THE INTRODUCTION OF COLD-BENDING RING BUCKLING MARGINS INTO PANDA2

This is an important news item. Often cylindrical shells are fabricated by cold bending a flat plate or sheet into a cylindrical form. NASA fabricates some light-weight stiffened cylindrical shells with use of the following steps:

1. A regular array of "pockets" is "hogged out" of a rather thick flat plate. The "hogged out" flat plate is then a "stiffened" flat plate, with the stiffeners (stringers and rings) being the thicker material between the "pockets".

2. The "hogged out" plate is then cold bent into a cylindrical shell, or rather into a part of a cylindrical shell, say a panel subtending 120 degrees or something like that. The cold-bending is applied usually with the stiffeners on the inside. In this process the plate is bent into a radius, call it "RCOLD", which is smaller than the design radius, call it "RCYL". Upon elastic spring-back the final radius should be close to RCYL.

3. An annealing process is applied to the cold-bent cylindrical panel.

4. A number of cylindrical panels fabricated in this way are welded together to form a complete (360-degree) cylindrical shell.

This news item is concerned with Step 2. The question arises, "How can the designer determine the best "slenderness" (height-to-thickness ratio) of the rings, which experience potentially destabilizing hoop compression during the cold-bending process?" The designer wants the rings to be thick enough so that they do not buckle during the cold-bending process, but not so thick as to represent an unnecessary weight penalty. This question does not arise in the case of the stringers because they do not bend during the cold bending process.

In order to provide an answer to this question, three new design margins have been introduced into PANDA2 in Load Set 1, Subcase 1. In the list of margins presented for Load Set 1, Subcase 1 the three new margins (for an optimized shell without internal sub-rings) appear as follows:

17 6.59E-01 Cold-bending ring buckling, closed form soln; N=154; FS=1.1
in which \( N \) is the number of circumferential halfwaves in the critical buckling mode. If there are internal sub-rings, then there is an additional cold-bending buckling margin of the form:

\[ -2.46 \times 10^{-4} \text{ Cold-bending subring buckling, closed form soln; } N=102; FS=1.1 \]

The first of the three margins listed above (Margin 17) is derived from a "closed form" analysis in which it is assumed that the skin plays no role in the buckling. A polynomial expression in assumed for the distribution of normal deflection of the internal ring web. This polynomial has three undetermined coefficients. An eigenvalue problem of rank 3 is set up and solved for the three real eigenvalues, the lowest positive one of which is the critical ring web buckling load factor. These computations are carried out in **SUBROUTINE COLDBD**, which is called from **SUBROUTINE STRUCT**.

The second of the three margins listed above is derived from a discretized single-module "skin"-ring module of the type described in the paper, "Additional buckling solutions in PANDA2", AIAA Paper AIAA-99-1233, 40th AIAA Structures Meeting, 1999, pp. 302-345; (See pp. 318 - 321 and Fig. 30). The word, "skin", is in quotes because "skin" represents the skin plus smeared stringers (and smeared substiffeners, if any). The prebuckling distribution of stress resultants over the "skin" ring module is derived in the new **SUBROUTINE COLDBD**. Also, the elastic-plastic integrated 6 x 6 constitutive matrices \( C_{ij} \) (called \( C_{X\text{COLDBD}} \) and \( C_{Y\text{COLDBD}} \)) for the "skin"-ring module segments ("skin", ring web, ring outstanding flange) are computed in **SUBROUTINE COLDBD**. The critical elastic-plastic cold-bending buckling load factor is obtained over a range of circumferential wave numbers in search of the critical (lowest) eigenvalue (buckling load factor).

The third of the three margins listed above is derived from the same type of model as the second. In this case the stringers
are eliminated. Therefore, the buckling model is a discretized skin-ring single module model with smeared sub-stiffeners, if any. The word, skin, is not in quotes because in this model the shell skin without any smeared stiffeners forms the first part of the discretized module. This margin is usually the most critical of the three. Buckling load factors from this third model are more accurate than the first model, the "closed-form" model, because the deformation of the panel skin in the cold-bending buckling mode is accounted for, whereas in the "closed-form" model the skin is assumed to remain undeformed during buckling. The third model is more conservative than the second model because the stringers are neglected (sub-stiffeners are smeared). The third model is appropriate, however, because the critical circumferential wavelength of the cold-bending buckling mode is restricted to be less than or equal to the spacing between the stringers.

The cold-bending buckling analysis is entered only if the PANDA2 user chooses that he or she wants to provide input data for a stress-strain "curve" for Material Type 1. The nonlinear stress-strain curve is used ONLY in the cold-bending ring buckling analysis. All the other buckling and stress margins computed by PANDA2 are still based on elastic material behavior.

The prompting file, .../execute/PROMPT.DAT, was modified as follows:

------------------------------------------------------------
238.1 Want to supply a stress-strain "curve" for this mat'l (H)?
238.2

Please use elastic properties only unless you want to simulate the cold-bending fabrication process. (See Item No. 790 in the file, ...panda2/doc/panda2.news).

The stress-strain curve is used ONLY in the analysis of cold-bending. (See Item No. 790 in ...panda2/doc/panda2.news). PANDA2 does not account for plasticity or nonlinear stress-strain curves in its ordinary buckling and stress analyses, even if you choose to provide a stress-strain curve here.

The stress-strain curve is used ONLY for generating a buckling constraint condition for buckling of an INTERNAL Tee-shaped or rectangular ring during the cold-bending of a flat plate with "hogged out" pockets that form stiffeners into a cylindrical panel.

