BOSOR5—PROGRAM FOR BUCKLING OF COMPLEX, BRANCHED SHELLS OF REVOLUTION INCLUDING LARGE DEFLECTIONS, PLASTICITY AND CREEP

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ABSTRACT

BOSOR5 performs axisymmetric collapse and nonsymmetric bifurcation buckling including elastic-plastic material behaviour and creep. It does not supercede BOSOR4, as it has no modal vibration capability or linear nonsymmetric stress analysis capability. It will handle segmented or branched, multmaterial, stiffened shells. The wall may be layered. Smeared stringers and smeared or discrete rings are permitted to go plastic. Only static analysis is performed by BOSOR5. The strategy for solution of the nonlinear prebuckling problem is such that the user obtains reasonably accurate answers even if very large load or time steps are used. This strategy is based on a subincremental iteration method in which the size of the subincrement is automatically determined so that the change in stress is less than a certain prescribed percentage of the effective stress. Discrete rings of arbitrary cross section are considered to be assemblages of thin rectangular elements. The input is interactive as with BOSOR4.

THEORETICAL BACKGROUND AND PROGRAM OVERVIEW

BOSOR5 is based on finite difference energy minimization; trigonometric variation is assumed for the circumferential variable; Newton's method is used to solve nonlinear prestress equilibrium; inverse power iterations with spectral shifts are used for eigenvalue extraction; Lagrange multipliers are used for constraint conditions; BOSOR5 is based on thin shell theory.

BOSOR5 has been widely used since 1974. In 1983 extensive user-friendly features were added to the VAX version to make provision of input data easy and reliable. An interactive input session generates a completely annotated file that can be used for documentation and for input for future similar cases. A MODIFY utility makes updating a case much easier than before. These features are essentially the same as those described in connection with BOSOR4. (See the paper on BOSOR4 in Volume 2 for details.)

The complex segmented and branched shells of revolution may have various
meridional geometry, wall construction, boundary conditions, ring reinforcements, stringer reinforcements, and types of loading. Pressure and surface traction may vary along the meridian; temperature may vary along the meridian and through the thickness. Line loads may be applied at discrete ring centroids. Each load may have its own quasi-static variation in time, so that sequential loadings, such as a thermal cycle followed by an external pressure, may be applied. In this way fabrication effects followed by service loads can be simulated with BOSOR5. Examples of this are given below. All loads must be axisymmetric.

BOSOR5 is currently in widespread use and is maintained by the developer. Notices of bugs found are distributed to all known users. BOSOR5 has been thoroughly checked out by comparisons with other known solutions, tests, and by extensive use at many different institutions the world over for about 10 years. The characteristics and status of BOSOR5 are similar to those of BOSOR4. Therefore, the reader is referred to Table 1 in the paper on BOSOR4 for details on restrictions, language, documentation, I/O, hardware, size, availability, and maintenance of BOSOR5.

FIELD OF APPLICATION

BOSOR5 performs the following analyses:

1. a nonlinear stress analysis for axisymmetric behaviour of axisymmetric shell systems (moderately large deflections, elastic-plastic, creeping material). Axisymmetric collapse is a special case of this type of analysis.

2. an eigenvalue analysis in which the eigenvalues represent buckling loads of axisymmetric shell systems subjected to axisymmetric loads. Eigenvectors correspond to axisymmetric and to nonaxisymmetric buckling modes.

BOSOR5 will handle segmented and branched shells with the same geometries as those handled by BOSOR4. (See Fig. 1 in the paper on BOSOR4.)

ASSUMPTIONS, METHOD, DISCRETIZATION, USER-FRIENDLY FEATURES OF BOSOR5

The governing assumptions on which BOSOR5 is based are the same as those on which BOSOR4 is based, except that the material can creep and can go plastic (with elastic unloading). The plasticity model is von Mises yield and associated flow law with isotropic strain hardening. Deformation theory is used for the in-plane shear modulus, which is needed in the analysis governing nonaxisymmetric bifurcation buckling. The strains are assumed to be small.

The method and discretization scheme are the same as in the case of BOSOR4, except that the presence of nonlinear and irreversible material behaviour necessitates the use of the principle of virtual work rather than the principle of minimum potential energy; there is no kinetic energy involved, since BOSOR5 handles only statics problems; and a double-iteration loop is required during the prebuckling analysis phase because both nonlinear geometric (moderately large deflection) and nonlinear material behaviours are present.

Details of the analysis are presented in references 1-4.

The discretization scheme in BOSOR5 is identical to that in BOSOR4. Figure 2 in the paper on BOSOR4 gives details.

