Buckles, Creases & Wrinkles

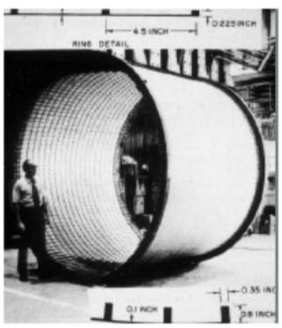
In shell structures & soft materials

John W. Hutchinson

School of Engineering & Applied Sciences, Harvard University

Motived by the space effort and other large structural applications, I worked on the stability of shell structures early in my career in the 1960's. The technical challenge centered on the fact that many technically important shells, such as cylindrical shells under axial compression and externally pressurized spherical shells, buckle in a highly unstable manner and collapse loads are extremely sensitive to small imperfections. Recently, I have been working on instabilities of soft materials such as elastomers and gels. Compression instabilities of soft materials have much in common with shell bucking and this is the story I will outline with the aid of some pictures.

Shell structures: Buckling & catastrophic collapse



Stiffened cylindrical shell for space rocket (NASA)

Unstiffened spherical shell for LNG tanker

Buckled & collapsed cylindrical shell water tower

Buckled huge steel wine barrel in CA failed during earthquake

When an unstiffened cylindrical shell is compressed to the point that it buckles, it undergoes a dramatic dynamic snapping process and collapses into a long wavelength buckling mode as shown in the figure on the right below. Concomitant with the catastrophic collapse is the fact that the shell is highly sensitive to imperfections in its geometry, such as small thickness variations or slight surface undulations. Collapse loads for thin cylinders are typically as low as one quarter the buckling load of the perfect shell. It will be relevant to the connection to soft materials to note that the short wavelength buckling mode shown below on the left is the mode that evolves at the onset of buckling. Usually, this mode is never observed. It is seen here because a cylindrical mandrel inside the shell has arrested the buckles before they have a chance to develop.

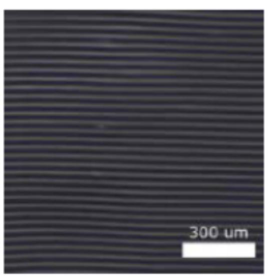
Short wavelength mode of an unstiffened cylindrical shell arrested by internal mandrel

Long wavelength mode of an unstiffened cylindrical shell in the collapse state

Soft materials such as elastomers (compliant rubber-like materials) and gels have become an important area of study with many applications. A soft material usually responds to compression by undergoing surface wrinkling or creasing. When a thin stiff film is attached to a thick substrate of soft material, highly regular wrinkle patterns form when the system is subject to in-plane compression, as illustrated on the left in the figure below. The wavelength of the wrinkles is typically ten to twenty times the film thickness and thus wavelengths in the nano- to micro-meter range can be readily produced. Moreover, the wrinkle patterns can be manipulated by altering the level and direction of compression, and efforts are underway to control and exploit these patterns for various applications.

WRINKLING

Systems with thin stiff film



Sinusoidal wrinkling of stiff thin film on compliant substrate compressed in vertical direction.

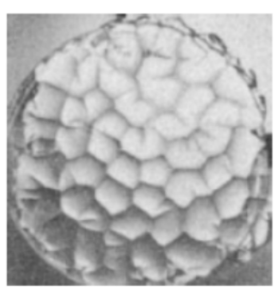
Herringbone wrinkling of stiff gold film on compliant substrate compressed equi-biaxially.

CREASING

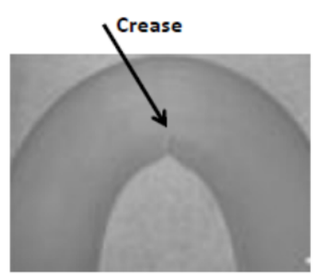
Homogeneous substrates with no films

Creases (sulci) on surface of human brain (compressed by growth nearly equi-biaxially)

Crease on surface of a homogeneous gel (Jello) compressed borizontally between hands.



Creases on surface of a swelling gel constrained within a circular container



Crease on the compressive side of a rubber bent rubber bar

A homogeneous block of elastomer or gel, without a film on its surface, does not wrinkle when it is compressed but, instead, is observed to form surface creases as illustrated by the examples above on the right. Theoretical results of M. A. Biot from 1963 predict that a perfectly flat half-space of neo-Hookean material (the canonical elastomer) should remain flat until a compressive strain of 45% is imposed at which point sinusoidal surface wrinkles should appear. However, for more than two decades, experimentalists (e.g., T. Tanaka, A. N. Gent & their co-workers) observed surface creases as the first departure from flatness at compressive strains about 35% and never observed wrinkling.

The first theoretical understanding of this anomaly emerged from the work of E.B. Hohlfeld and L. Mahadevan in 2008 followed by additional numerical simulations of W. Hong, X. Zhao and Z. Suo a year later. A crease is a surface mode involving finite strain changes which becomes energetically favorable compared to the perfectly flat configuration at compressive strains above 35% (for the neo-Hookean material). The scale of the crease, or the wavelength of Biot's wrinkle, is arbitrarily according to the nonlinear elastic continuum theory. A surface imperfection in the form of an undulation with finite slope is required to trigger the crease, and it seems reasonable to assume such imperfections will always be present at some scale to trigger creases at, or slightly above, the point where they become energetically favored.

Additional insight as to why wrinkles are not observed on the surface of a compressed half-space emerged from the work of Y. Cao and the present author in 2012. By exploring the nonlinear behavior of Biot's wrinkling mode, we showed that wrinkling is both highly unstable and extremely imperfection-sensitive with close parallels to the behavior of the cylindrical shell under axial compression. The mathematics underlying the nonlinear wrinkling behavior is very similar to that W.T. Koiter uncovered for cylindrical shell buckling. Our analysis suggests that a nearly perfectly flat surface would collapse dynamically into a creased state before appreciable wrinkling can be observed, in much the same way that the cylindrical shell snaps into the collapsed state without revealing its short wavelength mode.

The shell pictures have been taken from shellbuckling.com. I am indebted to Z. Suo for some of the wrinkling and creasing pictures which are from various sources.