If you want to simulate the cold-bending process:

1. The entire structure must be fabricated from the same
isotropic material. (You can supply more than one material type in the PANDA2 model, but all material types must have the same isotropic elastic properties.)

2. If in the PANDA2 model you plan to introduce more than one material type (in order to identify what the maximum stresses are in the various parts of the shell structure) the stress-strain curve must be supplied ONLY FOR THE FIRST MATERIAL TYPE in the PANDA2 model, that is, for material index no. 1. All other material types must correspond to elastic, isotropic materials with the same elastic properties as those of material type 1.

NOTE: The stress-strain curve is assumed to be the same for both tension and compression.

The stress-strain curve is used ONLY in the analysis of cold-bending. (See Item No. 790 in ...panda2/doc/panda2.news). PANDA2 does not account for plasticity or nonlinear stress-strain curves in its ordinary buckling and stress analyses. The stress-strain curve is used ONLY for generating a buckling constraint condition for buckling of an INTERNAL Tee-shaped or rectangular ring during the cold-bending of a flat plate with "hogged out" pockets that form stiffeners into a cylindrical panel.

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NOTE: The stress-strain curve is assumed to be the same for both tension and compression.

Next, supply a table of (strain, stress) values, starting with (0., 0.). Maximum of 20 (strain, stress) pairs permitted.
240.1 strain coordinate for stress-strain "curve", strain
240.2
NOTE: the first strain entry must be zero.
NOTE: the second strain entry (the first non-zero entry)
must have the value: strain(2) = stress(proportional limit)/E.

241.1 stress coordinate for stress-strain "curve", stress
241.2
NOTE: the first stress entry must be zero.
NOTE: the second stress entry (the first non-zero entry)
must have the value: stress(2) = stress(proportional limit).

242.1 any more (strain, stress) pairs (Y or N)?

(lines skipped to save space)

270.1 Does cold bending include the outstanding ring flange?
270.2
If there are no outstanding ring flanges, answer "N". If the final fabricated panel or shell has rings with outstanding flanges, then read on.

The cylindrical shell may be cold bent before outstanding flanges are welded on to the tips of the ring webs, or the outstanding ring flanges may be present during the cold bending process. If only the ring webs exist when the cold bending process takes place, then the analysis of buckling during the cold-bending process will be undertaken with the assumption that the flanges are made of very, very soft material and experience no prebuckling compression at all.

271.1 ring web radial compression, FN1WEB
271.2
When the flat plate with "hogged out" pockets is formed into a cylindrical panel by cold bending, there may, during the cold-bending process, occur a compressive radial resultant, FN1WEB, generated. FN1WEB (units=force/length) is the maximum radial compression in a ring web during the cold-bending process.

It is best for now to use zero: FN1WEB = 0.

An example of the input file for BEGIN appropriate for a case in which cold-bending will be simulated follows (*.BEG file):
$ Do you want a tutorial session and tutorial output?
$ Panel length normal to the plane of the screen, L1
$ Panel length in the plane of the screen, L2
$ Identify type of stiffener along L1 (N,T,J,Z,R,A,C,G)
$ stiffener spacing, b
$ width of stringer base, b2 (must be > 0, see Help)
$ height of stiffener (type H for sketch), h
$ Are the stringers cocured with the skin?
$ What force/(axial length) will cause web peel-off?
$ Is the next group of layers to be a "default group" (12 layers!)?
$ number of layers in the next group in Segment no.( 1)
$ Can winding (layup) angles ever be decision variables?
$ layer index (1,2,...), for layer no.( 1)
$ Is this a new layer type?
$ thickness for layer index no.( 1)
$ winding angle (deg.) for layer index no.( 1)
$ material index (1,2,...) for layer index no.( 1)
$ Any more layers or groups of layers in Segment no.( 1)
$ Is the next group of layers to be a "default group" (12 layers!)?
$ number of layers in the next group in Segment no.( 2)
$ Can winding (layup) angles ever be decision variables?
$ layer index (1,2,...), for layer no.( 1)
$ Is this a new layer type?
$ Any more layers or groups of layers in Segment no.( 2)
$ Is the next group of layers to be a "default group" (12 layers!)?
$ number of layers in the next group in Segment no.( 3)
$ Can winding (layup) angles ever be decision variables?
$ layer index (1,2,...), for layer no.( 1)
$ Is this a new layer type?
$ thickness for layer index no.( 2)
$ winding angle (deg.) for layer index no.( 2)
$ material index (1,2,...) for layer index no.( 2)
$ Any more layers or groups of layers in Segment no.( 2)
$ Is the next group of layers to be a "default group" (12 layers!)?
$ number of layers in the next group in Segment no.( 3)
$ Can winding (layup) angles ever be decision variables?
$ layer index (1,2,...), for layer no.( 1)
$ Is this a new layer type?
$ Are the rings cocured with the skin?
$ Is the next group of layers to be a "default group" (12 layers!)?
$ number of layers in the next group in Segment no.( 3)
$ Can winding (layup) angles ever be decision variables?
$ layer index (1,2,...), for layer no.( 1)
$ Is this a new layer type?
$ Is the panel curved in the plane of the screen (Y for cyls.)?
$ Radius of curvature (cyl. rad.) in the plane of screen, R
$ Is panel curved normal to plane of screen? (answer N)
$ Is this material isotropic (Y or N)?
$ Young's modulus, E( 1)
$ Poisson's ratio, NU( 1)
$ transverse shear modulus, G13( 1)
$ Thermal expansion coeff., ALPHA( 1)
0 $ residual stress temperature (positive), TEMPTUR( 1)
0 $ Want to supply a stress-strain "curve" for this mat'l (H)?---
y $ any more (strain, stress) pairs (Y or N)? N
0.636360E-02 $ strain coordinate for stress-strain "curve", strain( 2)
y $ any more (strain, stress) pairs (Y or N)? E
70000.00 $ stress coordinate for stress-strain "curve", stress( 2) I
100000.0 $ stress coordinate for stress-strain "curve", stress( 3) U
0.1909090E-01 $ strain coordinate for stress-strain "curve", strain( 4)
y $ any more (strain, stress) pairs (Y or N)? A
0.000000 $ ring web radial compression, FN1WEB( 1) A
70000.00 $ Maximum allowable effective stress in material type( 1)
n $ Do you want to take advantage of "bending overshoot"?
0.9500000E-01 $ weight density (greater than 0!) of material type( 1)
 n $ Is lamina cracking permitted along fibers (type H(elp))?
y $ Is this material isotropic (Y or N)?
70000.00 $ Maximum allowable effective stress in material type( 2)
n $ Do you want to take advantage of "bending overshoot"?
0.9500000E-01 $ weight density (greater than 0!) of material type( 2)
 n $ Is lamina cracking permitted along fibers (type H(elp))?
y $ Is this material isotropic (Y or N)?
0.1100000E+08 $ Young's modulus, E( 2)
0.3000000 $ Poisson's ratio, NU( 2)
4230769. $ transverse shear modulus, G13( 2)
0 $ Thermal expansion coeff., ALPHA( 2)
 n $ residual stress temperature (positive), TEMPTUR( 2)
y $ Want to supply a stress-strain "curve" for this mat'l? (N)
y $ Want to specify maximum effective stress ?
70000.00 $ Maximum allowable effective stress in material type( 3)
n $ Do you want to take advantage of "bending overshoot"?
0.9500000E-01 $ weight density (greater than 0!) of material type( 3)
n $ Is lamina cracking permitted along fibers (type H(elp))?
y $ Is this material isotropic (Y or N)?
0.1100000E+08 $ Young's modulus, E( 3)
0.3000000 $ Poisson's ratio, NU( 3)
4230769. $ transverse shear modulus, G13( 3)
0 $ Thermal expansion coeff., ALPHA( 3)
 n $ residual stress temperature (positive), TEMPTUR( 3)
y $ Want to supply a stress-strain "curve" for this mat'l? (N)
y $ Want to specify maximum effective stress ?
70000.00 $ Maximum allowable effective stress in material type( 3)
n $ Do you want to take advantage of "bending overshoot"?
0.9500000E-01 $ weight density (greater than 0!) of material type( 3)
n $ Is lamina cracking permitted along fibers (type H(elp))?
0 $ Prebuckling: choose 0=bending included; 2=use membrane theory
1 $ Buckling: choose 0=simple support or 1=clamping