The user-friendly features described in the paper on BOSOR4 also apply to BOSOR5. Please see Tables 3, 4, and 5 in the paper on BOSOR4 for details.
EXAMPLES OF APPLICATION

DETAILED EXAMPLE CASE: The style of input and files generated with use of BOSOR5 are so similar to those generated with use of BOSOR4 that no such example will be repeated here. Please see Tables 4 and 5 of the paper on BOSOR4.

However, BOSOR5 input/output are a bit different from BOSOR4 I/O. With BOSOR5 the user executes the pre-, main-, and postprocessors explicitly. (With BOSOR4 these three processors are all executed by the user's typing the one command BOSORALL.) The explicit execution of each processor in BOSOR5 is better because more computer time is required to solve problems in which both geometric and material nonlinearity are present, and loading history is often important because of path dependence in problems involving plasticity. Therefore, the user generally wants to be able to interact with the analysis more often than is the case with BOSOR4, in which only geometric nonlinearity is present.

With BOSOR5, data that determine the state of the plastically deformed shell at each load step are saved, so that the user may restart the mainprocessor analysis at any load step for which this state has been determined in a previous run.

The nature of nonlinear problems treated with BOSOR5 makes frequent use of the restart feature the rule rather than the exception that the use of this feature tends to be with BOSOR4.

BUCKLING OF INTERNALLY PRESSURIZED SHELLS: Ellipsoidal and torispherical heads with internal pressure can buckle because the material in the knuckle region is drawn in toward the axis of revolution as the internal pressure is increased. This deformation is displayed in Fig. 1. The material in the knuckle region is therefore under a biaxial stress field that is tensile in the meridional direction and compressive in the circumferential direction. The buckling mode consists of wrinkles in the knuckle region, as shown in Fig. 2. BOSOR5 calculates the axisymmetric prebuckled state and the lowest bifurcation pressure. Both geometric and material nonlinearity must be included to solve problems of this type accurately. Extensive comparisons with tests performed at the University of Liverpool, the University of Manchester, and in France are given in references 8, 9 and 10. In reference 10 this interesting problem is described in detail.

BUCKLING OF A WATER TANK: In 1972 in Belgium a large water tank collapsed upon being filled for the first time. The geometry of the tank is shown in Fig. 3(a). Failure appeared to be due to meridional buckles that formed in the conical region represented by Segment 9 in Fig. 3(b). This region was subjected to high meridional compression combined with circumferential tension, just the opposite of the biaxial stress state in the internally pressurized ellipsoidal head shown in Fig. 2. Accordingly, the buckles were long in the circumferential direction and short in the meridional direction, in contrast to the wrinkles displayed in Fig. 2. Figure 3(c) shows the axisymmetric prebuckling deformation of the tank predicted by BOSOR5 at the predicted bifurcation buckling load factor, \( \lambda = 1.8 \) times the load present at the moment of collapse. The prediction is higher than the actual because no allowance is made in the BOSOR5 model for geometric imperfections or welding prestresses. Figure 3(d) shows a redesigned tank and predicted axisymmetric deformations at load factors \( \lambda = 1.0 \) and at the predicted bifurcation buckling load factor, \( \lambda = 2.65 \).

AXISYMMETRIC COLLAPSE OF ROCKET BOOSTER STAGE: Figure 4 shows the rather complex rocket interstage geometry. The basically cylindrical shell is under uniform axial compression. Figure 5 displays the multi-segment model treated with BOSOR5. Plastic collapse with increasing axial compression \( V \) occurs because of the large amount of meridional bending caused by the inward excursion of the axial load path in the region of the joint at Station 176. This rocket interstage was tested and failed at a load within 1% of the critical load predicted with BOSOR5.

MODELING FABRICATION EFFECTS WITH BOSOR5: The BOSOR5 program has been used for calculation of bifurcation buckling of cold-bent and welded ring-stiffened
Fig. 1. Axisymmetric deformations predicted by BOSOR5 for an internally pressurized ellipsoidal pressure vessel head. Elastic-plastic bifurcation buckling is caused by the relatively narrow band of circumferential compression that occurs in the knuckle region where Δr is less than zero. Accurate predictions of bifurcation buckling require inclusion of both nonlinear geometric and material behavior in this case.

