrick, W. H. and Williams, F.W. A general algorithm for computing natural frequencies of elastic structures. *Q. J. Mech. Appl. Math.*, 1971, 24(3), 263-284.

F.W. Williams (Department of Civil Engineering, University of Birmingham, Birmingham B15 2TT, England), “Computation of natural frequencies and initial buckling stresses of prismatic plate assemblies”, *Journal of Sound and Vibration*, Vol. 21, No. 1, March 1972, pp. 87-106, doi:10.1016/0022-460X(72)90208-8
ABSTRACT: Many structures consist of a set of thin rectangular flat plates of uniform thickness which are rigidly connected together along their longitudinal edges. Two computer programs which are applicable to such structures are described. They are called gasvip and vipal and they use an exact method of analysis, either to find natural frequencies in the presence of uniform longitudinal stress, or to find the initial buckling stress in uniform longitudinal compression. Gasvip sets up the overall stiffness matrix of the structure, whereas vipal enables substructures to be used. There are some types of problem which cannot be solved by using Vipal, but where it can be used it often takes much less computer time than gasvip. vipal also has the advantage that there is virtually no limit on the number of nodes (i.e., line junctions between component plates) which can be handled within about 4K of core store.

Wittrick, W. H. and Williams, F. W. An algorithm for computing critical buckling loads of elastic structures. *J. Struct. Mech.*, 1973, 1(4), 497-518.

W. H. Wittrick and F. W. Williams (Department of Civil Engineering, University of Birmingham, Edgbaston, Birmingham, England), “Buckling and vibration of anisotropic or isotropic plate assemblies under combined loadings”, *International Journal of Mechanical Sciences*, Vol. 16, No. 4, April 1974, pp. 209-239, doi:10.1016/0020-7403(74)90069-1

ABSTRACT: This paper describes the underlying theory, and a general-purpose computer program, VIPASA, for determining the critical buckling stresses or natural frequencies of vibration of thin prismatic structures, consisting of a series of plates rigidly connected together along longitudinal edges. Each plate may be either isotropic or anisotropic and may carry a basic stress system consisting of longitudinal and transverse direct stress combined with shear. The structure is assumed to be subjected to a “dead load” system which does not cause buckling; in addition a “live load” system, defined in magnitude by a single load factor, may be applied and the value of the load factor at buckling is determined. Alternatively the natural frequencies of vibration of the structure when subjected to the dead load system are determined. Any number of critical load factors or

natural frequencies can be obtained. The theory is based upon the assumption that all modes are sinusoidal, in the sense that all three components of displacement vary sinusoidally along any longitudinal line, but phase differences are incorporated to allow for the effects of anisotropy and shear. Apart from this assumption no further approximations are made other than those inherent in thin plate theory.

Williams, F.W., Wittrick, W.H. and Plank, R.J., "Critical buckling loads of some prismatic plate assemblies", Proc. IUTAM Symposium on Buckling of Structures, 1974, edited by B. Budiansky, Springer-Verlag, pp. 17-26, (1976)

Plank, R.J. and Williams, F. W., "Critical buckling of some stiffened panels in compression, shear and bending", Aeronautical Quarterly, Vol. XXV, pt. 3, August 1974, pp. 165-179

M.S. Anderson (1), F.W. Williams (2) and C.J. Wright (3)

(1) Structural Concepts Branch, NASA Langley Research Center, Hampton, VA 23665, U.S.A.

(2) Department of Civil Engineering and Building Technology, University of Wales Institute of Science and Technology, Cardiff CF1 3EU, Wales UK

(3) British Aerospace, Bristol BS99 7AR, England

"Buckling and vibration of any prismatic assembly of shear and compression loaded anisotropic plates with an arbitrary supporting structure", International Journal of Mechanical Sciences, Vol. 25, No. 8, 1983, pp. 585-596, doi:10.1016/0020-7403(83)90050-4

ABSTRACT: The VIPASA computer program accurately treats buckling and vibration of prismatic plate assemblies with a response that varies sinusoidally in the longitudinal direction. In-plane shear loading of component plates produces skewed mode shapes that do not conform to desired support conditions, and this has placed a limitation on the general applicability of VIPASA. This problem is overcome in the present paper by coupling the VIPASA stiffness matrices for different wavelength responses through the method of Lagrangian Multipliers. Supports at arbitrary locations, including support provided by any elastic structure, are included in the theory. Examples illustrate the accuracy and convergence of the method and some of the principal features of the solution. The complete generality and capability of VIPASA have been retained in a computer program VICON that permits constraints and a supporting structure consisting of any number of transverse beam-columns.

