"IMPERFECTION SENSITIVITY" by David Bushnell, 2011

This frequently used term means "the sensitivity of the **load** at which a shell buckles to imperfections in the shape of the shell".

Most of the pictures in this file are from the 1981 "Pitfalls" lecture: "Buckling of shells – Pitfall for designers", by David Bushnell, AIAA Journal, Vol. 19, No. 9, September, 1981, Presented as AIAA Paper 80-0665R, the SDM Lecture, at the AIAA/ASME/ASCE/AHS 21st Structures, Structural Dynamics and Materials Conference, Seattle, Washington, May12-14, 1980

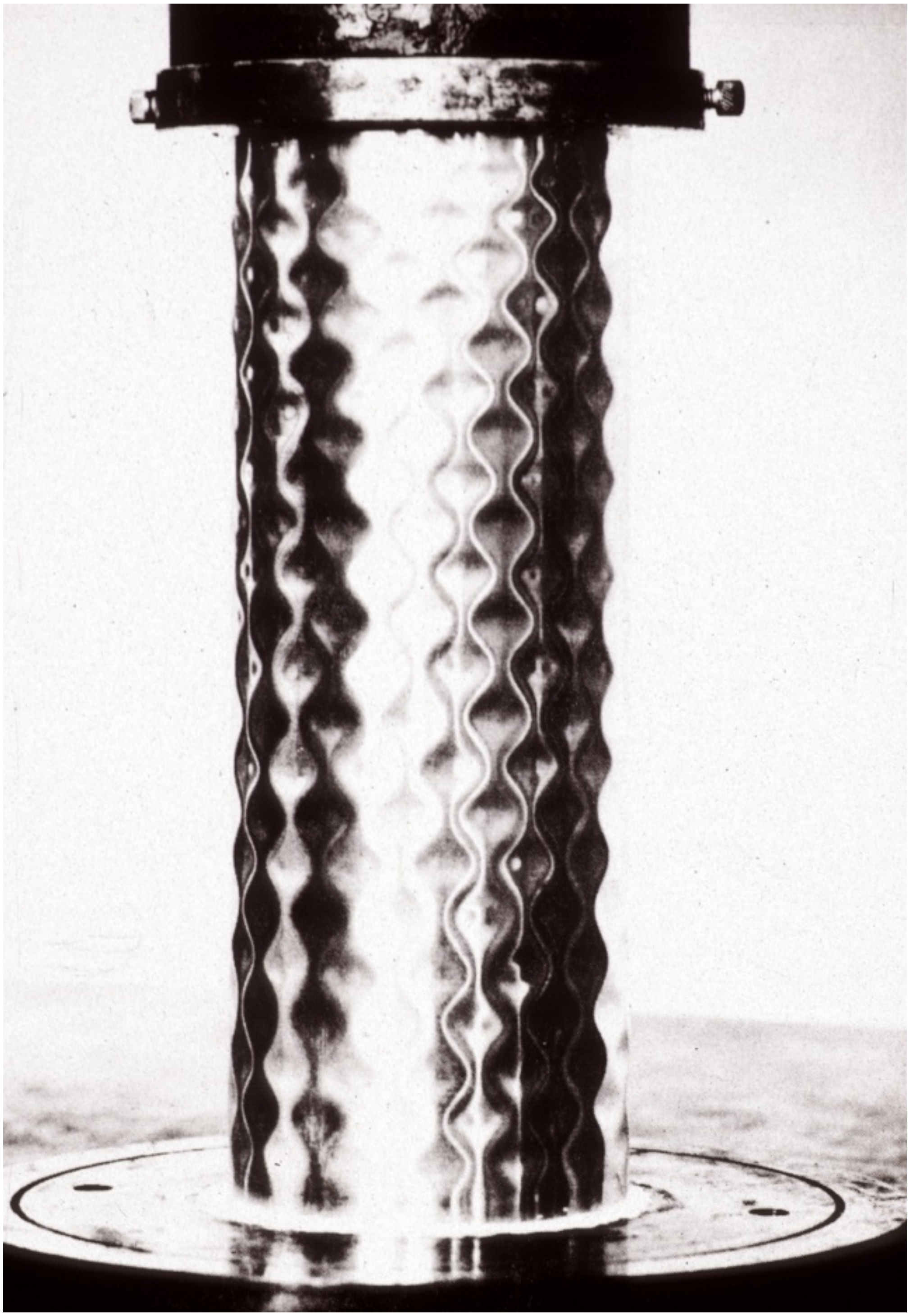
The picture captions contain the string, "Slide". The slide numbers correspond to the slides shown during the 1980 "Pitfalls" lecture. The references cited in the figure captions are listed at the end of this document.

SOME ADVICE:

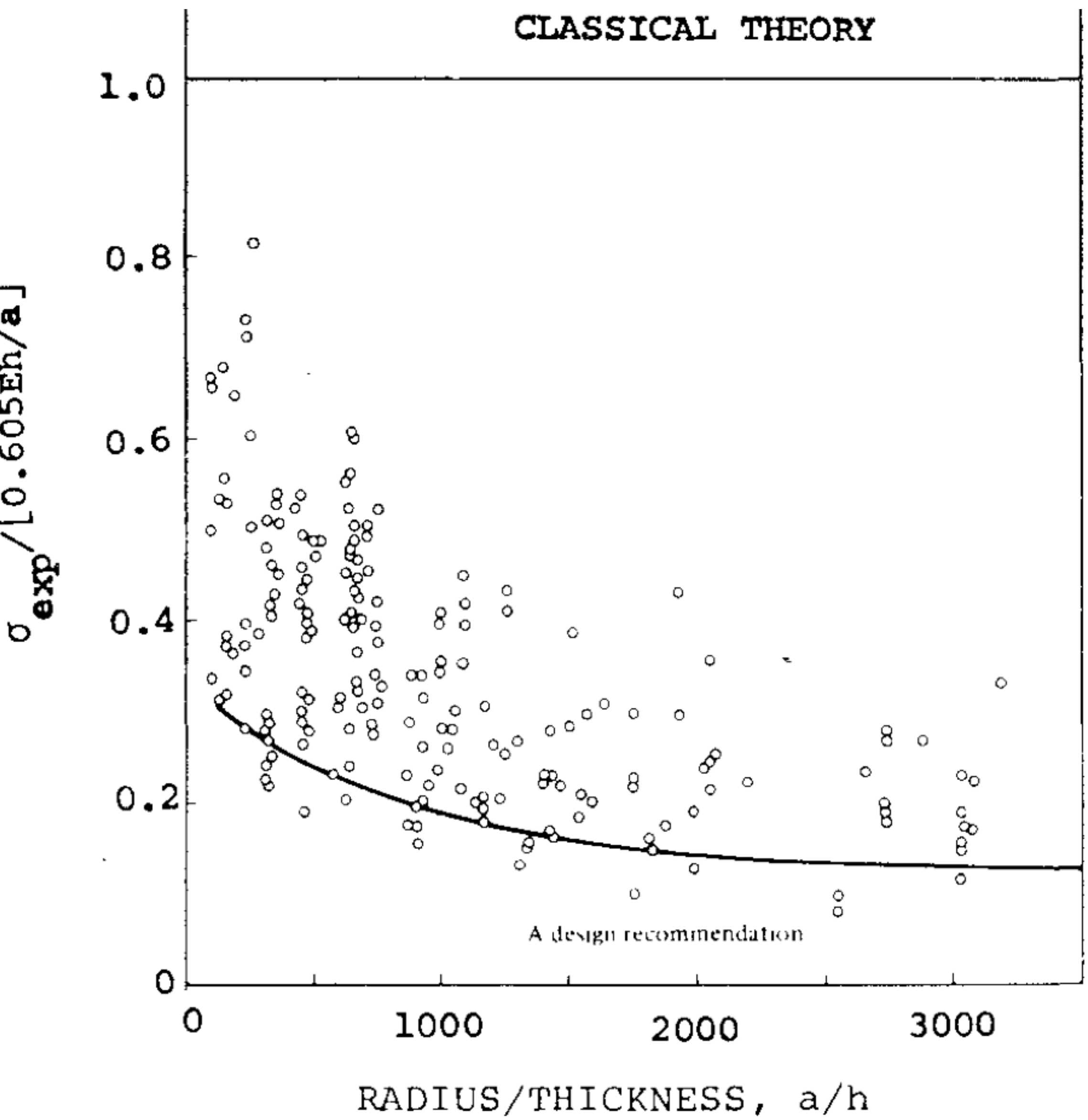
Most of the figures are taken from published papers and reports. Please see the relevant complete documents for more details, especially 1981pitfalls.pdf , the PowerPoint slide show, pitfallsnasa.ppt, and bosor5.papers/1982decade.pdf.

PART 1 Shells that are highly imperfection-sensitive

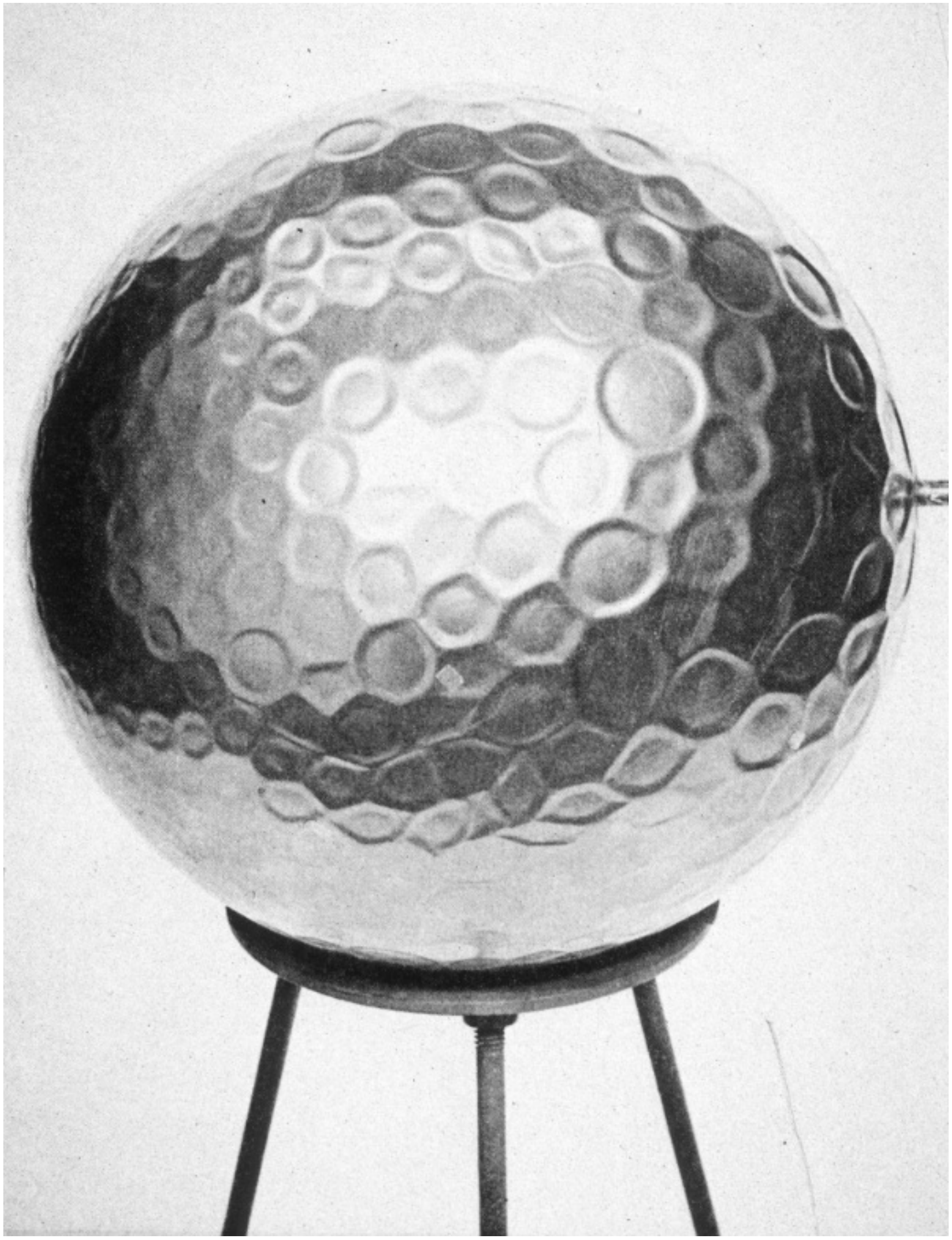
These shells tend to have uniform properties and uniform pre-buckling stress over a large part of their surfaces. They also tend to have buckling modes the characteristic wavelengths of which are very small compared to the size of the shell. Axially compressed thin monocoque cylindrical shells and externally pressurized thin monocoque spherical shells are of this type, as the next figures demonstrate. Any very small dent in the shell wall is likely to initiate local buckling at a load far below the buckling load of the perfect shell. That initial local buckle in the imperfect shell is very likely to propagate over the entire surface of the shell at a load only slightly higher than the initial buckling load of the imperfect shell.