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Note on stress-strain curve for Material Type 1
It is by providing the stress-strain curve for Material Type 1 that
the PANDA2 user is "telling" PANDA2 to perform the various
cold-bending ring buckling analyses, if appropriate.
********************************************************************************
The input for DECIDE and MAINSETUP remains unchanged.

It is appropriate to conduct cold-bending ring buckling analyses only under the following conditions:

1. The entire structure must be fabricated from a single isotropic material. As described in the modified PROMPT.DAT file above, the PANDA2 user can still use multiple materials in order to generate different stress constraints corresponding to different segments of the structure, but all these materials must have the same isotropic elastic properties and only Material Type 1 has the stress-strain curve. IT IS ASSUMED THAT THE STRESS-STRAIN CURVE FOR MATERIAL TYPE 1 IS THE SAME FOR TENSION AND FOR COMPRESSION. If your material behaves differently in tension and compression, use the compression curve for input data to PANDA2.

2. The rings must be internal and must have either rectangular or Tee-shaped cross sections.

3. The base under the ring has no faying flange. In other words, the base under the ring has the same dimensions and properties as the panel skin midway between rings.

Two types of cold-bending fabrication are covered for shells with rings that have an outstanding flange:

1. The cold-bending process may occur for rings with outstanding flanges present.

2. The cold-bending process occurs before the outstanding flanges are welded to the tips of the ring webs. In this case the simulation of the cold-bending process occurs for a ring with an outstanding flange, but the elastic modulus of this flange is set equal to \(FMULT \times EELAST\), in which \(FMULT\) is a very small number and \(EELAST\) is the Young's modulus, and the prebuckling compression in the outstanding ring flange due to cold bending is set equal to zero.

The cold-bending ring buckling capability was incorporated into PANDA2 by modifying the following source libraries: arrays.src, begin.src, stoget.src, and store.src. The prompting file, ...panda2/execute/PROMPT.DAT was modified as described above.

In order to incorporate the cold-bending simulation in PANDA2 new dimension statements and labeled common blocks were added, and a rather long section of coding was added to SUBROUTINE STRUCT. This new coding follows:

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(lines skipped to save space)
C BEG APR 2009