F.W. Williams (1) and M.S. Anderson (2)

(1) Department of Civil Engineering, UWIST, Cardiff CF1 3EU, U.K.

(2) Structural Dynamics Branch, NASA Langley Research Center, Hampton, VA 23665, U.S.A.

"Incorporation of Lagrangian Multipliers into an algorithm for finding exact natural frequencies or critical buckling loads", International Journal of Mechanical Sciences, Vol. 25, No. 8, 1983, pp. 579-584, doi:10.1016/0020-7403(83)90049-8

ABSTRACT: An existing algorithm enables natural frequencies or critical load factors to be found with certainty when "exact" stiffness matrices are used. This algorithm is extended to permit Lagrangian Multipliers to be used to couple the "exact" stiffness matrices of component structures to represent connections between the structures. The new algorithm also permits coupling of the stiffness matrices for different assumed wavelengths of sinusoidal response of a given structure with the stiffness matrices of other structures to satisfy required constraint conditions. The algorithm applies to problems formulated using real or complex arithmetic.

F. W. Williams and J. R. Banerjee (Department of Civil Engineering, University of Wales Institute of Science and Technology, Cardiff CF1 3 EU, U.K.), "Accurately computed modal densities for panels and cylinders, including corrugations and stiffeners", Journal of Sound and Vibration, Vol. 93, No. 4, April 1984, pp. 481-488, doi:10.1016/0022-460X(84)90417-6

ABSTRACT: The new computer program VISCAN enables exact modal densities to be computed very economically for any prismatic assembly of isotropic or anisotropic flat plates which are simply supported at their ends and are rigidly connected together along their longitudinal edges, so long as bending and in-plane displacements are uncoupled for the anisotropic plates. A description of how VISCAN was developed from the well established program VIPASA is followed by results for flat, corrugated and stiffened panels and for a cylindrical shell, a corrugated cylinder and a stiffened cylinder. Comparisons are made with existing

experimental results for all these structures except the stiffened panel. The method used to change VIPASA into VISCAN could be applied to other existing computer programs to enable exact modal densities to be found for many additional types of structures.

Williams, F. W., and Kennedy, D., Users Guide to VICON, VIPASA with Constraints, Dept of Civil Engineering and Building Technology, Univ. of Wales, Inst. of Science and Technology, Aug. 1984.

F.W. Williams (1) and M.S. Anderson (2)

(1) Department of Civil Engineering, UWIST, Cardiff CF1 3EU, U.K.

(2) Structural Dynamics Branch, NASA Langley Research Center, Hampton, VA 23665, U.S.A.

“Inclusion of elastically connected members in exact buckling and frequency calculations”, *Computers & Structures*, Vol. 22, No. 3, 1986, pp. 395-397, doi:10.1016/0045-7949(86)90043-X

ABSTRACT: A standard stiffness matrix procedure which permits any combination of rigid, elastic, pinned or sliding connections of the degrees of freedom at the ends of a member to the nodes of its parent structure is described, in order to show how easily it can be extended to allow an existing algorithm to be used to ensure that no eigenvalues of the parent structure can be missed even when “exact” member theory is used. The eigenvalues are the natural frequencies of undamped free vibration analyses or the critical load factors of buckling problems. The method preserves the exactness of the member theory and an efficient method for computer application is indicated. The theory also permits any combination of rigid, elastic, pinned or sliding connections between the freedoms of a substructure and those of its parent structure.

Butler, R. and Williams, F. W., "Optimum design features of VICONOPT, an exact buckling program for prismatic assemblies of anisotropic plates," AIAA Paper 90-1068-CP, Proceedings 31st AIAA/ASME Structures, Structural Dynamics, and Materials (SDM) Meeting, pp 1289-1299, 1990.