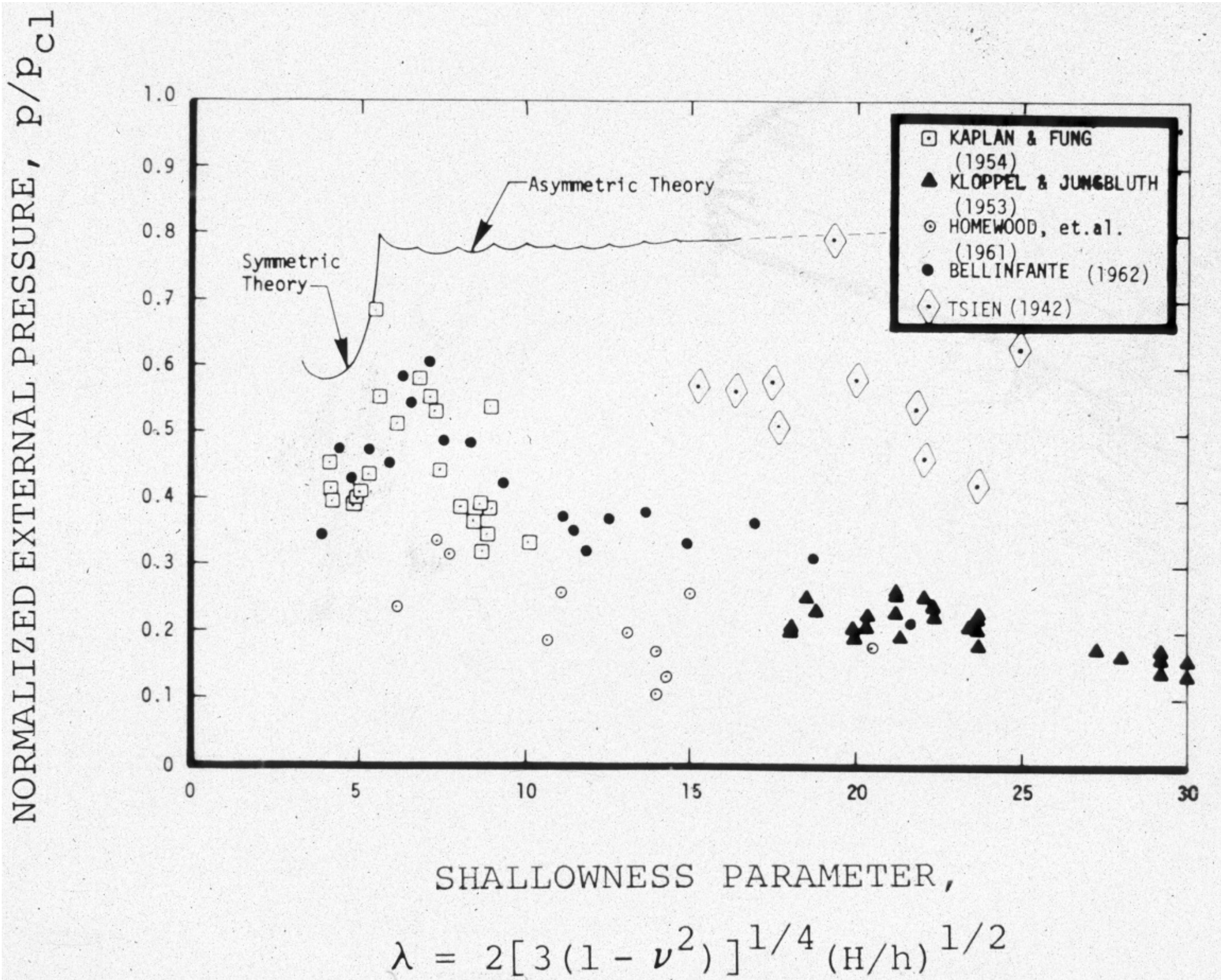
Slide 44 A buckled, uniformly axially compressed monocoque cylindrical shell with a large radius/thickness. In this photograph the post-buckling pattern is “artificially” stabilized because there is a solid mandrel inside the shell. (Photograph by Horton, et al [44]) (Fig. 3.1 in AIAA Journal, Vol. 19, No. 9, 1981)



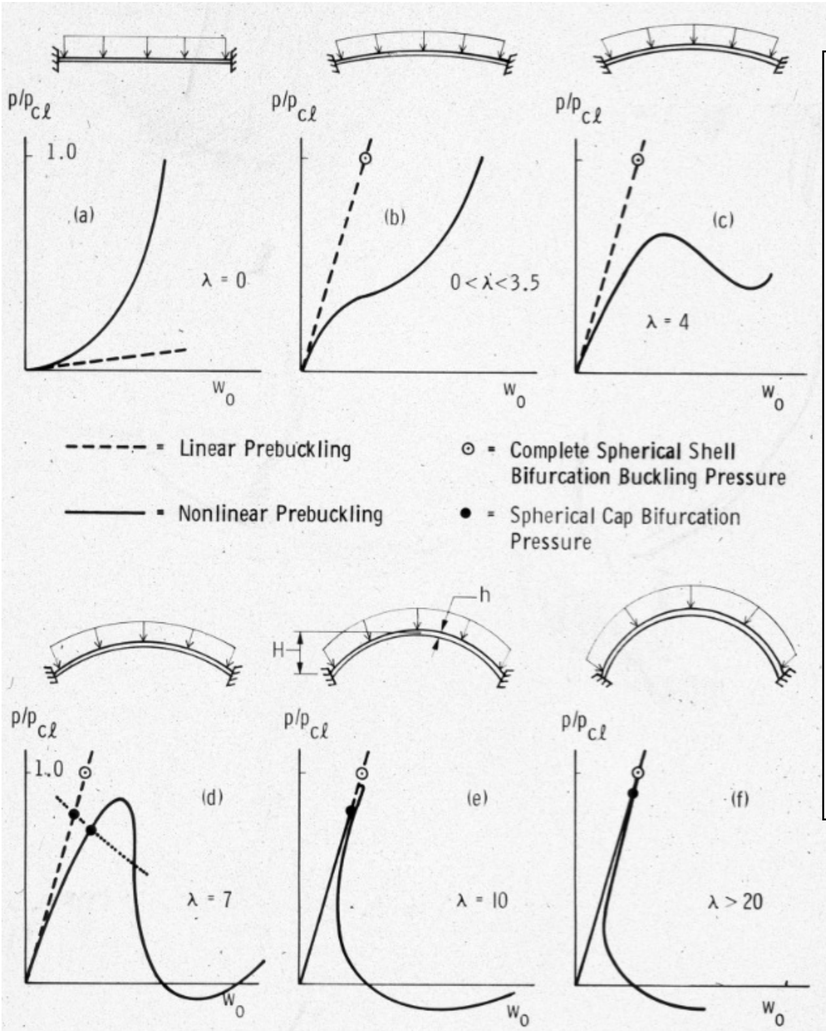
Slide 45 Comparison between test and theory for the buckling of axially compressed cylindrical shells. “a” is the radius; h is the wall thickness; E is the Young’s modulus; 0.605Eh/a is the “classical” buckling pressure of a perfect shell made of isotropic material with Poisson ratio equal to 0.3; $sigma_{exp}$ is the buckling stress from tests. The normalized buckling load of the perfect shell is 1.0. Most of the test points fall far below 1.0, especially for shells with very high radius/thickness ratio, a/h. The solid line corresponds to a design recommendation in which about 95 per cent of the test results fall above the curve. (from the book by Brush and Almroth [11]). (Fig. 3.2 in AIAA Journal, Vol. 19, No. 9, 1981)



Slide 46 Uniformly externally pressurized monocoque spherical shell with a large radius/thickness. As with the buckled axially compressed thin cylindrical shell displayed two slides ago, there are many, many buckles over the entire surface of the shell the characteristic size of which is very small compared to the dimensions of the test specimen. In this photograph the post-buckling pattern is “artificially” stabilized because there is a solid mandrel inside the shell. (Photograph by Carlson, et al [45]). (Fig. 3.9 in AIAA Journal, Vol. 19, No. 9, 1981)

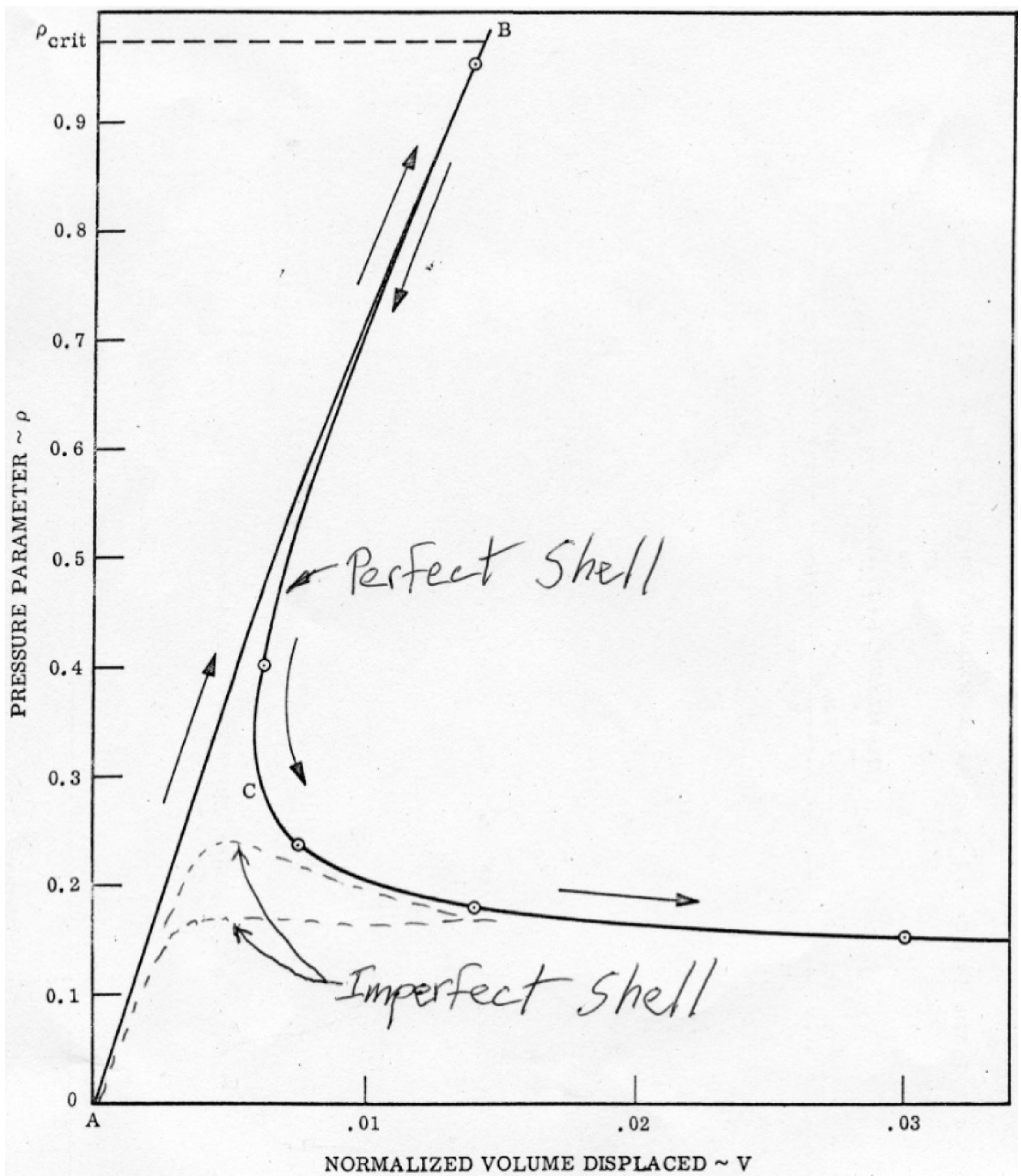


Slide 48 Comparison between test and theory for the collapse of externally pressurized spherical caps. p is the collapse pressure; p_{cl} is the theoretical buckling pressure of the perfect shell; H is the "rise" of the apex of the spherical cap above its base; h is the shell wall thickness. (from Abner Kaplan [43]). (Fig. 3.11 in AIAA Journal, Vol. 19, No. 9, 1981)

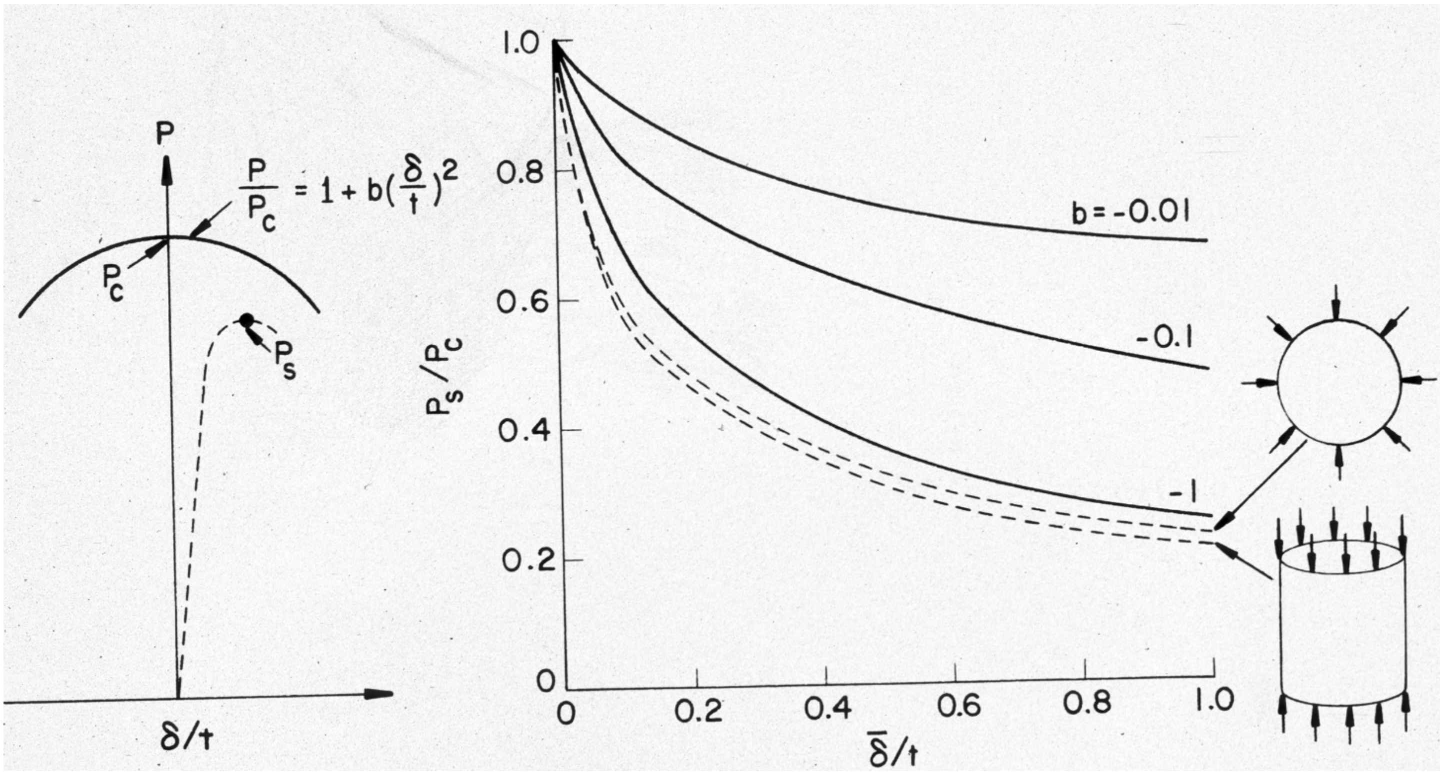


Slide 47 Load-deflection curves for externally pressurized perfect spherical caps with increasing "shallowness" parameter, lambda. Imperfection sensitivity increases with increasing lambda. p is the collapse pressure of the perfect shell; p_{cl} is the "classical" buckling pressure of a complete spherical shell. (Fig. 3.10 in AIAA Journal, Vol. 19, No. 9, 1981)