COMMON/CIJCLD/CXCOLD(6,6,5),CYCOLD(6,6,5),CY3CLD(6,6,11),
1              CXCLD0(6,6,5),CXCLD1(6,6,5)
COMMON/CNVARX/CNXVAR(23,8),CNYVAR(23,8)
COMMON/NCRITX/NCRIT1,NCRIT2,NCRIT3,NCRITA,NCRITB,NCRITC
COMMON/NCRITY/NCRIT4,NCRITD,NCRIT5,NCRITE
COMMON/LOCATR/ILOCPR,ILOCBR,IRWCPR(98),IRWCBR(98),IWXPR,IIWBR
COMMON/NCONDX/NCOND
COMMON/RING3R/D1R(98),D2R(98)
COMMON/JUNCTR/IFIXBR(588),IFXBR(588),ITYPER(98)
COMMON/FREEDG/IFREE
COMMON/ORBA/IFLGG,KTM,KROOTS
COMMON/COLBND/FLGCLD,FN1WEB,RCOLD
COMMON/YCOLDX/YCOLD1(400),YCOLD2(400)
DIMENSION DUMMYA(1000),DIAGR(*),EIGCLD(50),YCOLD(400),YCOLDS(400)
DIMENSION WDDD9(23,8),WFLANG(2),WFLNG0(2),B2COLD(2),B2CLD0(2)
DIMENSION WDDD10(23,8),WDDD11(23,8),BCOLD(2),BCOLD0(2)
COMMON/MODL11/WW9(23,8),WD9(23,8),WDD9(23,8),
1              UU9(23,8),VV9(23,8),VP9(23,8)
COMMON/MODL12/WW10(23,8),WD10(23,8),WDD10(23,8),
1              UU10(23,8),VV10(23,8),VP10(23,8)
COMMON/MODL13/WW11(23,8),WD11(23,8),WDD11(23,8),
1              UU11(23,8),VV11(23,8),VP11(23,8)
COMMON/IFCTXX/IFCT13,IFCT14,IFCT15
CHARACTER*3 CCN
CHARACTER*7 CCN2
C END APR 2009

(lines skipped to save space)

C BEG APR 2009

shells with INTERNAL rings with rectangular or,
Tee-shaped cross sections. The entire shell must',
be fabricated of the same isotropic material.',
See Item No. 790 in ...panda2/doc/panda2.news.

CHAPTER 27 Compute the objective function (e.g. WEIGHT).
CHAPTER 28 Present design, loading, and margins for the',
current load set and subcase. (See Table 6 in [18])
Tee-shaped cross sections. The entire shell must be fabricated of the same isotropic material.
See Item No.790 in ...panda2/doc/panda2.news.

CALL COLDBD(IFILE, NPRT, IMOD, ILABEL, ILOADS, ICASE, INUMTT, ICAR,
PCWORD, CPLOT, IADDC, FSASFEP, CONMAX, IPOINC, ICONST, CONSTR, WORDB,
MAXCON, ITYPE, CXCOLD, CYCOLD, CY3CLD, CXCLD0, CXCLD1,
CNXVAR, CNYVAR, WAVCLD, KAYER(1,1), ISBCLD)

WFLANG(1) = W(1)
WFLANG(2) = W(2)
WFLNG0(1) = WW0(1)
WFLNG0(2) = WW0(2)
BCOLD(1) = B(1)
BCOLD(2) = B(2)
B2COLD(1) = B2(1)
B2COLD(2) = B2(2)
B2CLD0(1) = B20(1)
B2CLD0(2) = B20(2)

IF (FLGCLD.LT.0.1) WFLANG(2) = 0.01*W(2)
IF (FLGCLD.LT.0.1) WFLNG0(2) = 0.01*WW0(2)

CIRCLD = MIN(CIRC, 5.0*WAVCLD)
EIGCLM = 10.E+16
IF (NPRT.GE.0) WRITE(IFILE,'(/,A,/,A,/,A)')

***** BEGIN DISCRETIZED "SKIN"-RING MODULE MODEL OF *****
***** COLD-BENDING BUCKLING OF RING (smeared stringers) *****
***** This is Model No. 2 of cold-bending ring buckling *****

IF (IMOD.EQ.0) THEN

DO 9272 NAVE = 1,10

DUMMY = 0.
CALL MOVER(0., 0, DUMMYA, 1, 1000)
IPRINT = 0

CALL ARYS2(IFILE, IWR, ILOC, DSR, NWAVE, ASR, BSR, R, CIRCLD,
DUMMY, 0, 0, DUMMYA, DUMMYA, DUMMYA, DUMMYA,
DUMMYA, DUMMYA, DUMMYA, DUMMYA, DUMMYA, IPRINT, 1, IMOD,
DUMMY, ILABEL,
B, B2, B0, B20, H, H0, WFLANG, WFLNG0, W2, W20, IZSTIF, ISTIF,
NSEGR, I5R, M3R, NCONDR, D1R, D2R, IFIXBR,
IFXBR, ITYPER, IMAXBR, KMAXBR, ILRBR,
IRWCBR, IWB, IDR, NBLK, NGBKR, NKF,
DUMMYA, DUMMYA, DUMMYA, DUMMYA, DUMMYA, DUMMYA,
CXCOLD, 1, WFOUND, ISANDW, NLAYER, IFREE,
KROOTS, TX0, DUMMYA,
CALL EBAND2(IFILE, 0, NWAVE, ILOCR, DIAGR, ASR, BSR, CRX, DRX, DIR, 
XR, YR, ZR, 0, M3R, NBLKR, IDRWR, NGBKR, NKFR, IMAXBR, KMAXBR, 1, 
IPV2R, IBCVR, 1000, 2, 1, 0, -1.E-16, DUMMY1, IFLGG, KTM, KROOTS)

EIGCLD(NWAVE) = ROOTX(1)

EIGCLM = MIN(EIGCLM, ABS(EIGCLD(NWAVE)))