Williams, F. W., Kennedy, D., Anderson, M.S., "Analysis features of VICONOPT, an exact buckling and vibration program for prismatic assemblies of anisotropic plates," AIAA Paper 90-0970-CP, Proceedings 31st AIAA/ASME Structures, Structural Dynamics, and Materials Meeting, pp 920-929, 1990.

Williams FW, Kennedy D, Butler R and Anderson MS. VICONOPT: program for exact vibration and buckling analysis or design of prismatic plate assemblies. *AIAA Journal*, Vol. 29, No. 11, pp 1927- 1928, 1991.

Butler, R., and Williams, F. W., “Optimum Design Using VICONOPT, a Buckling and Strength Constraint Program for Prismatic Assemblies of Anisotropic Plates,” *Computers and Structures*, Vol. 43, No. 4, 1992, pp. 699-708.

Williams, F. W., and Jianqiao, Y., “Optimum Mass Design of Prismatic Assemblies of Plates with Longitudinal Voids,” *Computers and Structures*, Vol. 44, No. 3, 1992, pp. 557-565.

F.W. Williams (University of Wales College of Cardiff, Cardiff CF2 1YF, U.K.), “Review of exact buckling and frequency calculations with optional multi-level substructuring”, *Computers & Structures*, Vol. 48, No. 3, August 1993, pp. 547-552, doi:10.1016/0045-7949(93)90334-A

ABSTRACT: This review covers the many applications of the Wittrick-Williams algorithm, which ensures that no critical buckling loads, or natural frequencies of undamped free vibration, are missed even when using the ‘exact’ member equations obtained by solving the appropriate differential equations. The review includes: plane and space frames; prismatic assemblies of isotropic or anisotropic plates, including in-plane plate shear loads; exact multi-level substructuring; design; damping; efficient solution of rotationally or linearly repetitive structures; use of Lagrangian multipliers; programmable pocket calculator methods; program listings for small computers and; references to large computer programs.

C.B. York (1), F.W. Williams (1), D. Kennedy (1) and R. Butler (2)

(1) School of Engineering, University of Wales College of Cardiff, Newport Road, Cardiff, CF2 1YF, U.K.

(2) School of Mechanical Engineering, University of Bath, Claverton Down, Bath, BA2 7AY, U.K.

“A parametric study of optimum designs for benchmark stiffened wing panels”, *Composites Engineering*, Vol. 3, Nos. 7-8, 1993, pp. 619-632, Special Issue: Use of Composites in Aircraft
doi:10.1016/0961-9526(93)90086-Y

ABSTRACT: Results are presented for the most heavily and lightly loaded of eight benchmark stiffened laminated wing panels defined from a Dornier wing by a GARTEUR (Group for Aeronautical Research and Technology in Europe) working party. These benchmark panels had three identical and equally spaced blade stiffeners. The results were chosen to help designers to understand many important aspects of the choice of design variables, and of the effects of changing the sophistication of modelling and theory used, for a wide range of wing panels. The percentage changes of (global) optimum mass are presented, along with the final values of the design variables. Some examples of mass histories and of (rejected) local optimum masses are also given. The principal design variables are skin and blade ply thicknesses and blade height. Additional factors considered include the effects of adding flanges to the blades whose plies either matched those of the blades or were allowed to vary independently, varying the number of stiffeners, allowing the stiffeners to differ from each other, varying stiffener spacing, varying some ply angles, including the stiffening effect of adjacent spars, including the effects of continuity with laterally adjacent panels, including through thickness shear deformation in the panel analysis and analysing the panel with its true skewed shape rather than approximating it as rectangular in plan.