This slide demonstrates the transition in behavior from a flat plate under uniform pressure to a deep spherical cap or a complete spherical shell under uniform external pressure. The flat plate (a) exhibits increasing stiffness as the pressure is increased and membrane tension develops as the flat plate bulges downward under the pressure. A slightly curved plate (b) initially softens, then stiffens as the external pressure is increased, but there is no local maximum load-carrying capacity. The more curved plate (shell) (c) exhibits the type of nonlinear buckling called "snap-through": The shell softens until it has zero stiffness, then "snaps" into an inverted position, after which it stiffens with further increase in pressure. A shell with still more curvature (d) exhibits bifurcation buckling (black points) before axisymmetric "snap-through". Yet deeper (or thinner) shells (e, f) exhibit the same type of behavior as (d), but the characteristic equilibrium paths have an increasing degree of the "doubling back" feature typical of shells the behaviors of which are extremely sensitive to initial imperfections.



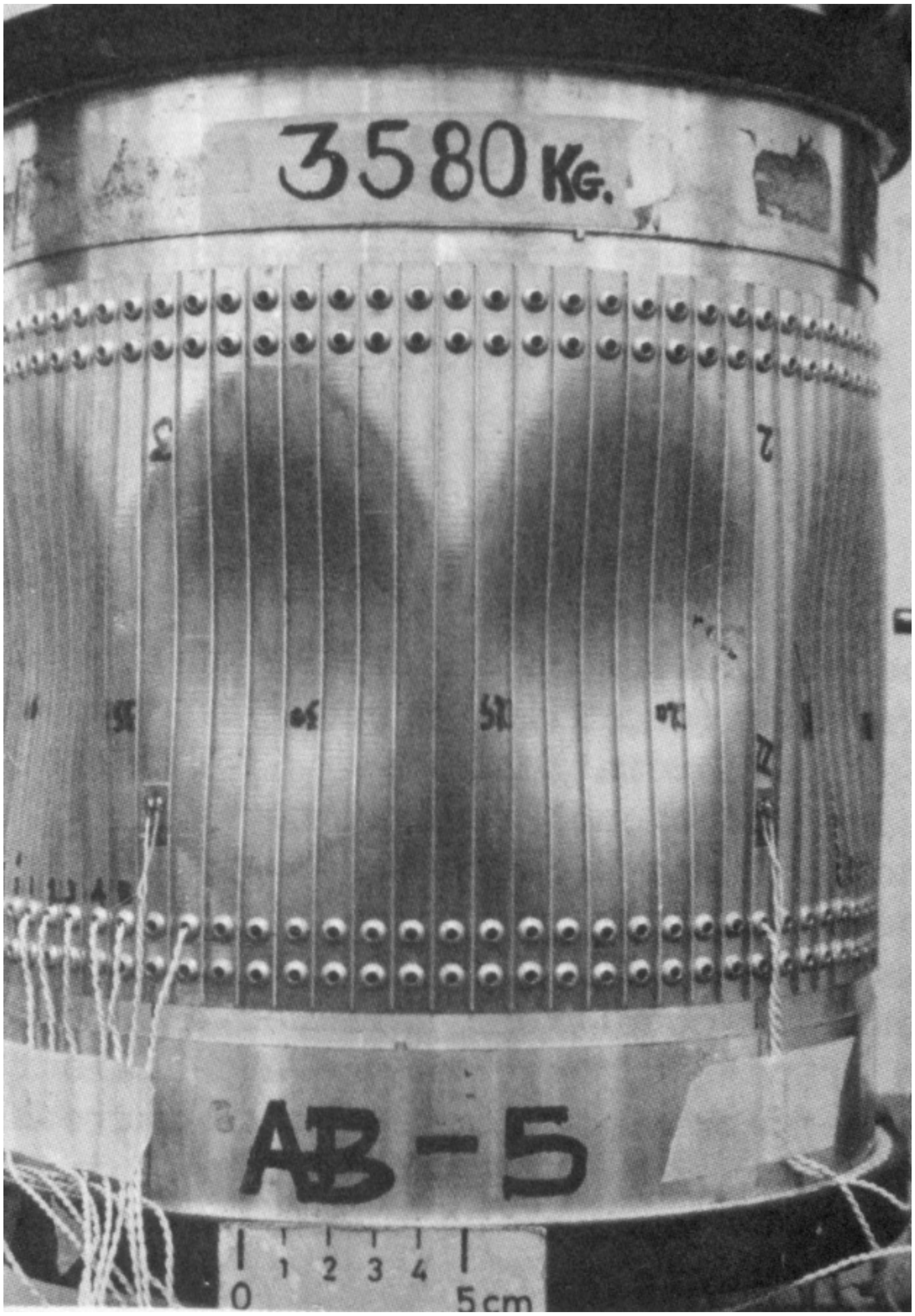
Slide 42 Load-deflection curves for a perfect and imperfect axially compressed monocoque thin cylindrical shell or externally pressurized monocoque thin spherical shell. Note that there is a dramatic effect of an initial imperfection on the load-carrying capacity of the shell. The perfect shell buckles at a normalized load of 1.0. (From the PowerPoint file: pitfallsnasa.ppt)



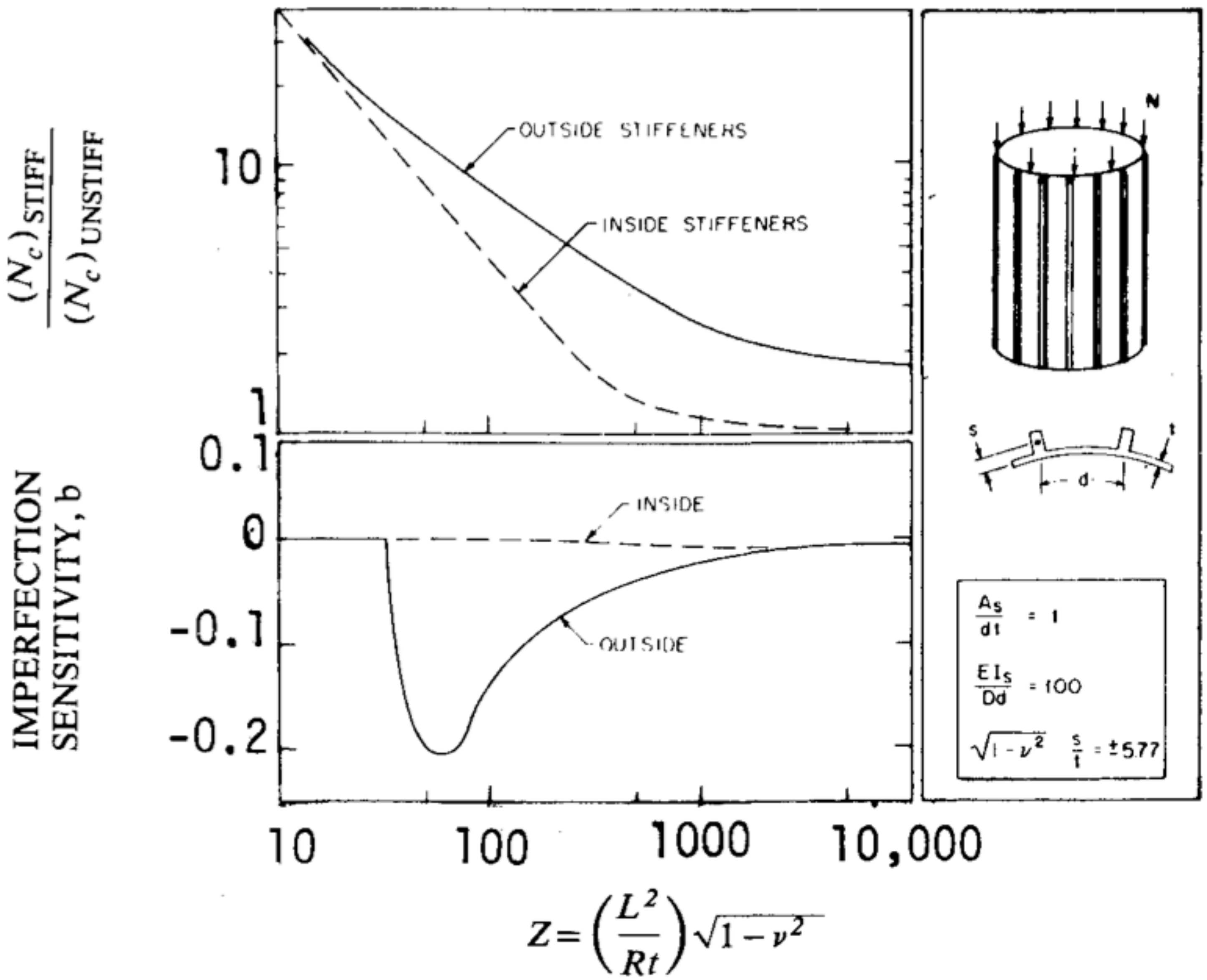
Slide 49 Predictions from Koiter's asymptotic theory for the collapse of perfect and imperfect cylindrical shells under axial compression and spherical shells under external pressure. P_C is the buckling load of the perfect shell; P_S is the collapse load of the imperfect shell; t is the shell wall thickness; delta is the amplitude of the imperfection; b is the Koiter "imperfection sensitivity" coefficient. (from Budiansky and Hutchinson [53]) (Fig. 3.3 in AIAA Journal, Vol. 19, No. 9, 1981))

PART 2 Shells that are moderately “imperfection-sensitive”

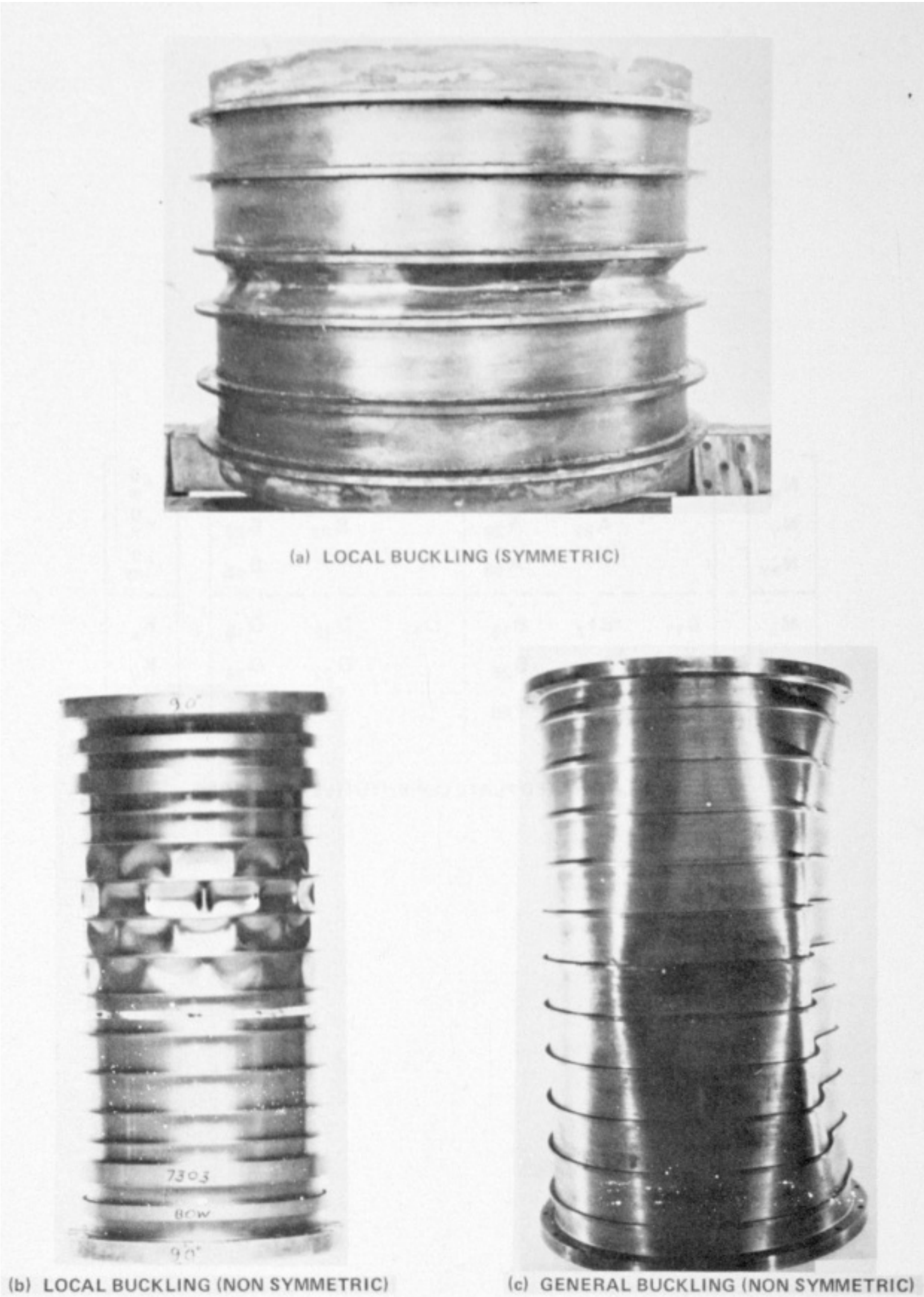
Shells that are moderately “imperfection sensitive” include all shells for which the characteristic dimension of a buckling wave is of the same order as a typical overall dimension of the shell. Shells that fall into this category are axially compressed stiffened cylindrical shells, all externally pressurized cylindrical shells, and all cylindrical shells under uniform torsion. Also moderately “imperfection sensitive” are shells in which buckling occurs at loads for which much of the material in the shell wall has been stressed well beyond its proportional limit, shells in which destabilizing (compressive) loads are fairly concentrated over a relatively small part of the surface of the shell (such as internally pressurized ellipsoidal and torispherical shells), shells that are stabilized by internal pressure, stiffened shells that are designed for service in their locally post-buckled states, and optimized axially compressed, axially stiffened flat plates.