IF (NWAVE.GT.1) CALL TRANS3(M3R, YCOLD, YCOLDS)
CALL TRANS1(M3R, YR, YCOLD)
NWAVE1 = NWAVE - 1
IF (NWAVE.GT.1.AND.EIGCLD(NWAVE).GT.EIGCLD(NWAVE1)) GO TO 9273
CONTINUE
9273 CONTINUE
EIGCLM = ABS(EIGCLD(NWAVE1))
NCRITA = NWAVE1*CIRC/CIRCLD
NCRIT1 = NWAVE1
WAVLEN = CIRC/FLOAT(NCRITA)
CALL TRANS3(M3R, YCOLDS, YCOLD1)

IF (ISTIF(1).NE.0.AND.IMOD.EQ.0.AND.NPRT.GE.0) THEN
  WRITE(IFILE,'(/,A,1P,E12.4,/,A,1P,E12.4)')
  ' circumferential spacing of the stringers=', B(1),
  ' circumferential half-wavelength of the critical buckling mode=', WAVLEN
ENDIF

CALL TRANS2(M3R, YCOLDS, YR)
IF (NPRT.GE.0) WRITE(IFILE, '(/,A)')
1
**** BEGIN SUB. MODE ("SKIN"-RING MODULE COLD BENDING 1) ****
CALL MODE(IFILE, NPRT, NSEGR, I5R, IWR, DSR, M3R, YR, ISKN13, 1, 
2, ZPARTY, 0., CYCOLD, 1, WPRES, EIGCLM, 
RMAX, ITIPPL, ICWBRC, IMOD, 
WW9, WD9, WDD9, UU9, VP9, VV9, ZREFRG, NWAVE1, FKNSRG(1), 
FKNDDM, ICRNRG, P, IFCT13, WDDD9, 14, 0, TY, 2, 1, 
INTSNG, RAD2, TY, IZSTIF, B, B2, H, WFLANG, W2, 
B0, B20, H0, WFLNG0, W20, ISTIF, INTEXT, IFAY, IBEAM, IONEST,
IF (IFCT13.NE.0) THEN
  WRITE(IFILE,'(/,A,/,A,/,A,/,A,/,A,/)')
  ' ******* NOTE ******* NOTE ******* NOTE ******',
  ' Since the mode is FICTITIOUS, the discretized',
  ' "skin"-ring module cold-bending buckling model',
  ' will not be used.',
  ' ****** END NOTE *** END NOTE **** END NOTE ***'
ENDIF

ELSE
  End of (IMOD.EQ.0) condition

  DUMMY = 0.
  CALL MOVER(0.,0,DUMMYA,1,1000)
  IPRINT = 0

  CALL ARRS2(IFILE,IWR,ILOCR,DSR,NCRIT1,ASR,BSR,R,CIRCLD,
    DUMMYA,0,0,0,DUMMYA,DUMMYA,DUMMYA,DUMMYA,
    DUMMYA,DUMMYA,DUMMYA,DUMMYA,DUMMYA,DUMMYA,
    B,B2,B0,B20,H,H0,WFLANG,WFLNG0,W2,W20,IZSTIF,ISTIF,
    NSEGR,I5R,M3R,NCONDR,D1R,D2R,IFIXBR,
    IFXBR,ITYPER,IMAXB,IMAXB,ILOCR,
    IRWCBR,IIWBR,IDRWR,NBLKR,NGBK,R,NKFR,
    DUMMYA,DUMMYA,DUMMYA,DUMMYA,DUMMYA,DUMMYA,
    CXCOLD,1,WFOUN,D,ISDNL,ISDL,NLABEL,
    IR,CRX,DRX,DIR,1
  IPRN = 0
  CALL TRANS2(M3R,YCOLD1,YR)
  CALL EBAND2(IFILE,0,NCRIT1,ILOCR,DIAGR,ASR,BSR,CRX,DRX,DIR,
    XR,YR,ZR,1,M3R,NBLKR,IDRWR,NGBK,R,NKFR,1,1,
    IPV2R,IBVCR,1000,2,1,0,-1.E-16,DUMMYA,IFLGG,KTM,KROOTS)

  EIGCLM = ROOTX(1)

  IF (NPRT.GE.2) WRITE(IFILE,'(A,1P,E12.4,A,I3,/,A,1P,E12.4,A)')
  ' circumferential waves over the circ.length=',CIRCLD,'=','NCRIT1,
  ' ring cold buckling load factor from a discretized module=',
  ' EIGCLM,' (smeared strings')