C.B. York and F.W. Williams (Division of Structural Engineering, School of Engineering, University of Wales College of Cardiff, Cardiff, CF2 1YF, UK), “Theory and buckling results for infinitely wide, stiffened skew plate assemblies”, *Composite Structures*, Vol. 28, No. 2, 1994, pp. 189-200,
doi:10.1016/0263-8223(94)90048-5

ABSTRACT: Existing theory and the associated computer program VICONOPT deal with infinitely wide plate assemblies given that boundary conditions on all sides of each panel form a rectangle. They also deal with cases when the four supports form a parallelogram so that the plate is a skew plate. This is true provided the panel is of finite width, i.e. isolated from any adjacent panels, which is the case commonly modelled in practice. It does not represent what happens in the real structure, however, where normally there is continuity with the adjacent panel. The present paper shows how the theory and the computer program VICONOPT can be modified so that skewed plate assemblies that are infinitely wide and repeat at transverse intervals can now be modelled exactly. The paper also shows that the theory can be used, if a small measure of approximation is accepted, to model this situation by analysing only one of the identical stiffeners with associated panel skin in the common situations where the panel has equally spaced, identical, longitudinal stiffeners between each adjacent pair of longitudinal lines of support. Illustrative results are given.

C.B. York and F.W. Williams (Division of Structural Engineering, School of Engineering, University of Wales College of Cardiff, Cardiff, CF2 1YF, UK), “Buckling analysis of skew plate assemblies: Classical plate theory results incorporating Lagrangian multipliers”, *Computers & Structures*, Vol. 56, No. 4, August 1995, pp. 625-635, doi:10.1016/0045-7949(94)00568-N

ABSTRACT: A procedure is presented for the buckling analysis of prismatic skew plate assemblies subject to invariant in-plane stresses. Based on the exact solution of the plate differential equations, the method of Lagrangian multipliers is used to enforce the transverse skew boundaries by a sufficient number of point constraints. Analysis assumes that the plate is infinitely long and that supports repeat at bay length intervals, typifying the continuity found in aircraft wing construction. Following a brief derivation of the formulation adopted, results are presented and comparisons are made with other analyses for an unstiffened isotropic skew plate, subject to pure compression loading with both simply supported and clamped boundary conditions. Results for four benchmark stiffened panels, i.e. plate assemblies, incorporating composite material and combined loading are also given for a range of skew angles.

Bennett, P. N. and Williams, F. W. Wave propagation along prismatic plate structures with masses attached by springs. In *Proceedings of the 6th International Conference on Recent Advances in Structural Dynamics*, Southampton, 1997, vol. 3, pp. 1-16 (Univ. Southampton).

G. Sun (1) and F. W. Williams (2)

(1) School of Civil Engineering and Mechanics, Shanghai Jiao Tong University, Shanghai, China

(2) Division of Structural Engineering, Cardiff School of Engineering, University of Wales, Cardiff, PO Box 686, Cardiff CF2 3TB, Wales, U.K.

“An initial post-buckling analysis for prismatic plate assemblies under axial compression”, *International Journal of Solids and Structures*, Vol. 34, No. 28, October 1997, pp. 3705-3725, doi:10.1016/S0020-7683(96)00204-1
ABSTRACT: This paper provides a Koiter-type initial post-buckling analysis for prismatic plate assemblies made of isotropic materials. The structures are assumed to consist of a series of long flat strips rigidly connected together at their edges, subjected to longitudinal in-plane compressive stress. The transcendental eigenvalue problems, which arise when exact solutions to the member equations are used to form the stiffness matrix of the plate assemblies, are first solved to obtain the buckling load and corresponding buckling mode of the structure. The analysis then obtains exact solutions to the post-buckling member equations and the a-coefficient and b-coefficient which characterize the initial post-buckling behavior. The post-buckling characteristics of the stiffened plate are found to be influenced significantly by the height of the stiffener.

S.M. Powell, D. Kennedy and F.W. Williams (Cardiff School of Engineering, University of Wales Cardiff, P.O. Box 917, The Parade, Cardiff CF2 1XH, U.K.), “Efficient multi-level substructuring with constraints for buckling and vibration analysis of prismatic plate assemblies”, *International Journal of Mechanical Sciences*, Vol. 39, No. 7, July 1997, pp. 795-805, doi:10.1016/S0020-7403(96)00090-2

ABSTRACT: The use of substructuring in the buckling and vibration analysis of large structures permits very substantial improvements in computational efficiency. The exact multi-level substructuring capability of the widely used computer program VICONOPT for the analysis and optimum design of prismatic plate assemblies has been extended by the inclusion of new theory, presented in this paper, which permits constraints on any of the internal or external nodes of substructures. The computational savings by using substructuring in this way are shown to be typically 50–70% compared with previous VICONOPT solutions. The theory is applicable to any method or computer code for structures whose buckling or vibration modes combine responses of different half-wavelengths, with VICONOPT being used as an example.