Slide 55 A buckled axially compressed, axially stiffened cylindrical shell. Note the buckles are large because of the large “effective” axial bending stiffness of the stringer-stiffened shell wall. This behavior greatly reduces the imperfection sensitivity of stiffened shells compared with that for monocoque cylindrical shells under axial compression. (from Singer and Abramovich [40]). (Fig. 3.4 in AIAA Journal, Vol. 19, No. 9, 1981)



Slide 56 Predictions from Koiter's asymptotic theory for the imperfection sensitivity of axially compressed stringer-stiffened cylindrical shells. Z is the "Batdorf parameter"; L is the length of the shell; R is the radius; t is the wall thickness of the skin of the shell; N_C(stiff) is the buckling load of the perfect stiffened shell; N_C (unstiff) is the buckling load of the unstiffened shell, and b is the Koiter asymptotic imperfection sensitivity factor. (from Budiansky and Hutchinson [53]). (Fig. 3.12 in AIAA Journal, Vol. 19, No. 9, 1981)

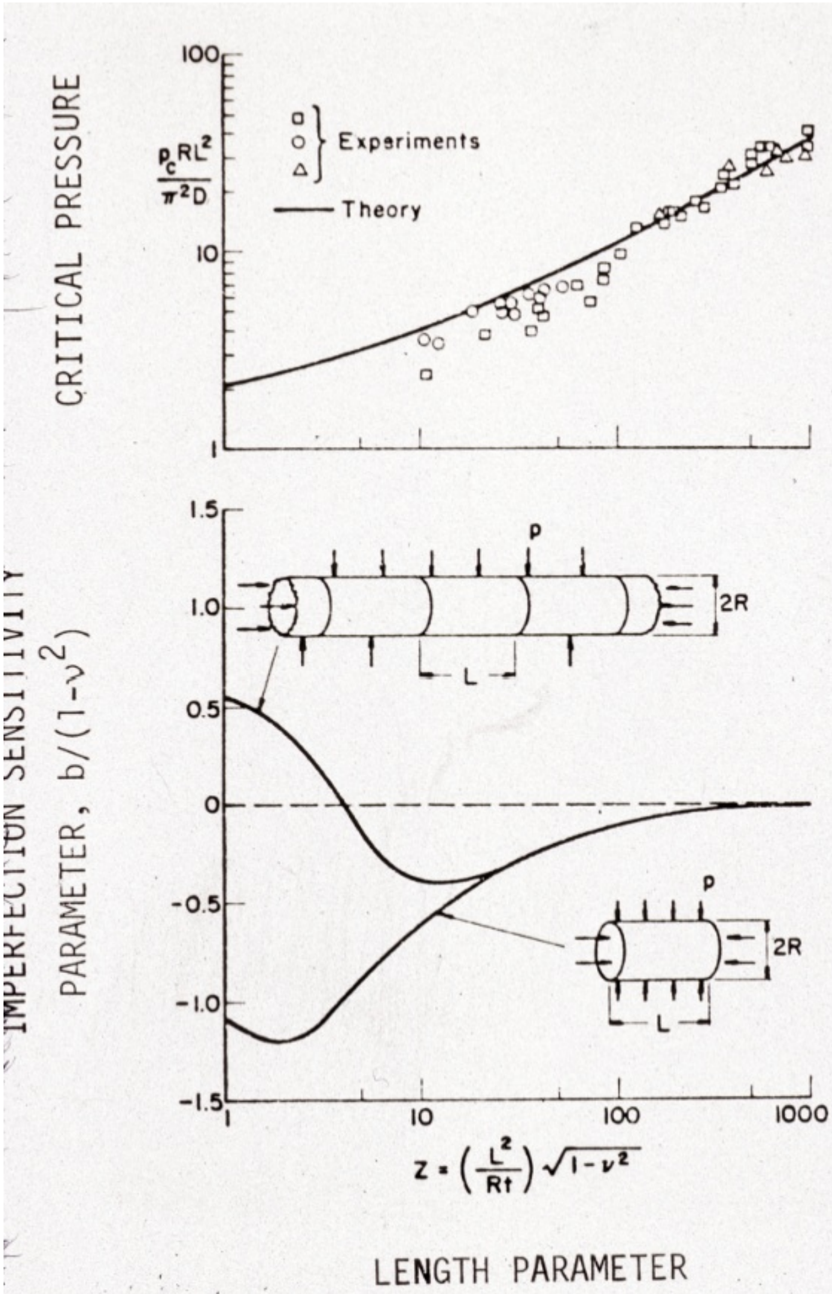


(a) LOCAL BUCKLING (SYMMETRIC)

(b) LOCAL BUCKLING (NON SYMMETRIC) (c) GENERAL BUCKLING (NON SYMMETRIC)

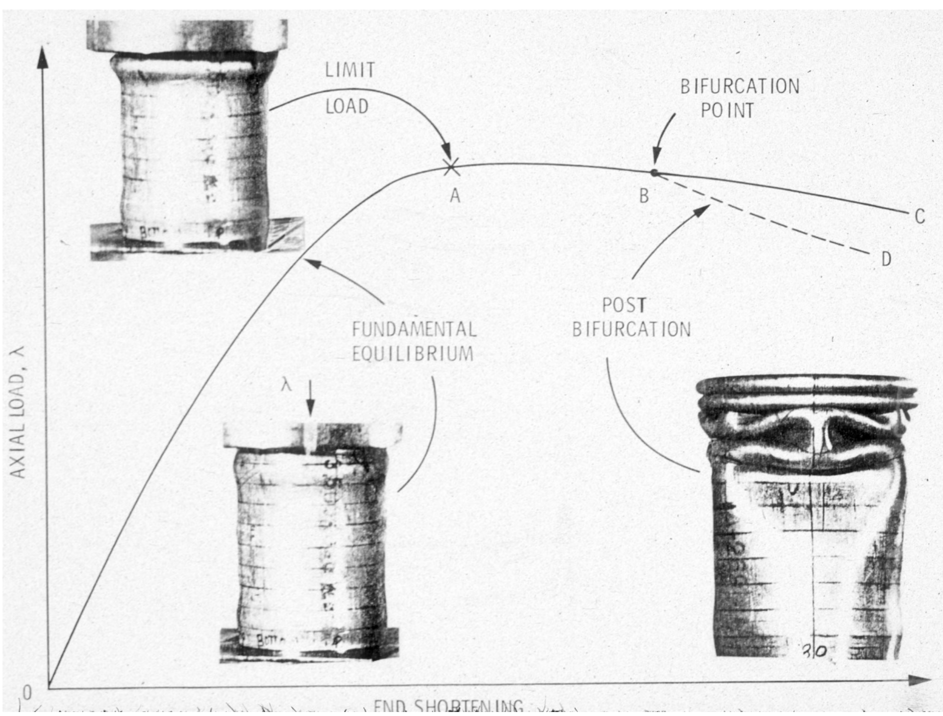
FIGURE 1 BUCKLING MODES FOR RING STIFFENED CYLINDERS
(REPRINTED WITH PERMISSION OF MR. T. E. REYNOLDS, DTNSRDC)

Slide 53 Buckled externally pressurized ring-stiffened cylindrical shells. (a) heavily stiffened (axisymmetric plastic buckling), (b) moderately stiffened (local buckling between rings), (c) lightly stiffened (general buckling) (Photographs courtesy of T. E. Reynolds, David Taylor Model Basin). (from the PowerPoint file, pitfallsnasa.ppt)

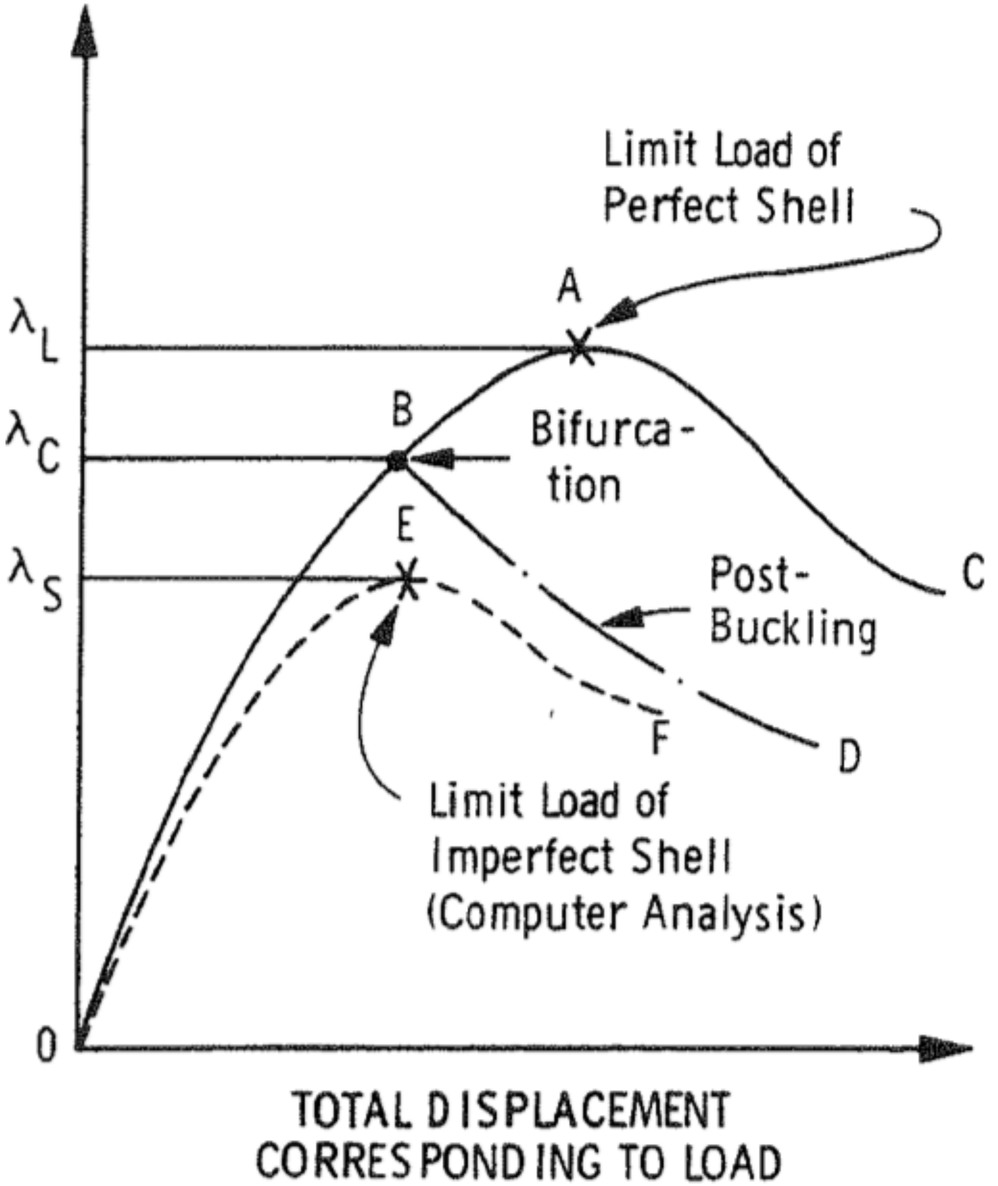


Slide 54 Comparison of test and theory and the prediction from Koiter's asymptotic theory for the imperfection sensitivity of externally pressurized ring-stiffened cylindrical shells. Z is the "Batdorf parameter"; L is the length of the shell between rings; R is the radius; t is the wall thickness of the skin of the shell; p_c is the bifurcation buckling pressure; D is the bending stiffness of the skin of the shell, and b is the Koiter asymptotic imperfection sensitivity factor. (from Hutchinson and Koiter [16]). (Fig. 3.13 in AIAA Journal, Vol. 19, No. 9, 1981)