ENDIF
End of IMOD condition

C Constraint condition for ring cold-bending buckling (yes stringers):
   IF (IFCT13.NE.0) GO TO 9276
   INUMTT = INUMTT + 1
   FSAFTY = 1.1
   CALL CONVRF(FSAFTY,CCN2)
   CALL CONVRT(NCRITA,CCN)
   IF (IMOD.EQ.0.AND.(EIGCLM/FSAFTY).LT.MAXCON) THEN
      ICAR = ICAR + 1
      PCWORD(ICAR) = 1 'Cold-bending ring buckling, "skin"-ring module'
      CPLOT(ICAR) = EIGCLM/FSAFTY - 1.
      IADDCC(ICAR) = 0
      FSAFEP(ICAR) = FSAFTY
   ENDIF
   IF (IMOD.EQ.0.AND.(EIGCLM/FSAFTY).GT.CONMAX) GO TO 9276
   IF (IMOD.EQ.1.AND.IPOINC(INUMTT).EQ.0) GO TO 9276
   ICONST = ICONST + 1
   IF (IMOD.EQ.0) IPOINC(INUMTT) = 1
   CONSTR(ICONST) = EIGCLM/FSAFTY
   WORDB(ICONST) = 1 'Cold-bending ring buckling, "skin"-ring module; N='//CCN//' 1 'Margin=',CONSTR(ICONST)-1.,WORDB(ICONST)
   IF (NPRT.GE.2) WRITE(IFILE,'(A,,A,,A,,A,I3,A,I2,A,I2)')
      ' *** THE FOLLOWING COLD-BENDING BUCKLING MARGIN JUST COMPUTED:', 1 'CONSTRAINT NO.',ICONST,'; LOAD SET NO.',ILOADS,';
      'SUBCASE NO.',ICASE
   9276 CONTINUE
      IF (ITYPE.EQ.1.AND.NPRT.GE.2)
       WRITE(IFILE,*') AFTER 4440 C: IMOD,INUMTT,ICONST=',
       IMOD,INUMTT,ICONST
   C
   IF (NPRT.GE.0) WRITE(IFILE,*') End of computation of cold-bending ring buckling load factor', 1' in SUBROUTINE STRUCT from "skin"-ring discretized module.', 1'******************************************************************************'
   C
   IF (ISTIF(1).EQ.0) GO TO 9284
   C
   CIRCLD = MIN(CIRC,10.0*WAVCLD)
   CIRCLD = MIN(CIRCLD,B(1))
   EIGCLM = 10.E+16
   IF (NPRT.GE.0) WRITE(IFILE,'(/,A,,A,,A,,A)')
IF (IMOD.EQ.0) THEN

DO 9277 NWAVE = 1,10

DUMMY = 0.
CALL MOVER(0.,0.,DUMMYA,1,1000)
IPRINT = 0

CALL ARRS2(IFILE,IWR,ILOCR,DSR,NWAVE,ASR,BSR,R,CIRCLD,
1 DUMMYA,0,0,0,DUMMYA,DUMMYA,DUMMYA,DUMMYA,
1 DUMMYA,DUMMYA,DUMMYA,DUMMYA,DUMMYA,IPRINT,1,IMOD,
1 DUMMY,ILABEL,
1 B,B2,B0,B20,H,H0,WFLANG,WFLNG0,W2,W20,T2STIF,ISTIF,
1 NSEGR,T5R,M3R,NCONDR,D1R,D2R,IFIXBR,
1 IFXBR,ITYPER,IMAXBR,KMAXBR,ILOCRBR,
1 IRWCRBR,IIWBR,IDRWR,NBLKR,NGBK,R,NKFR,
1 DUMMYA,DUMMYA,DUMMYA,DUMMYA,DUMMYA,DUMMYA,
1 CXCLD1,1,WFOUND,ISANDW,NLAYER,IFREE,
1 KROOTS,TX0,DUMMYA,
1 CY3CLD,INSRNG,IFAY,TY0,CYCOLD,NPRT,DUMMY, DUMMY,
1 0,DUMMYA,DUMMYA,DUMMYA,DUMMYA,0,PEDG,
1 CNXVAR,CNYVAR,DUMMYA,DUMMYA,DUMMYA,DUMMYA,DUMMYA)

CALL EBAND2(IFILE,0,NWAVE,ILOCR,DIAGR,ASR,BSR,CRX,DRX,DIR,
1 XR,YR,ZR,0,M3R,NBLKR,IDRWR,NGBK,R,NKFR,IMAXBR,KMAXBR,1,
1 IPV2R,IBVCR,1000,2,1,0,-1.E-16,DUMMY,IFLGG,KTM,KROOTS)

EIGCLD(NWAVE) = ROOTX(1)

IF (NPRT.GE.0) WRITE(IFILE,'(A,1P,E12.4,A,1P,E12.4,A)')
1 ' circumferential waves over the circ.length=' ,CIRCLD,'=',NWAVE,
1 ' ring cold buckling load factor from a discretized module=',
1 EIGCLD(NWAVE)', (no stringers)'

EIGCLM = MIN(EIGCLM,ABS(EIGCLD(NWAVE)))

IF (NWAVE.GT.1) CALL TRANS3(M3R,YCOLD,YCOLD)
CALL TRANS1(M3R,YR,YCOLD)
NWAVE1 = NWAVE - 1
IF (NWAVE.GT.1.AND.EIGCLD(NWAVE).GT.EIGCLD(NWAVE1)) GO TO 9278

C
9277 CONTINUE
9278 CONTINUE
EIGCLM = ABS(EIGCLD(NWAVE1))
\begin{verbatim}
NCRITB = NWAVE1*CIRC/CIRCLD
NCRIT2 = NWAVE1
WAVLEN = CIRC/FLOAT(NCRITB)
CALL TRANS3(M3R,YCOLDS,YCOLD2)

C
IF (ISTIF(1).NE.0.AND.IMOD.EQ.0.AND.NPRT.GE.0) THEN
  WRITE(IFILE,'(/,A,1P,E12.4,/,A,1P,E12.4)')
  1' circumferential spacing of the stringers= ', B(1),
  1' circumferential half-wavelength of the critical buckling mode=', WAVLEN
ENDIF

