Dr. Todd O. Williams

Selected Publications:

Todd O. Williams and Frank L. Addessio (Los Alamos National Laboratory, Theoretical Division, Fluid Dynamics T-3, MSB216, Los Alamos, NM 87545, U.S.A.), “A general theory for laminated plates with delaminations”, International Journal of Solids and Structures, Vol. 34, No. 16, June 1997, pp. 2003-2024, ABSTRACT: An approximate analytical model for the behavior of a laminated composite plate in the presence of delaminations and other local effects is presented. The model is based on a generalized displacement formulation implemented at the layer level. The governing equations for a layer are obtained using the principle of virtual work. These governing equations for a layer are used in conjunction with the explicit satisfaction of both the interfacial traction continuity and the interfacial displacement jump conditions between layers to develop the governing equations for a laminated composite plate, including delaminations. The fundamental unknowns in the theory are the displacements in the layers and the interfacial tractions. The theory is sufficiently general that any constitutive model for the interfacial fracture (i.e. delamination) as well as for the layer behavior can be incorporated in a consistent fashion into the theory. The interfacial displacement jumps are expressed in an internally consistent fashion in terms of the fundamental unknown interfacial tractions. The current theory imposes no restrictions on the size, location, distribution, or direction of growth of the delaminations. Therefore, the theory can predict the initiation and growth of delaminations at any location as well as interactive effects between delaminations at different locations within the laminate. Pagano's exact solution for the cylindrical bending of a laminated plate has been modified to include the effects of delamination. An interface model, which expressed the displacement jump as a linear function of the surface tractions, was implemented into this modification of the exact solution. This extension was used to validate the approximate plate theory. The correlation between the approximate approach and the exact solution is seen to be excellent. The approximate plate theory is seen to give very accurate predictions for the interfacial tractions in a direct and consistent fashion, i.e. without the need to use integration of the pointwise equilibrium equations. This allows the interfacial displacement jumps in the presence of delaminations to be modeled accurately. It is seen that these displacement jumps have a significant effect on both the macroscopic and microscopic behavior of a laminated plate.

Todd O. Williams and Frank L. Addessio (Theoretical Division, Los Alamos National Laboratory, Los Alamos, NM, U.S.A.), “A dynamic model for laminated plates with delaminations”, International Journal of Solids and Structures, Vol. 35, Nos. 1-2, January 1998, pp. 83-106, doi:10.1016/S0020-7683(97)00055-3 ABSTRACT: A generalized theory for laminated plates, including delamination, is developed. The laminate model is based on a generalized displacement formulation implemented at the layer level. The equations of motion for a layer, which are explicitly coupled with both the interfacial traction continuity and the interfacial displacement jump conditions between layers, are used to develop the governing equations for a laminated composite plate. The delamination behavior can be modeled using any general constitutive fracture law. The interfacial displacement jumps are expressed in an internally consistent fashion in terms of the fundamental unknown interfacial tractions. The current theory imposes no restrictions on the size, location, distribution, or direction of growth of the delaminations. Therefore, the theory can predict the initiation and growth of delaminations at any location as well as interactive effects between delaminations at different locations within the laminate. The proposed theory is used to consider the dynamic response of laminated plates in cylindrical bending. First it is shown that the dynamic implementation agrees well with the exact predictions of a plate

under static loading conditions. Static, cylindrical bending is considered to validate the numerical implementation. Next, different dynamic loading cases are considered. First, the required level of discretization through the thickness of the laminate necessary to accurately capture the wave propagation characteristics for monotonic tensile loading transverse to the plate is determined. Next, the influence of the delamination on the free vibration behavior of a plate is considered. It is shown that the presence of delaminations can result in significant deviations from the perfectly bonded free vibration behavior. Finally, the plate is subjected to dynamic loading conditions that demonstrate the influence of internal wave interactions on the overall behavior of the plate.

Todd O. Williams (Theoretical Division, T-3, Los Alamos National Laboratory, Los Alamos, NM 87545, USA), "A generalized multilength scale nonlinear composite plate theory with delamination", International Journal of Solids and Structures, Vol. 36, No. 20, July 1999, pp. 3015-3050,
doi:10.1016/S0020-7683(98)00138-3
ABSTRACT: A new type of plate theory for the nonlinear analysis of laminated plates in the presence of delaminations and other history-dependent effects is presented. The formulation is based on a generalized two length scale displacement field obtained from a superposition of global and local displacement effects. The functional forms of global and local displacement fields are arbitrary. The theoretical framework introduces a unique coupling between the length scales and represents a novel two length scale or local-global approach to plate analysis. Appropriate specialization of the displacement field can be used to reduce the theory to any currently available, variationally derived, displacement based (discrete layer, smeared, or zig-zag) plate theory. The theory incorporates delamination and/or nonlinear elastic or inelastic interfacial behavior in a unified fashion through the use of interfacial constitutive (cohesive) relations. Arbitrary interfacial constitutive relations can be incorporated into the theory without the need for reformulation of the governing equations. The theory is sufficiently general that any material constitutive model can be implemented within the theoretical framework. The theory accounts for geometric nonlinearities to allow for the analysis of buckling behavior. The theory represents a comprehensive framework for developing any order and type of displacement based plate theory in the presence of delamination, buckling, and/or nonlinear material behavior as well as the interactions between these effects. The linear form of the theory is validated by comparison with exact solutions for the behavior of perfectly bonded and delaminated laminates in cylindrical bending. The theory shows excellent correlation with the exact solutions for both the inplane and out-of-plane effects and the displacement jumps due to delamination. The theory can accurately predict the through-the-thickness distributions of the transverse stresses without the need to integrate the pointwise equilibrium equations. The use of a low order of the general theory, i.e. use of both global and local displacement fields, reduces the computational expense compared to a purely discrete layer approach to the analysis of laminated plates without loss of accuracy. The increased efficiency, compared to a solely discrete layer theory, is due to the coupling introduced in the theory between the global and local displacement fields.