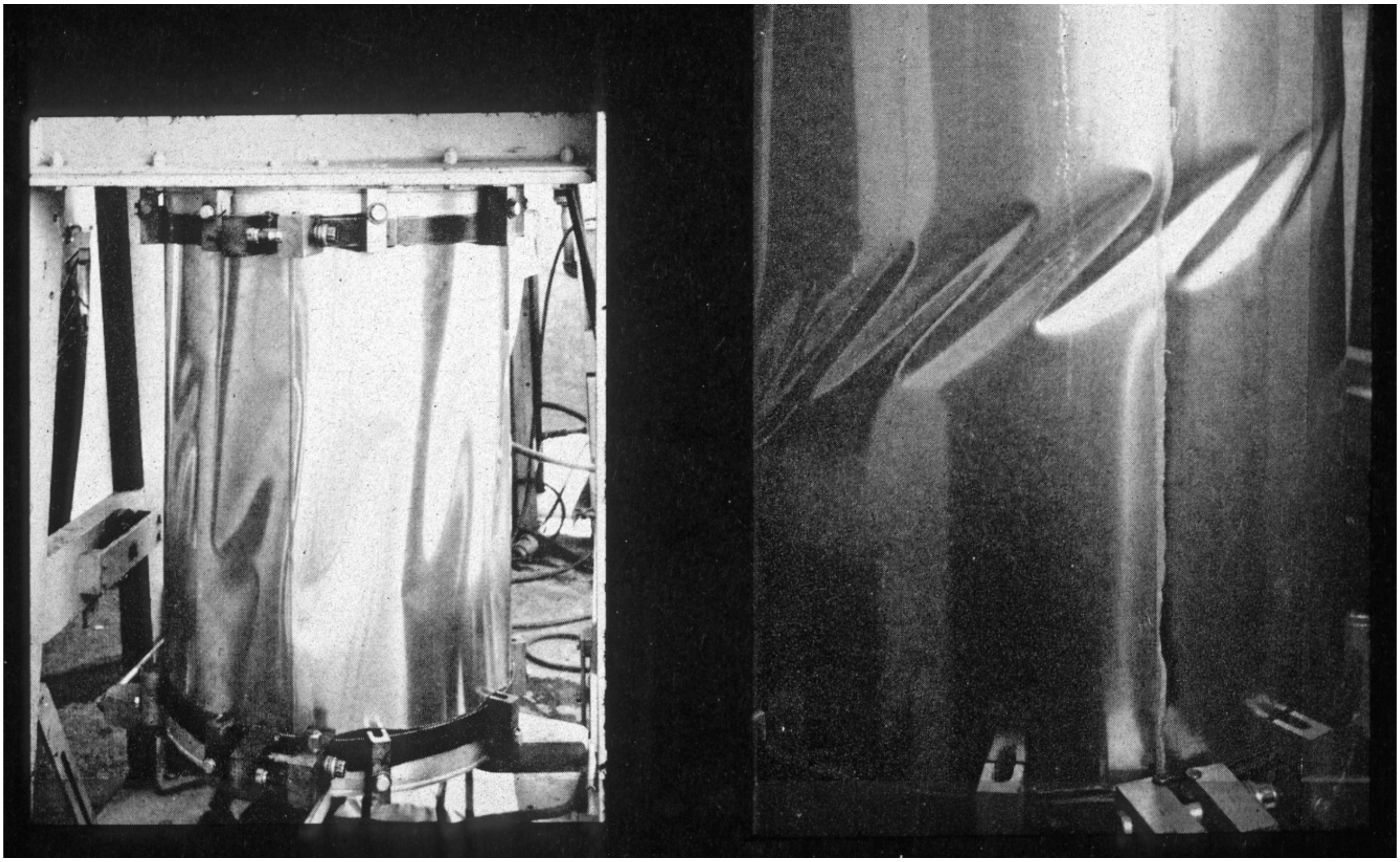
Buckling of externally pressurized cylindrical shells of intermediate or long length exhibits less imperfection sensitivity than does buckling of axially compressed cylindrical shells or externally pressurized spherical shells. The dimensions of the individual buckles of externally pressurized cylindrical shells are much larger than those of an axially compressed cylindrical shell, and buckling eigenvalues corresponding to different modes of buckling are not densely clustered as they are for axially compressed cylindrical shells. Here, in the top frame we see a comparison of test v. theory as a function of the "Batdorf" parameter Z. Note that in the region where test points fall below theory the asymptotic imperfection sensitivity parameter b is negative, indicating a degree of imperfection sensitivity (bottom frame). The curve with the most negative values of b for small Z arises because, for small Z (short shells), the axial component of compression generated by the external hydrostatic pressure produces behavior similar to that of a uniformly axially compressed cylindrical shell.



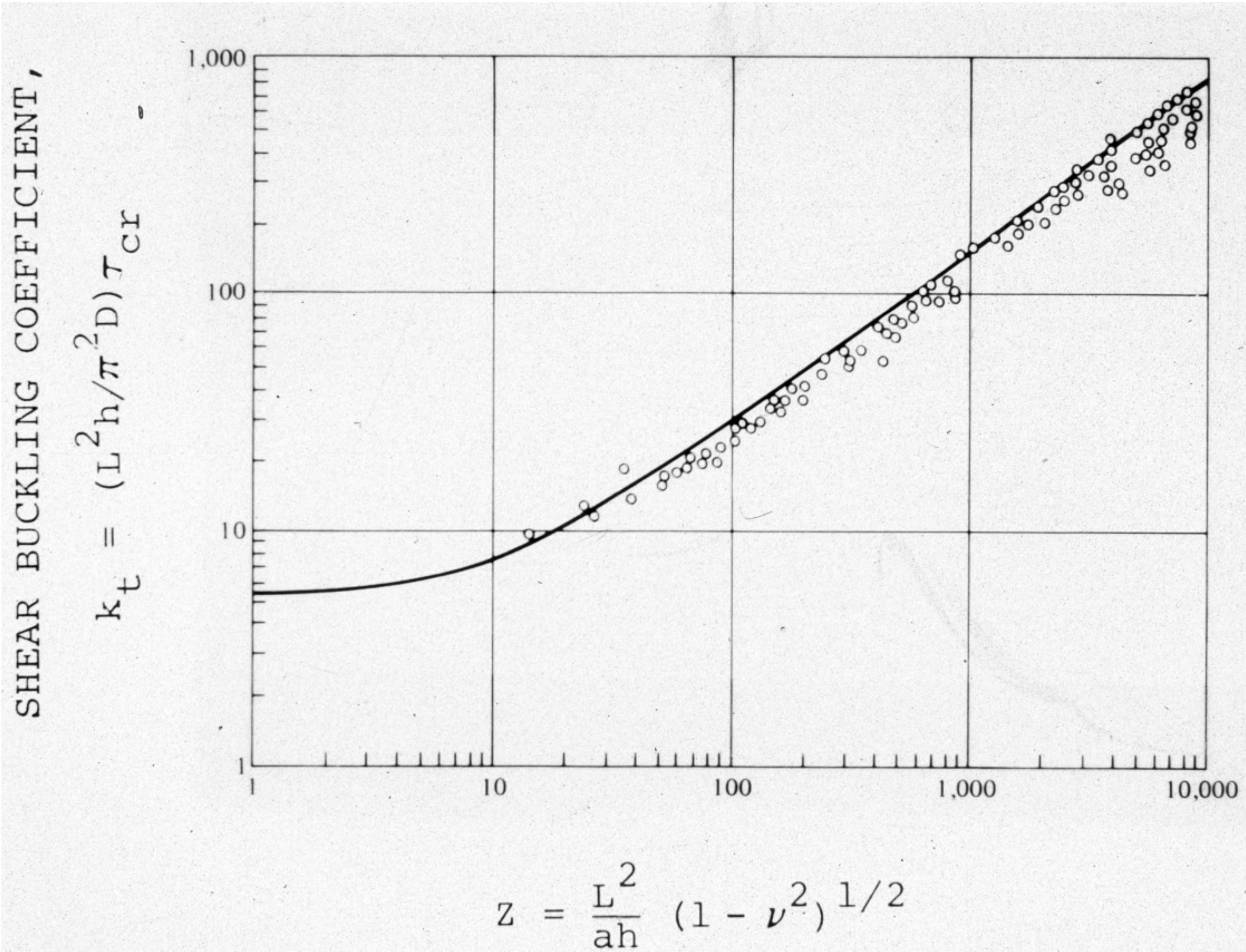
Slide 40 Elastic-plastic deformations of an axially compressed, rather thick cylindrical shell and the load-end-shortening curve with limit point A, bifurcation buckling point B, and post-bifurcation equilibrium path, BD. (from Sobel and Newman [87]). (Fig. 2.1 in AIAA Journal, Vol. 19, No. 9, 1981)



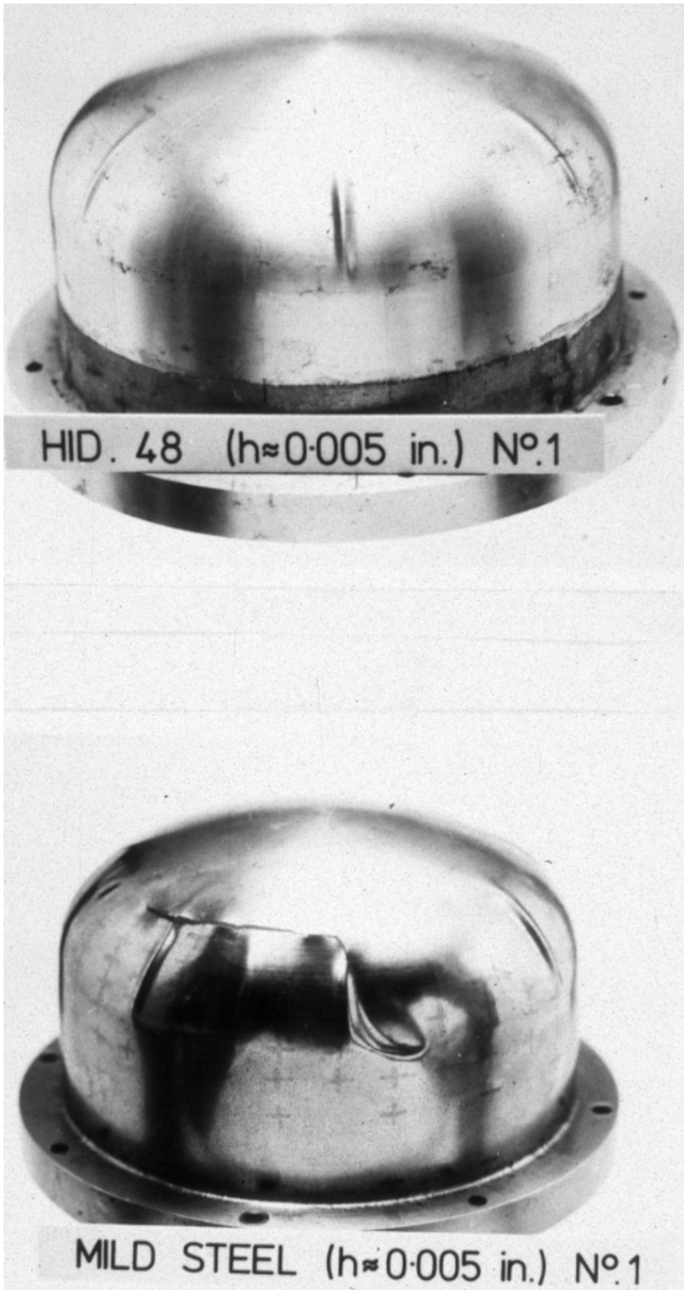
Slide 41 Load-deflection curves for a shell with moderate imperfection sensitivity. The bifurcation point, B, occurs before the limit point, A, is reached. In other words, non-axisymmetric buckles start to appear before axisymmetric collapse. There is some sensitivity to initial imperfections, that is, the maximum load-carrying capacity of the imperfect shell (dashed line) is moderately (but not dramatically) below the load-carrying capacity of the perfect shell. (Fig. 2.2a in AIAA Journal, Vol. 19, No. 9, 1981)



Slide 51 Thin monocoque cylindrical shells under uniform torque either without (left) or with (right) internal pressure exhibit much less imperfection sensitivity than do the same shells under uniform axial compression. (Photographs from Harris, et al [42].) (Fig. 3.7 in AIAA Journal, Vol. 19, No. 9, 1981)



Slide 52 Comparison between test and theory for the buckling of monocoque cylindrical shells under torsion. Z is the "Batdorf parameter"; L is the shell length; a is the shell radius; h is the wall thickness; D is the shell wall bending stiffness; tau_{cr} is the bifurcation buckling stress. (from the book by Brush and Almroth [11]) (Fig. 3.8 in AIAA Journal, Vol. 19, No. 9, 1981)



Slide 65 Elastic-plastic buckling of internally pressurized torispherical shells. Similar buckling patterns for internally pressurized aluminum and mild steel torispherical test specimens. With metallic specimens the nonlinear behavior is complicated by the presence of both moderately large pre-buckling deformation and elastic-plastic material behavior. The behavior of these and other pressure vessel heads has been extensively studied by Gerry Galletly and his colleagues at the University of Liverpool. (Photographs by Professor Gerald Galletly and colleagues at the University of Liverpool). (from the PowerPoint file, pitfallsnasa.ppt)