C. B. York (1) and F. W. Williams (2)

(1) Department of Civil and Environmental Engineering, The University of Edinburgh, Crew Building, The King's Buildings, Edinburgh, EH9 3JN, UK

(2) Division of Structural Engineering, Cardiff School of Engineering, Queens Buildings, University of Wales Cardiff, PO Box 917, Cardiff, CF2 1XH, UK

“Aircraft wing panel buckling analysis: efficiency by approximations”, *Computers & Structures*, Vol. 68, No. 6, September 1998, pp. 665-676, doi:10.1016/S0045-7949(98)00050-9

ABSTRACT: A comparison of ‘exact’ and approximate methods for the determination of critical buckling loads of prismatic benchmark metal and composite panels is presented. The panels are stiffened by either J-, blade- or hat-stiffeners and are representative of typical aircraft wing panel configurations, with in-plane shear and compression load combinations. Buckling design curves and modes are illustrated, and associated CPU times are given to demonstrate the accuracy and efficiency of the approximations adopted. Initial results for the benchmarks, which are rectangular in plan-form, are compared with rigorous finite element solutions. Thereafter, attention is focused on results for the same panels but with parallelogram plan-form. Two analysis methods based on Classical Plate Theory are used as follows: an existing, ‘exact’ method, incorporating Lagrangian multipliers to constrain the transverse (or skew) boundary conditions; and a recently developed approximate infinite width technique, based on the previous one but analysing only a repeating portion of the plate assembly.

R. Mao (1) and F. W. Williams (2)

(1) Department of Engineering Mechanics, Shanghai Jiao Tong University, Shanghai 200030, People's Republic of China

(2) Division of Structural Engineering, School of Engineering, The University of Wales Cardiff, Cardiff, CF2 3TB, Wales

“Post-Critical Behaviour Of Orthotropic Circular Cylindrical Shells Under Time Dependent Axial Compression”, *Journal of Sound and Vibration*, Vol. 210, No. 3, February 1998, pp. 307-327, doi:10.1006/jsvi.1997.1151

ABSTRACT: A non-shallow non-linear shell theory is used to analyze the parametric resonance of orthotropic circular cylindrical shells under harmonically varying axial compression. As special cases, post-buckling and non-linear vibration problems are also studied. In the analysis the non-linear terms and the inertias contributed by both normal displacement and circumferential displacement are included. Therefore the final dynamic system includes two equations in v and w . The transverse shear deformation is taken into account by a first order theory. The spatial variables in the governing equations are eliminated by the Galerkin procedure. The final ordinary differential equations are solved by an asymptotic method. Numerical results show the dependence of the post-critical behaviour on the properties of material, geometry and excitation.

Renjie Mao (1) and F. W. Williams (2)

(1) Department of Engineering Mechanics, Shanghai Jiao Tong University, Shanghai 200030, Peoples' Republic of China

(2) Division of Structural Engineering, School of Engineering, The University of Wales Cardiff, Cardiff CF2 3TB, U.K.

“Nonlinear analysis of cross-ply thick cylindrical shells under axial compression”, *International Journal of Solids and Structures*, Vol. 35, No. 17, June 1998, pp. 2151-2171, doi:10.1016/S0020-7683(97)00169-8

ABSTRACT: Buckling, postbuckling, nonlinear vibration and parametric resonance of thick circular cylindrical shells under axial compression are analyzed in this paper. The theory developed is based on a nonlinear and non-shallow thick shell theory, with its final equations involving two unknowns, the circumferential displacement v and the radial displacement w . The shell wall is cross-ply laminated. The plies are specially orthotropic, but the lamination can be unsymmetric. The axial load is assumed to be harmonically time dependent, or constant as a special case. The governing nonlinear partial differential equations are reduced to nonlinear ordinary differential equations in terms of time by the Galerkin procedure. Then, an asymptotic method is used to solve the resulting nonlinear ordinary differential equations. The numerical results for buckling loads are shown to compare very well with those of three-dimensional theories in the literature, even for very thick shells. The effects of lay-up and thickness on postbuckling equilibrium, nonlinear vibration and parametric resonance are demonstrated by examples.