R. Gilat (1), T. O. Williams (2) and J. Aboudi (3)
(1) Faculty of Engineering, Academic College JAS, Ariel 44837, Israel
(2) Theoretical Division, T-3, Los Alamos National Laboratory, Los Alamos, NM 87545, USA
(3) Department of Solid Mechanics, Materials and Structures, Faculty of Engineering, Tel Aviv University, Ramat Aviv 69978, Israel
"Buckling of composite plates by global–local plate theory", Composites Part B: Engineering, Vol. 32, No. 3, April 2001, pp. 229-236, doi:10.1016/S1359-8368(00)00059-7
ABSTRACT: The bifurcation buckling problem of laminated composite plates is formulated within the framework of a multilength scale plate theory. This theory is a combination of single-layer and layer-wise

theories. It is generated by representing the displacement as the sum of global and local effects that introduce a coupling between the two length scales. Comparisons between the presently predicted buckling loads of homogeneous and orthotropic laminated plates and the exact solutions show a very good correlation. Furthermore, the theory accurately predicts the buckling load of symmetric cross-ply plates as compared with the results of a layer-wise approach. This accuracy is achieved with reduced computation expense. The global–local plate theory is general enough to incorporate delamination effects. As a result of the inclusion of these effects, the buckling loads of plates with imperfect interlaminar bonding are predicted.

Todd O. Williams (Theoretical Division, T-3, Los Alamos National Laboratory, MS B216, Los Alamos, NM 87545, USA), "A generalized, multilength scale framework for thermo-diffusional-mechanically coupled, nonlinear, laminated plate theories with delaminations", International Journal of Solids and Structures, Vol. 42, Nos. 5-6, March 2005, pp. 1465-1490, doi:10.1016/j.ijsolstr.2004.08.007
ABSTRACT: A new type of plate theory based on a general, unified, theoretical framework for the response of (von Karman) nonlinear, delaminated plate theories in the presence of thermo-diffusional-mechanical coupling is presented. The theory is based on the unique use of two length scale expansions obtained from a superposition of global and local effects for the displacement, temperature, and solute concentration fields. The orders and forms of these local and global displacement, temperature, and solute fields are arbitrary. The theory incorporates delamination and/or nonlinear elastic or inelastic interfacial behavior for the mechanical, thermal, and concentration effects in a unified fashion through the use of generalized interfacial constitutive (cohesive) relations. The mathematical framework introduces new types of coupling effects between the different length scale effects of all three fields. The resulting unified theoretical framework can be used to consider the general thermo-diffusionally-mechanically coupled response of laminated (or homogeneous) plates in the presence of delaminations, buckling, and/or nonlinear material behavior. The author is unaware of any previous attempts to develop plate theory formulations capable of considering the multitude of effects incorporated into the proposed framework. It is shown that existing displacement-based plate theories for both the mechanical as well as thermo-mechanical behavior of laminated plates can be obtained via suitable specializations of the proposed framework. New types of plate theories can be obtained through various specializations of the proposed general theory.

Todd O. Williams (Theoretical Division, T-3, Los Alamos National Laboratory, MS B216, Los Alamos, NM 87545, USA), "A new theoretical framework for the formulation of general, nonlinear, multiscale plate theories", International Journal of Solids and Structures, Vol. 45, No. 9, May 2008, pp. 2534-2560, doi:10.1016/j.ijsolstr.2007.12.006
ABSTRACT: A new type of general, theoretical framework for the development of comprehensive, nonlinear, multiscale plate theories for laminated structures is presented. The theoretical framework utilizes a generalized two scale description of the displacement field based on a superposition of global and local effects where the functional forms for the global and local displacement fields are arbitrary. The two scale nature of theory allows it to explicitly consider the layered nature of the structure. The subsequent development of the governing equations for the theory is carried out using the general nonlinear equations of continuum mechanics referenced to the initial configuration. The equations of motion and the lateral surface boundary conditions for the theory are derived using the method of moments over the different scales subject to an orthogonality constraint. The theory satisfies the interfacial constraints and the top and bottom surface boundary conditions in a strong sense. Delamination effects are incorporated into the theory through the use of cohesive zone models (CZMs). Arbitrary CZMs can be incorporated into the theory without the need for reformulation of the governing equations. The theory is formulated in a sufficiently general fashion that any type of history-dependent material can be used to describe the inelastic response of the materials composing the layers. Furthermore, as a result of

the multiscale nature of the theory it can be specialized to single scale theories of the equivalent single layer (ESL) or discrete layer (DL) types in a unified fashion and without the need for any reformulation. While the starting point for the proposed theory is the same as used by Williams [Williams, T.O., 1999. A generalized multilength scale nonlinear composite plate theory with delamination. Int. J. Solid Struct. 36, (20) 3015–3050; Williams, T.O., 2001. Efficiency and accuracy considerations in a unified plate theory with delaminations. Comp. Struct. 52, (1) 27–40; Williams, T.O., 2005. A generalized, multilength scale framework for thermo-diffusionally-mechanically coupled, nonlinear, laminated plate theories with delaminations. IJSS 42, (5–6) 1465–1490] the subsequent formulation is significantly different. The differences in the two theories allow the currently proposed theory to improve on the capabilities of the previous theory; particularly in the satisfaction of the traction continuity constraints at the interfaces. It is shown that the theory is capable of providing accurate predictions for all of the fields in perfectly bonded and delaminated plates even for relatively low orders of displacement approximations. In particular, the theory is shown to provide accurate predictions for the transverse stresses that are continuous across the interfaces directly from the constitutive relations.