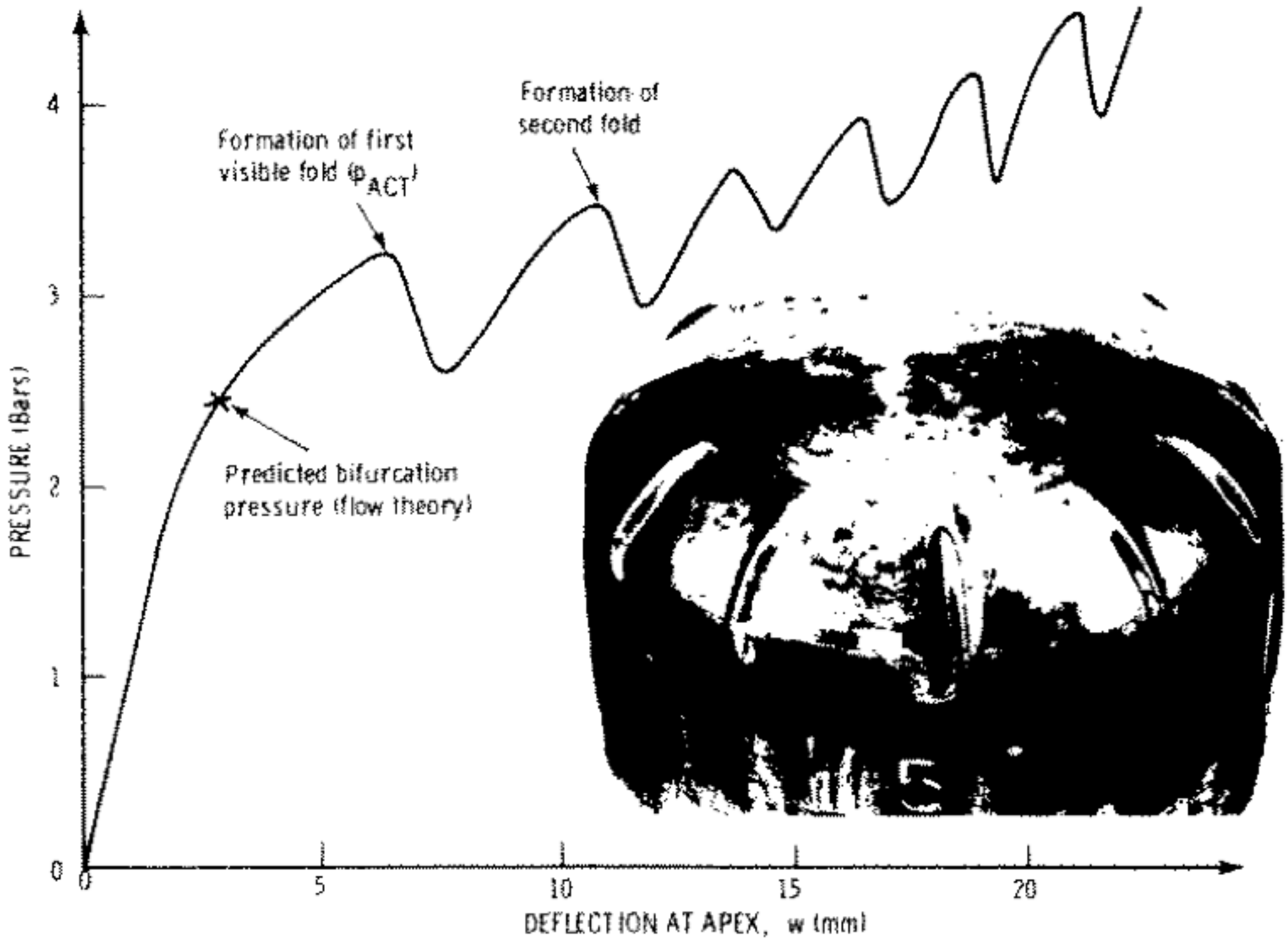
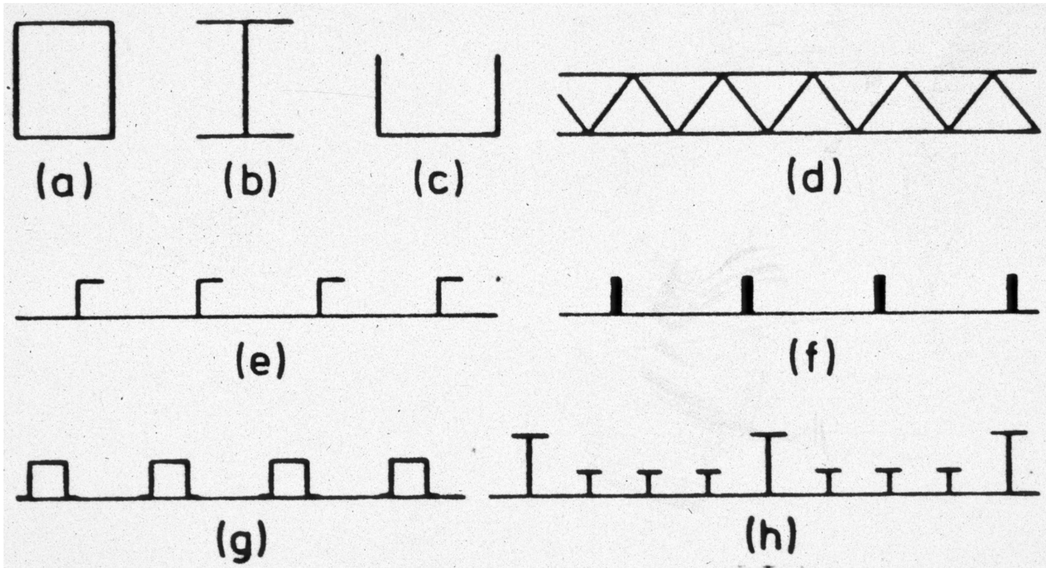
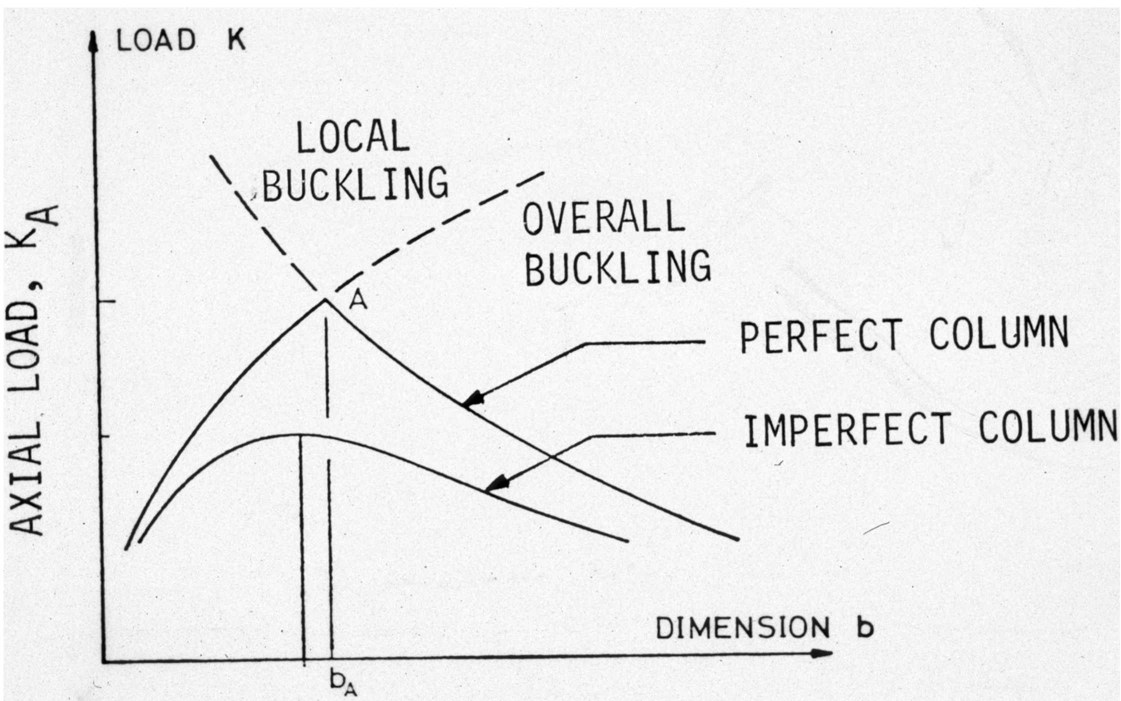


Fig. 2 Pressure-deflection curve for an internally pressurized ellipsoidal pressure vessel head. Each local peak in the pressure-deflection curve is mildly sensitive to initial geometrical imperfections. (Photo by Gill and his colleagues at the University of Manchester.) (from STRUCTURAL ANALYSIS SYSTEMS, Vol.2, A. Niku-Lari, editor, Pergamon Press, paper on BOSOR5 by David Bushnell, pp. 55-67, 1986)



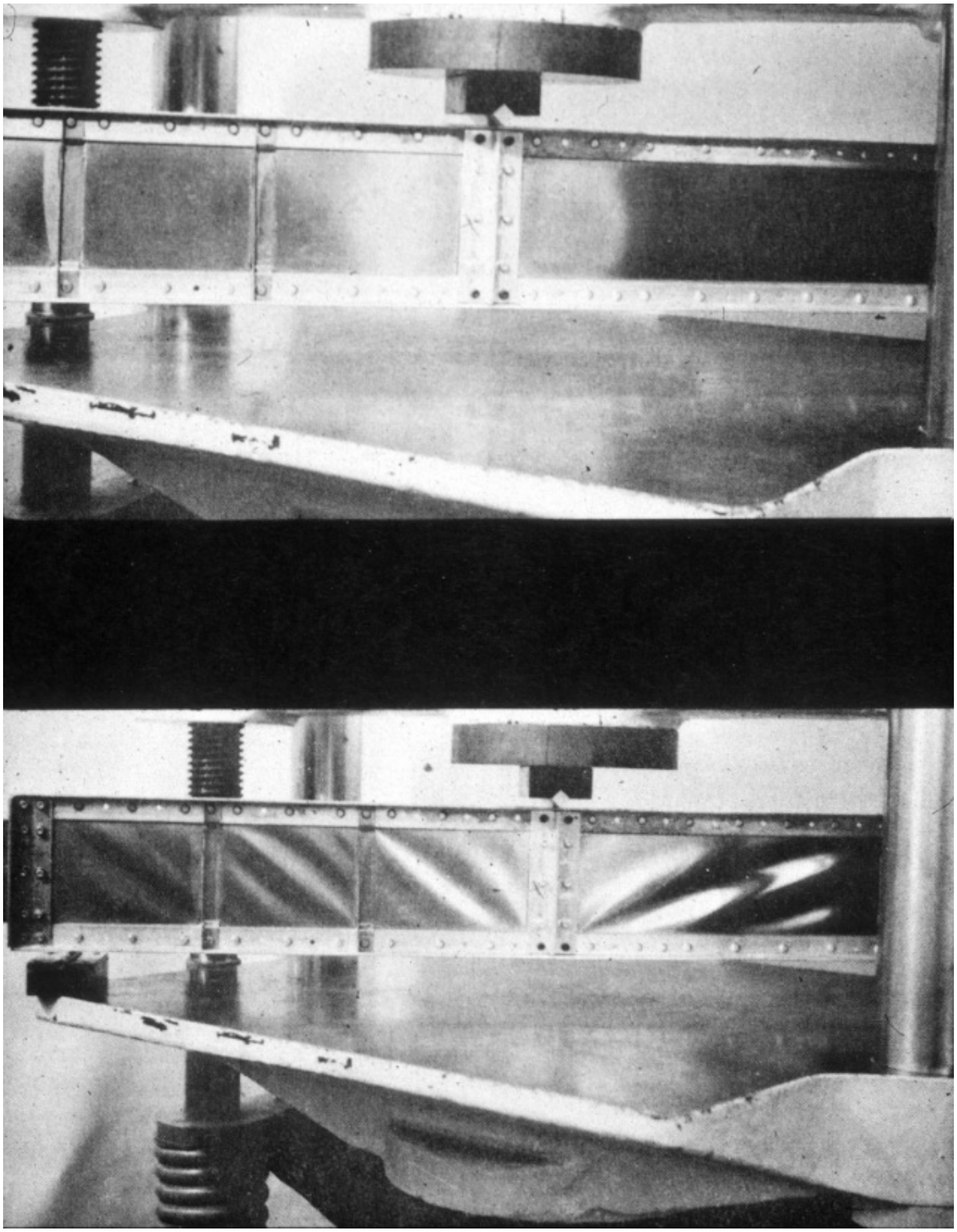
Slide 77 Axially compressed stiffened panels (drawing after van der Neut.) (from the PowerPoint file, pitfallsnasa.ppt)