York, C. B., and Williams, F. W., “Aircraft Wing Panel Buckling Analysis: Efficiency by Approximations,” *Computers and Structures*, Vol. 68, 1998, pp. 665- 676.

Edwards, D. A., Williams, F. W., and Kennedy, D., “Cost Optimization of Stiffened Panels using VICONOPT,” *AIAA Journal*, Vol. 36, No. 2, Feb. 1998, pp. 267-272.

Powell SM, Williams FW, Askar A-S and Kennedy D. Local postbuckling analysis for perfect and imperfect longitudinally compressed plates and panels. Proc. of 39th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference, Long Beach, CA, Part I, pp 595-603, 1998.

Kennedy, D., Ong, T. J., O'Leary, O. J., and Williams, F.W. Practical optimisation of aerospace panels. In *Proceedings of the 1st ASMO UK/ISSMO Conference*, Ilkley, 1999, pp. 217-224 (MCB University Press, Bradford).

G. Sun (1), D. Kennedy (2) and F. W. Williams (2)

(1) School of Civil Engineering and Mechanics, Shanghai Jiao Tong University, Shanghai, 200030, People's Republic of China

(2) Division of Structural Engineering, Cardiff School of Engineering, Cardiff University, P.O. Box 686, Cardiff CF2 3TB, Wales, UK

“A post-buckling analysis for isotropic prismatic plate assemblies under axial compression”, *International Journal of Mechanical Sciences*, Vol. 42, No. 9, September 2000, pp. 1783-1803, doi:10.1016/S0020-7403(99)00055-7

ABSTRACT: This paper presents a post-buckling analysis for prismatic plate assemblies made of isotropic materials. The structures are assumed to consist of a series of long flat strips rigidly connected together at their edges, subjected to longitudinal in-plane compressive load. The buckling load and corresponding buckling mode of the structure are first obtained as the results of transcendental eigenvalue problems, which arise when

exact solutions to the member differential equations are used to form the stiffness matrix of the plate assemblies. The other post-buckling field functions are also obtained analytically as exact solutions to the member differential equations. Results for the load end-shortening and load-deflection relationships for long prismatic plate assembly examples are obtained and compared with results obtained by other authors.

O'Leary, O. J., Williams, F. W., and Kennedy, D. Optimum stiffened panel design with fundamental frequency constraint. *Thin-Walled Struct.*, 2001, 39(7), 555-569.

M. Lilloco (1), R. Butler (1), G.W. Hunt (1), A. Watson (2), D. Kennedy (2) and F.W. Williams (2)

(1) University of Bath, Bath, BA2 7AY, UK

(2) Cardiff University, Cardiff, Wales CF24 3TB, UK

“Analysis and testing of a postbuckled stiffened panel”, *AIAA Journal*, Vol. 40, No. 5, 2002, pp. 996-1000, doi: INIST-CNRS, Cote INIST : 214, 35400010064955.0230

ABSTRACT: The suitability of using the efficient, linear elastic design software VICONOPT for the analysis of a stiffened panel with a postbuckling reserve of strength is investigated. A longitudinally compressed panel, which initially buckled in a local skin mode, was analyzed with allowance being made for the effects of an initial overall imperfection. The panel was also analyzed using the nonlinear finite element package ABAQUS. and four laboratory specimens that represent the panel were tested to failure. The similarity of the experimental failure with the VICONOPT and ABAQUS predictions indicates that VICONOPT can give satisfactory analysis results for use in preliminary design.