C
CALL TRANS2(M3R,YCOLDS,YR)
IF (NPRT.GE.0) WRITE(IFILE,'(/,A)')
  1' **** BEGIN SUB. MODE (SKIN-RING MODULE COLD BENDING 2) ****'
  CALL MODE(IFILE,NPRT,NSEGR,I5R,IWR,DSR,M3R,YR,ISKN14,1,
            1 2, ZPARTY,0.,YCOLD,1,WPRES,EIGCLM,
            1 RMAX,ITIPPL,ICWBRG,IMOD,
            1 WW10,WD10,WDD10,UU10,VP10,ZREFRG,NWAVE1,FKNSRG(1),
            1 INTSNG,RAD2,TY,IZSTIF,B,B2,H,WFLANG,W2,
            1 B0,B20,H0,WFLNG0,W20,ISTIF,INTEXT,IFAY,IBEAM,IONEST,
            1 AXIAL,ICRIP,ISEGC,WRATIO,WTIPWS,WRATTP,
            1 WRATC,WRATCN,WRWIDE,WRATCN,ISOGRD,RESULT,IFLGPP)
C
  IF (IFCT14.NE.0) THEN
    WRITE(IFILE,'(/,A,/,A,/,A,/,A,/,A,/)
    1' ******* NOTE ******* NOTE ******* NOTE ******',
    1' Since the mode is FICTITIOUS, the discretized',
    1' skin-ring module cold-bending buckling model',
    1' will not be used.',
    1' ***** END NOTE *** END NOTE **** END NOTE ***'
  ENDIF
C
ELSE
End of (IMOD.EQ.0) condition
C
DUMMY = 0.
CALL MOVER(0.,0,DUMMYA,1,1000)
IPRINT = 0
C
CALL ARrys2(IFILE,IWR,ILOCR,DSR,NCRIT2,ASR,BSR,R,CIRCLD,
             1 DUMMYA,0,0,0,DUMMYA,DUMMYA,DUMMYA,DUMMYA,
             1 DUMMYA,DUMMYA,DUMMYA,DUMMYA,DUMMYA,IPRINT,1,IMOD,
             1 DUMMY,ILABEL,
             1 B,B2,B0,B20,H,H0,WFLANG,WFLNG0,W2,W20,IZSTIF,ISTIF,
             1 NSEGR,I5R,M3R,NCONDR,D1R,D2R,IFIXBR,
\end{verbatim}
CALL TRANS2(M3R,YCOLD2,YR)
CALL EBAND2(IFILE,0,NCRIT2,ILOCR,DIAGR,ASR,BSR,CRX,DRX,DIR,
    XR,YR,ZR,1,M3R,NBLKR,IDRWR,NGBKR,NKFR,IMAXBR,KMAXBR,1,
    IPV2R,IBVCR,1000,2,1,0,-1.E-16,DUMMY,IFLGG,KTM,KROOTS)

C
IF (NPRT.GE.0) WRITE(IFILE,'(/,A)')
1' **** BEGIN SUB. MODE (SKIN-RING MODULE COLD BENDING 2) ****'
   CALL MODE(IFILE,NPRT,NSEGR,I5R,IWR,DSR,M3R,YR,ISKN14,1,
       2,ZPARTY,0.,CYCOLD,1,WPRES,EIGCLM,
       1RMAT,ITIPPL,ICWBRG,IMOD,
       1WW10,WD10,WDD10,UU10,VP10,ZREFRG,NWAVE1,FKNSRG(1),
       1FKNDUM,ICRNNG,F,IFCT14,WDD10,15,0,TY,2,1,
       1INTSNG,RAD2,TY,IZSTIF,B,B2,H,WFLANG,W2,
       1B0,B20,H0,WFLNG0,W20,ISTIF,INTEXT,IFAY,IBEAM,IONEST,
       1AXIAL,CIRC,ICRIP,ISEGC,WRATIO,WTIPWS,WRATTP,
       1WRATWB,WRATCN,WRWIDE,WDLMM,PEDG,ISOGRD,RESULT,IFLGP)
C
   EIGCLM = ROOTX(1)
C
   IF (NPRT.GE.2) WRITE(IFILE,'(A,1P,E12.4,A,I3,/,A,1P,E12.4,A)')
   1' circumferential waves over the circ.length=',CIRCLD,'=',NCRIT2,
   1' ring cold buckling load factor from a discretized module=',
   1EIGCLM,' (no stringers)'
C
ENDIF
C
C Constraint condition for ring cold-bending buckling (no stringers):
   IF (IFCT14.NE.0) GO TO 9282
   INUMTT = INUMTT + 1
   FSAFTY = 1.1
   CALL CONVRF(FSAFTY,CCN2)
   CALL CONVRT(NCRITB,CCN)
   IF (IMOD.EQ.0.AND.(EIGCLM/FSAFTY).LT.MAXCON) THEN
       ICAR = ICAR + 1
       PCWORD(ICAR) =
       1'Cold-bending ring buckling, skin-ring module'
       CPLOT(ICAR) = EIGCLM/FSAFTY - 1.
Furthermore, three new subroutines were added to the struct.src library. A list of these three new subroutines follows:

C=DECK

SUBROUTINE COLDBD(IFILE,NPRT,IMOD,ILABEL,ILOADS,ICASE,INUMTT, ICAR,PCWORD,CPOST,IADDCC,FSAFEP,CONMAX,IPOINC,ICONST,CONSTR, WORDB,MAXCON,ITYPE,CXCOLD,CYCOLD,CY3CLD,CXCLD0,CXCLD1, CNXVAR,CNYVAR,WAVLEN,KLAYER,ISUB)
This subroutine is entered only if the rings are internal, have rectangular or Tee-shaped cross section, and if a stress-strain curve has been provided for material type 1. If the fabricated shell has Tee-shaped rings but the outstanding flange was welded on after the cold-bending process, then the outstanding flange is present in the model but its stiffness is reduced by a very small factor, $FMULT$, and its prebuckling compression is zero.

**Purpose:** To construct a constraint condition for buckling of an internal ring with a rectangular or tee-shaped cross section under the cold-bending process.