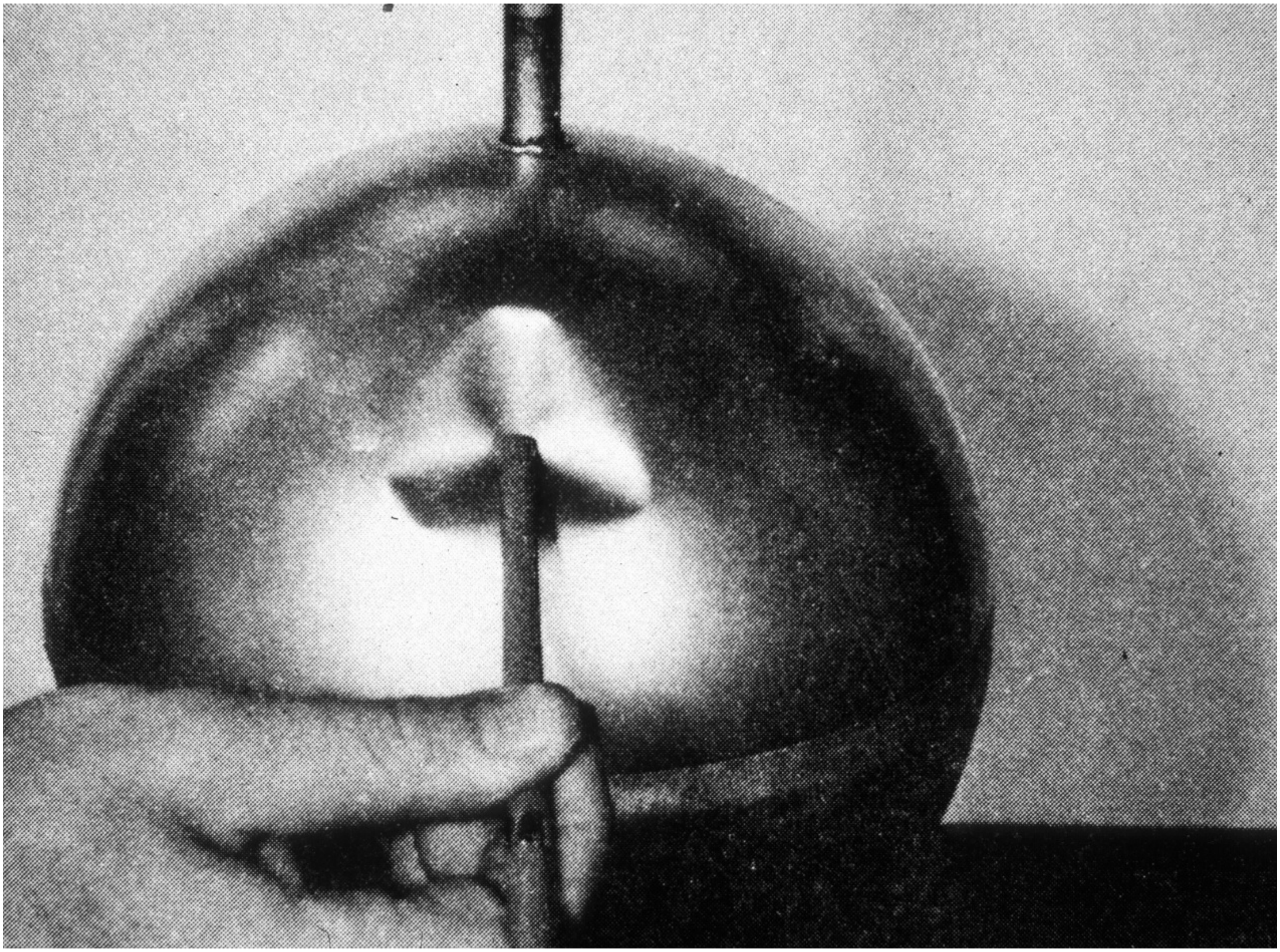
Slide 78 Imperfection sensitivity of an optimized axially compressed, axially stiffened panel. The imperfection sensitivity exists because a small, local deformation of the skin reduces its effective axial stiffness, which then reduces the general buckling load. (from Thompson and Lewis [172].) (Fig. 7.8 in AIAA Journal, Vol. 19, No. 9, 1981)

PART 3 Shells that are not imperfection-sensitive

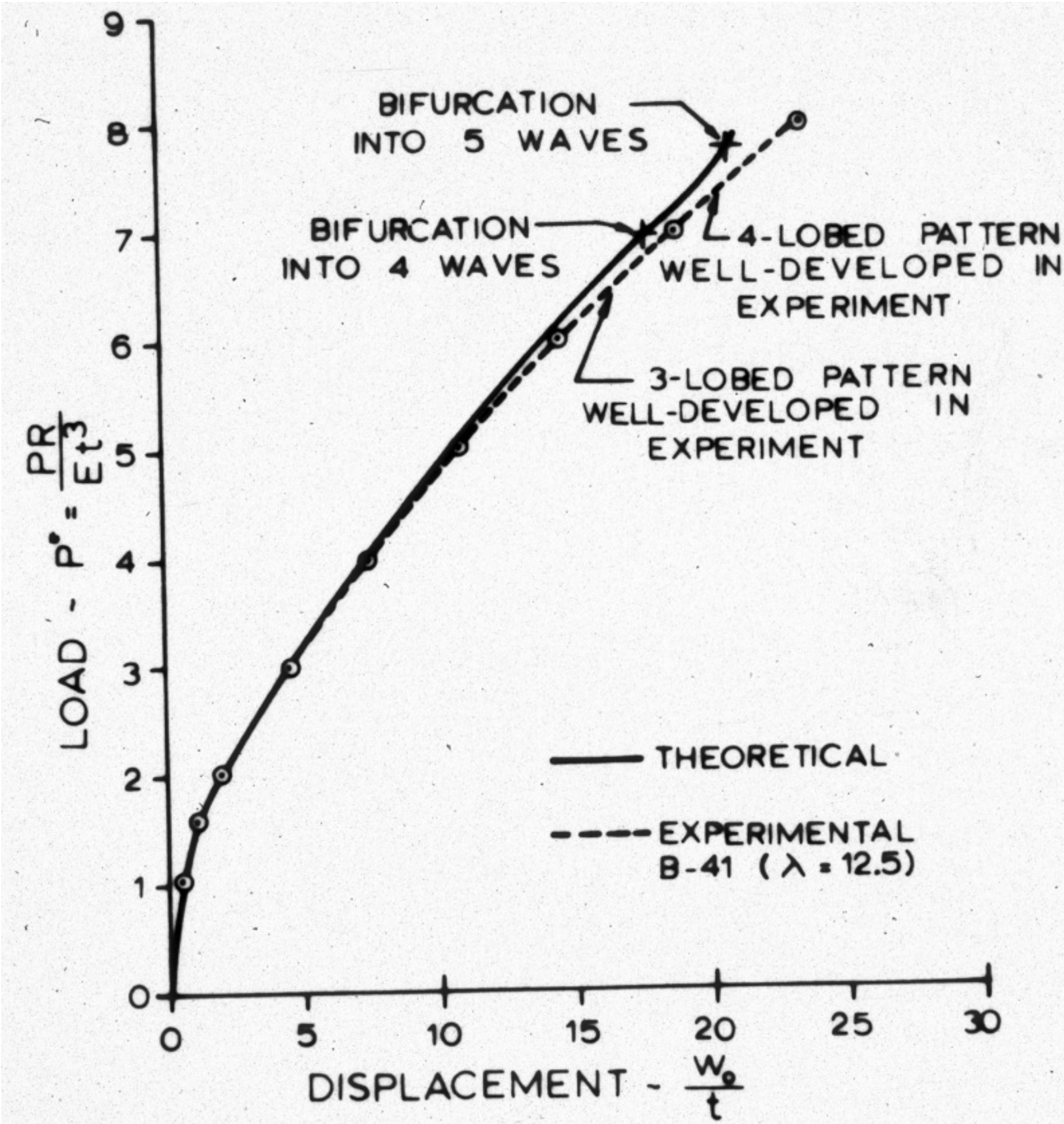
A beam in bending with a deep, thin web falls into this category. The locally buckled web continues to carry load far above that at which web buckling initially occurs. Diagonal tension develops in the buckled web. This diagonal tension acts as a truss. Also, a spherical shell with an inward-directed concentrated load is not imperfection sensitive. Unstiffened, axially compressed flat rectangular plates are not sensitive to initial imperfections. In an overall sense, internally pressurized ellipsoidal and torispherical shells are not imperfection sensitive because they can carry internal pressure far above that which causes initial buckling provided that fracture of the shell wall does not occur upon formation of the first few local buckles in the knuckle region where a relatively narrow band of circumferential membrane compression occurs.



Slide 70 The development of post-buckling diagonal tension in the web of a deep beam under a concentrated load that causes bending of the beam. Buckles in the web reduce somewhat the effective bending stiffness of the beam, but the beam does not fail. Instead, “diagonal tension” develops along the long axis of the buckles, simulating truss members and giving the beam an effective shear stiffness. (from the PowerPoint file, pitfallsnasa.ppt)



Slide 71 Concentrated inward load applied to a spherical shell. Initially a small axisymmetric inward dimple occurs. As the inward load is further increased a multi-lobed pattern gradually develops, indicating bifurcation buckling and post-buckling that is stable. (Photograph by Sendlebeck and Horton at Stanford University taken in the early 1960's.) (from the PowerPoint file, pitfallsnasa.ppt)

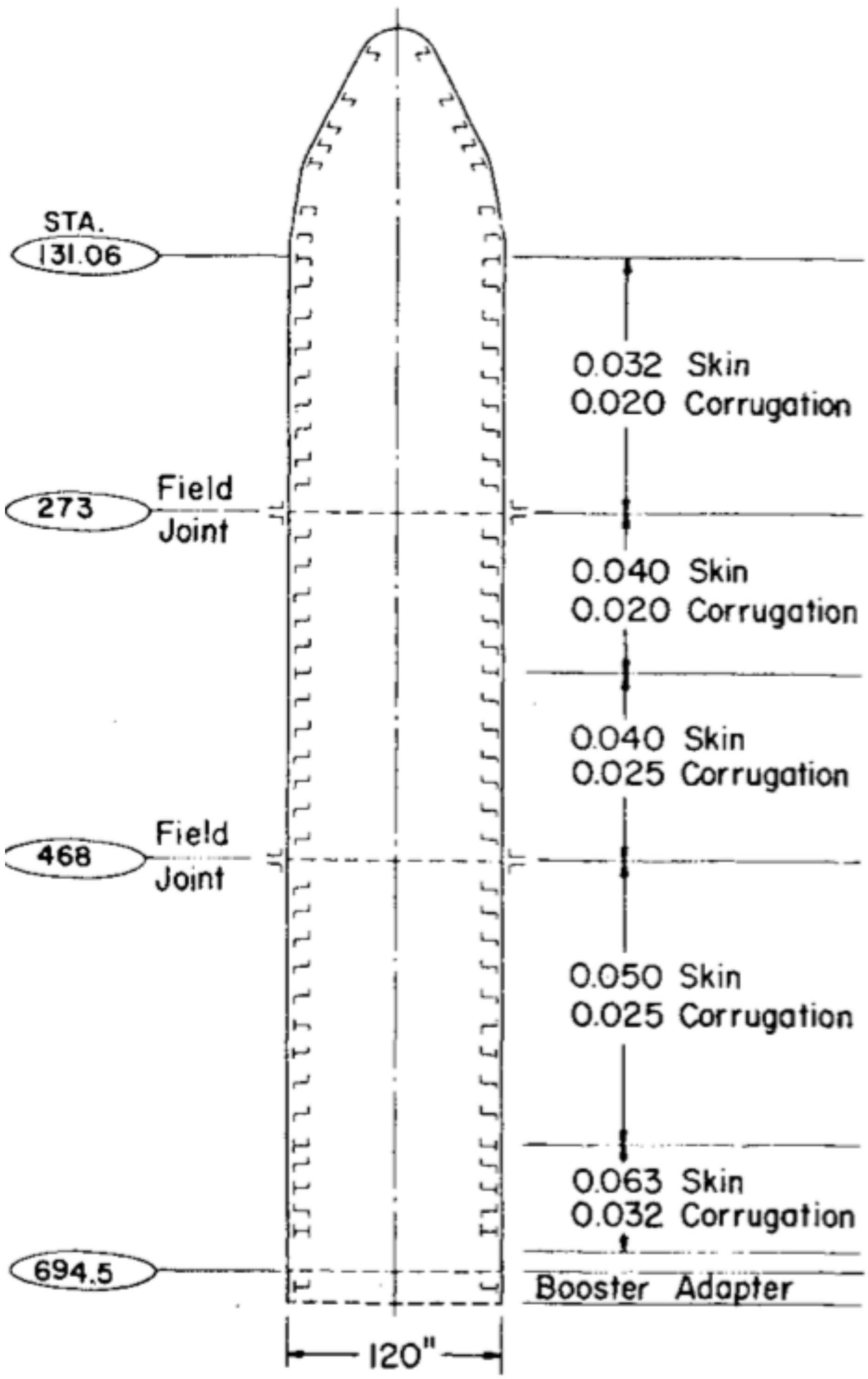


Slide 72 Comparison between test & theory for the spherical shell with the inward-directed normal concentrated load. As the load is increased the number of lobes in the non-axisymmetric pattern gradually increases. The pre-buckling behavior is very nonlinear. (from the PowerPoint file, pitfallsnasa.ppt)