Zhu Ju-fen (1), Zhang Gang (1), W. P. Howson (2) and F. W. Williams (3)

(1) Department of Engineering Mechanics, Dalian University of Technology, Dalian 116023, China

(2) Cardiff School of Engineering, Cardiff University, P.O. Box 925, Cardiff CF24 0YF, UK

(3) Department of Building and Construction, City University of Hong Kong, Kowloon, Hong Kong

“Reference surface element modelling of composite plate/shell delamination buckling and postbuckling”, *Composite Structures*, Vol. 61, No. 3, August 2003, pp. 255-264, doi:10.1016/S0263-8223(03)00016-3

ABSTRACT: A recently developed reference surface element technique is used to model the behaviour of the buckling and postbuckling of delaminated plates and shells. The technique can be easily incorporated into any finite element analysis programme for which the beam, plate and shell elements etc. satisfy the Reissner-Mindlin assumption. In this paper, the reference surface element formulation of a four-node Co quadrilateral membrane-shear-bending element (ZQUA24) is presented and numerical investigations are performed for composite plates and shells with various delamination shapes. The numerical results show that the present technique is simple, reliable and able to model delamination buckling and postbuckling behaviour of laminated plates or shells. Observations of practical engineering significance are obtained from the study.

Yuan, S., Ye, K., Williams, F. W., and Kennedy, D. Recursive second order convergence method for natural frequencies and modes when using dynamic stiffness matrices. *Int. J. Numer. Methods Eng.*, 2003, 56(12), 1795-1814.

A. Watson, D. Kennedy, F. W. Williams, C. A. Featherston, “Buckling and Vibration of Stiffened Panels or Single Plates with Clamped Ends”, *Advances in Structural Engineering*, Vol. 6, No. 2, May 2003, pp. 135-144, doi: 10.1260/136943303769013228

ABSTRACT: An efficient method for the buckling and vibration analysis of plates or stiffened panels with clamped ends is presented. The method uses Lagrangian multipliers to couple sinusoidal modes with appropriate half-wavelengths of response, thereby enforcing the end conditions at discrete point supports. Clamped ends can usually be modelled accurately using only a few point supports, while arguments from symmetry often enable some of the required end conditions to be satisfied without explicitly applying constraints. In such cases few half-wavelengths are needed to obtain excellent accuracy. Solutions obtained for the simple limiting case of single plates are exact or within 1% of the classical or other reported solutions. Solutions obtained for stiffened panels are in close agreement with those obtained using finite element analysis.

Hexiang, L. (1), Jinhua, H. (1) and Williams, F. W. (2, email: williamsfw@cf.ac.uk)

(1) Department of Engineering Mechanics, Key Laboratory of Structural Analysis for Industrial Equipment, Dalian University of Technology, Dalian 116023, Peoples Republic of China

(2) Department of Building and Construction, City University of Hong Kong, Kowloon, Hong Kong
“Calculation of critical buckling loads for finite length externally constrained thin circular cylinders”,
Communications in Numerical Methods in Engineering, Vol. 21, No. 5, May 2005, pp. 259–267. doi:
10.1002/cnm.746

ABSTRACT: The problem of the buckling of finite length externally constrained thin circular cylinders is solved to yield a generalized eigenvalue problem with constraint conditions. The solution is based on an inverse iteration procedure in which the non-linear complementary equation is solved by a non-smooth version of Newton's method that saves computer time and space when compared to a linear complementary algorithm. The numerical results show that the method proposed is effective and that the critical buckling load of the constrained cylinder is almost independent of the length of the cylinder, unlike the critical load of free (unconstrained) thin cylinders. In the direction of its axis, the shell is discretized by finite strips which circumferentially use straight bar and beam displacement functions. It is proved that when the widths of the strips approach zero, the geometrical relationships used in this paper approach those of Koiter–Sanders cylindrical shell theory.

Kennedy, D., O'Leary, O. J., and Williams, F. W. Optimum design of prismatic plate assemblies with spectral gap constraints. In Proceedings of the 5th International Symposium on Vibrations of Continuous Systems, Berchtesgaden, 2005, pp. 36-38 (Virginia Polytechnic Institute and State University, Blacksburg, VA).