cylindrical shells under external pressure. Residual stresses and deformations from cold bending a flat sheet into a cylindrical shell and subsequently welding rings to it can be incorporated into the model of buckling under service loads by introduction of these fabrication processes as functions of a time-like parameter, "time", which insures that the material in the analytical model experiences the proper sequence of loading prior to and during application of the service loads. The cold-bending process is first simulated by a thermal loading cycle in which the temperature varies linearly through the shell wall thickness, initially increasing in "time" to simulate cold bending around a die of radius $R_0$ and then decreasing in "time" to simulate springback to a final somewhat larger design radius $R$. The welding process is subsequently simulated by the assumption that the material in the immediate neighborhoods of the welds is cooled below the ambient temperature by an amount that leads to weld shrinkage amplitudes typical of those observed in tests. After these two fabrication processes have been simulated, the service load (e.g. external pressure) is applied in increasing
Fig. 2. Pressure-deflection curve for internally pressurized ellipsoidal pressure vessel head. BOSOR5 predicts the behaviour up to the bifurcation point. Photograph shows a head in the far postbuckled state. BOSOR5 does not predict postbifurcation behaviour steps until buckling is detected.

Figure 6 shows two nominally identical ring-stiffened cylindrical shells tested under external pressure. The specimen on the left was fabricated by cold bending the shell and then welding machined ring stiffeners to it. That on the right was carefully machined. In the BOSOR5 analysis the machined specimen is predicted to fail at 3.723 MPa (540 psi), precisely in agreement with the test. Simulation of only the cold-bending process leads to a prediction of $P_{cr} = 3.172$ MPa (460 psi). Simulation of both cold bending and welding does not change this result. The predicted radial shrinkage due to welding is maximum at the ring stiffeners and minimum midway between rings, a mode that converts the straight-walled cylinder into a sort of "caterpillar". The welding process apparently has little influence on the predicted buckling pressure in this case because of two counteracting effects: the residual welding stresses weaken the shell but the "caterpillar" type residual deformations strengthen it.

The fabricated specimen failed at a pressure about 15% below the prediction of $P_{cr} = 460$ psi. The discrepancy could be caused by initial geometric imperfections, the presence of the Bauschinger effect, which was not included in the BOSOR5 model, and residual stresses and other nonuniformities present in the sheet from which the cylinder was fabricated.

Figure 7 demonstrates the simulation with BOSOR5 of fabrication processes that precede application of the service load. Figure 8 shows an example from reference 11 in which the residual stresses predicted by BOSOR5 agree with those measured and calculated by Queener and DeAngelis for a cold-formed aluminium cylindrical shell. Figure 9 shows predicted residual deformation patterns in cylindrical
Fig. 3. Large steel water tanks and BOSOR5 models of them: (a) Geometry of original tank that failed in 1972; (b) BOSOR5 segmented and branched model of this tank; (c) BOSOR5 prediction of axisymmetric deformation just before predicted collapse; (d) Redesigned tank and deformations predicted by BOSOR5

shells caused by welding rings to the inner surface or to the outer surface of the cylindrical shell.

WORLDWIDE USE OF BOSOR5

Various versions of BOSOR5 have been in use worldwide since 1974. Table 6 in the paper on BOSOR4 lists many organizations that over the years have used BOSOR5 (and the elastic buckling program BOSOR4). Types of problems solved with the BOSOR programs are indicated.
Fig 4. Complex rocket interstage subjected to axial compression during launch. (Stability problem arises from the axial load path eccentricity at Station MS 176.)
Fig. 5. Discretized, segmented BOSOR5 model and deformed profile of the axially compressed rocket interstage with increasing axial compression $V$. 

$v = 600, 700, 800, 900, 1000, 1100, 1120, 1140, V_{cr} = 1160$ lb/in.
Fig. 6. Observed buckling patterns in externally pressurized, ring-stiffened cylindrical shells: (a) The cold bent and welded specimen on the left buckled at an external pressure $p_{cr} = 390$ psi. (b) The machined specimen buckled at $p_{cr} = 540$ psi.
Fig. 7. Loading functions of "time" for the BOSOR3 analysis of the bent and welded ring-stiffened cylinder: (A) cold bending process, including springback; (B) welding the rings to the cylindrical shell; (C) application of the external hydrostatic pressure.
Fig. 8. Comparison of BOSORS results with test and theory of Queener and deAngelis for residual stress in cold bent 6061 aluminium sheet
Fig. 9. Cylindrical shell with welded internal and external rings:
(a) Dimensions and BOSORS-discretized reference surface;
(b) predicted residual deformations after welding and before
loading of a specimen with internal rings; (c) predicted
residual deformation after welding and before loading of
a specimen with external rings.

REFERENCES BY D. BUSHNELL PERTAINING TO BOSORS

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