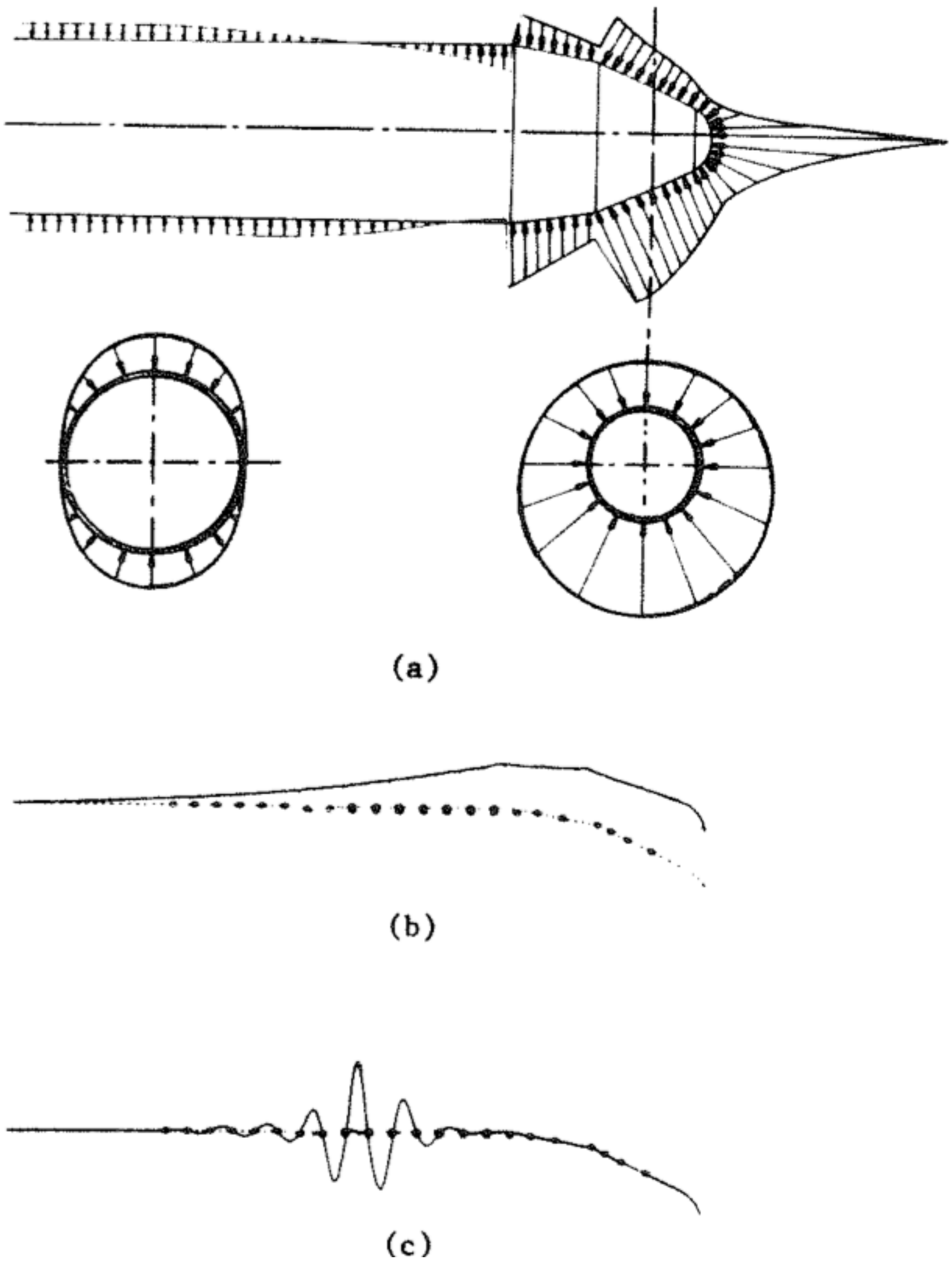

Photograph by Connie Indrebo, Crazy Creek Air Adventures, Middletown, California

Local buckling of the top surfaces of the wings of a glider. The top surfaces are like thin cylindrical shells under axial (span-wise) compression. The axial compression is largest near the roots of the wings and exists because the wings, clamped at their roots, are bent “upward” (toward the viewer) as the glider makes a tight turn around the peak of a nearby mountain from which this photograph was taken. The wings in their locally post-buckled state do not fail catastrophically because there exist strong internal spars that remain intact (unbuckled). The load at which the wing fails is not sensitive to initial imperfections even though the load at which the cylindrical skin of the upper surface of the wing is sensitive to initial imperfections because there is a strong internal structure that accepts the load shed by the skin of the wing as it buckles.

PART 4 A buckling failure that is not caused by an unavoidable imperfection, but is caused by a known flaw in the design. The design flaw can be thought of as an imperfection, but it is an "imperfection" that can be incorporated exactly into the analysis. Therefore, the shell is not "imperfection sensitive" in the "classical" sense of imperfection sensitivity in the presence of unknown and possibly random imperfections. The behavior of the shell with the design flaw is predictable if it is modeled correctly.



An externally corrugated, internally ring-stiffened payload shroud that failed during a test. The segmented stiffened payload shroud can buckle during launch. The skin thicknesses and external corrugation thicknesses increase in steps from tip to base of the shroud. In a test of this payload shroud buckling occurred unexpectedly at the field joint at Station 468. As demonstrated in the next three slides, buckling can occur from non-axisymmetric external dynamic pressure that generates primarily axial compression on the leeward side of the shroud that increases from its tip to its base as the shroud bends like a beam under the non-axisymmetric dynamic pressure loading encountered during launch. (Fig. 1.9a in AIAA Journal, Vol. 19, No. 9, 1981)



Slide 34 Buckling mode of a non-axisymmetrically loaded rocket payload shroud shown in Fig. 9(a): (a) Pressure distribution measured in a wind tunnel test; (b) Pre-buckling beam-type deflection; (c) Non-axisymmetric buckling mode. Buckling is between the discrete rings and occurs with 13 circumferential waves. (from the PowerPoint file, pitfallsnasa.ppt)

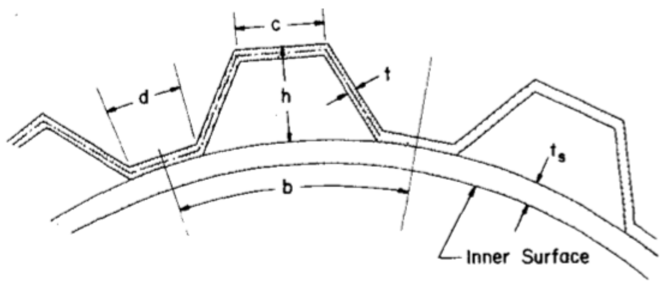


Fig. 1.9b in AIAA Journal, Vol. 19, No. 9, 1981
External axially oriented corrugations in the payload shroud.

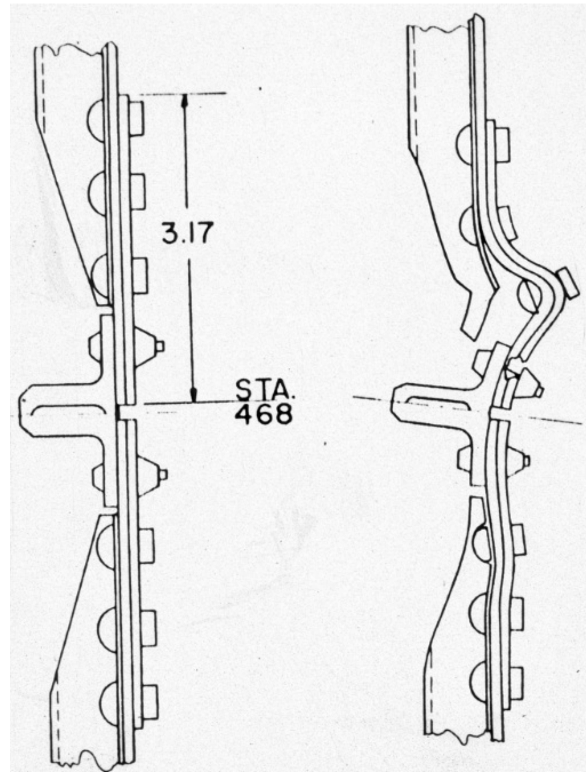
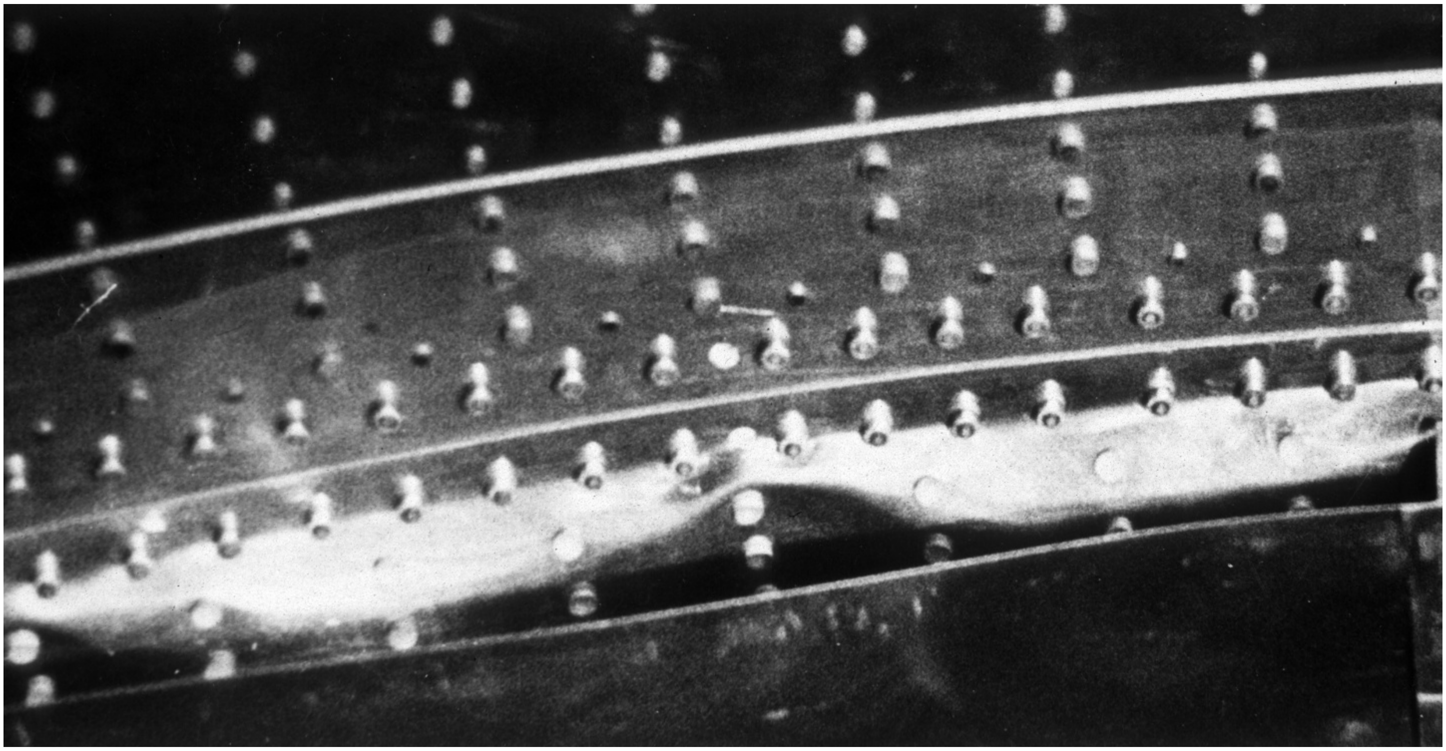


Fig. 1.9d in AIAA Journal, Vol. 19, No. 9, 1981
Schematic of local buckling failure at Station 468 in the payload shroud.

Slide 35 (right-hand sketch above) Local buckling failure is caused by an axisymmetric inward excursion of the load path of the axial compression seen by the payload shroud at Station 478. This is a known “imperfection” that can be included in the computerized model. This known “imperfection” is the most serious imperfection in the entire fabricated shell because during a test the payload shroud failed because of it and not because of some unknown and unknowable random imperfection at some other location. Therefore, one might say that the payload shroud as fabricated is not “imperfection sensitive” in the “classical” sense of that term. If the payload shroud had been designed taking account of the load path eccentricity at Station 468 (by the use of a thicker doubler, for example) then the stiffened cylindrical shell would be classed as “moderately imperfection sensitive” (See Part 2 above). (Fig. 1.9d in AIAA Journal, Vol. 19, No. 9, 1981)



Slide 36 Local buckling failure of the payload shroud at Station 468. (Fig. 1.9c in AIAA Journal, Vol. 19, No. 9, 1981)

REFERENCES CITED IN THE FIGURE CAPTIONS:

[11] Brush, D.O. and Almroth,B.O., Buckling of Bars, Plates, and Shells, McGraw-Hill Book Co., New York, 1975.

[16] Hutchinson, J.W. and Koiter,W.T., "Post-buckling Theory," *Applied Mechanics Review,* Vol.23, 1970, pp. 1353-1356.

[40] Singer, J. and Abramovich, H., "Vibration Techniques for Definition of Practical Boundary Conditions in Stiffened Shells," *AIAA Journal,* Vol. 17, July 1979, pp. 762-769.

[42] Harris, L. A., Suer, H. S., and Skene,W. T., "Model Investigations of Unstiffened and Stiffened Circular Shells", *Experimental Mechanics,* Vol. 1, No. 7, 1961, pp. 1-9.

[43] Kaplan, A., "Buckling of Spherical Shells," *Thin Shell Structures, Theory, Experiment, and Design,* edited by Y. C. Fung and E. E. Sechler, Prentice-Hall, Englewood Cliffs, N.J., 1974, p.248-288.

[44] Horton, W. H., Bailey, S. C., and McQuilkin, B. H., An In troduction to Stability," Stanford University report version of Paper 219 presented at ASTM Annual Meeting, Atlantic City, N.J., June 1966.

[45] Carlson, R. L., Sendlebeck, R. L., and Hoff, N. J., "Experimental Studies of the Buckling of Complete Spherical Shells," *Experimental Mechanics,* Vol.7, 1967, pp. 281-288.

[53] Budiansky, B. and Hutchinson, J. W., "A Survey of Some Buckling Problems," *AIAA Journal,* Vol. 4, Sept. 1966, pp. 1505- 1510.

[87] Sobel, L. H. and Newman, S. Z., "Plastic Buckling of Cylindrical Shells under Axial Compression," Paper presented at Third U.S. Congress on Pressure Vessels and Piping, San Francisco, June 25-29, 1979 (see also, *ASME Journal of Pressure Vessel Technology,* Vol. 102, Feb. 1980, pp. 40-44).

[172] Thompson, J.M.T. and Lewis, G. M., “On the Optimum Design of Thin-walled Compression Members," *Journal of the Mechanics and Physics of Solids,* Vol. 20, 1972, pp. 101-109.

FOR A MORE COMPLETE BIGLIOGRAPHY OF SHELL BUCKLING PAPERS PUBLISHED BEFORE 1980, SEE THE LIST OF REFERENCES AT THE END OF THE PAPER:

“Buckling of shells – Pitfall for designers”, by David Bushnell, AIAA Journal, Vol. 19, No. 9, September, 1981 (1981pitfalls.pdf).