**Note:** No faying flange (ring base of width $B2(RNG)$ thicker than the panel skin) is accounted for in this model.

**Input data:**
- `IFILE` = write out to file, `IFILE`
- `NPRT` = index for printing verbosity
- `IMOD` = 0 for current design; 1 for perturbed design
- `ILABEL` = statement label in SUBROUTINE STRUCT where SUBROUTINE COLDBD is called.
- `ILOADS` = load set number
- `ICASE` = load subcase number (1 or 2)
- `FSAFE` = factors of safety associated with design constraints
- `ITYPE` = 1 for optimization, 2 for analysis of "fixed" design
- `SIM`, `EIM` = coordinates of the material stress-strain curve
- `RCOLD` = minimum radius of curvature in the cold-bending process
- `FN1WEB` = stress resultant in the ring web in the x-direction (FN1WEB = 0 should probably be used as of this writing).
- `FLGCLD` = 0.0 = outstanding flange not included in the cold bending process
- 1.0 = outstanding flange is included in the cold bending process
- `B` = ring spacing
- `H` = ring web height
- `WFLANG` = width of outstanding ring flange
- `TSKIN` = thickness of shell skin
- `TWEB` = thickness of ring web
- `TFLANG` = thickness of ring flange
- `EELAST` = elastic modulus
- `KLAYER` = 1 there are substiffeners
- 0 there are no substiffeners

**Important data:**
- `BUCKLE` = load factor for buckling of the ring under cold bending.
- `WAVLEN` = wavelength of the critical buckling mode, hoop direction
Output data:

- \texttt{INUMTT,ICAR,PCWORD,CPLLOT,IADDCC,CONMAX,IPOINC,ICONST,CONSTR,}
- \texttt{WORDB,MAXCON} = quantities related to the design constraint from the "closed-form" solution.

- \texttt{CXCOLD(i,j,5)} = integrated constitutive matrices for the cold-bent state for the skin-stringer module for
  - \texttt{CXCOLD(i,j,1)} = skin with smeared substiffeners
  - \texttt{CXCOLD(i,j,2)} = skin + stringer faying flange + smeared substiffeners
  - \texttt{CXCOLD(i,j,3)} = stringer web
  - \texttt{CXCOLD(i,j,4)} = stringer outstanding flange
  - \texttt{CXCOLD(i,j,5)} = skin with smeared major stringers and smeared substiffeners

- \texttt{CYCOLD(i,j,5)} = integrated constitutive matrices for the cold-bent state for the "skin"-ring module for
  - \texttt{CYCOLD(i,j,1)} = skin with smeared substiffeners
  - \texttt{CYCOLD(i,j,2)} = \texttt{CYCOLD(i,j,1)} (no ring fay-flange)
  - \texttt{CYCOLD(i,j,3)} = ring web
  - \texttt{CYCOLD(i,j,4)} = ring outstanding flange
  - \texttt{CYCOLD(i,j,5)} = skin with smeared major rings and smeared substiffeners.

- \texttt{CY3CLD(i,j,k)} = integrated constitutive matrices for the cold-bent state for the ring web at k points along the web from root to tip, including the ring web root and the ring web tip.

- \texttt{CXCLD0(i,j,5)} = same as \texttt{CXCOLD(i,j,5)} except that \texttt{CXCLD0(i,j,1)} and \texttt{CXCLD0(i,j,5)} are for the panel skin alone, no stringers and no substringers or subrings.

- \texttt{CXCLD1(i,j,5)} = same as \texttt{CXCLD1(i,j,1)}

- \texttt{CNXVAR(i,j)} = meridional resultant for nodal point i, segment j in the "skin"-ring module

- \texttt{CNYVAR(i,j)} = hoop resultant for nodal point i, segment j in the "skin"-ring module

First, we must find the prebuckled state. Then we can determine the stability stiffness and load-geometric matrices.

We assume here that the stringers have no effect on the process, since they do not bend under the cold-bending process. Also, we assume that the height, H, of the web is small compared...
We use a skin-ring module as our geometry. This module has three parts:

- Part 1: Shell skin of axial length equal to the ring spacing $b$
- Part 2: Internal ring web of height $h$
- Part 3: Outstanding flange of the ring of width $w$

NOTE: It is assumed here that there is no thickened ring base.

The geometry is:

![Diagram of the geometry](image)

Cold bending puts the flange of width $w$ in hoop compression. In the sketch above, "d" is the distance from the middle surface of the panel skin to the neutral axis.

PREBUCKLING BEHAVIOR

It is assumed that the elastic-plastic material is isotropic and has the same stress-strain curve in tension and compression.
The theory used in this subroutine is taken from the paper, Bushnell, David, "Theoretical basis of the panda computer program for preliminary design of stiffened panels under combined in-plane loads", Computers & Structures, Vol. 27, No. 4, pp. 541-563, 1987.

For example, Eqs.(1-3) below are Eqs.(33,34) on p. 550 of that paper. The formulas for the instantaneous stiffnesses used here are Eqs.(42-45) on p. 551 of that paper.

This subroutine is based on J2 deformation theory. As of this writing, we assume that the stress generated during the cold-bending process is uniaxial: only hoop stress exists. Therefore, we assume that the stress at any point in the skin-ring module cross section is given by $E \times$ (hoop strain), in which $E$ is the hoop plastic modulus:

$$E = E_{22}; \quad E_{22} = a/d\ell; \quad d\ell = a^2 - b^2 \quad (1)$$

in which

$$a = (1 + 2g/3)/E(\text{elastic}); \quad b = -(\nu + g/3)/E(\text{elastic}) \quad (2)$$

and

$$g = 1.5*(E(\text{elastic})/E_s - 1). \quad (E_s = \text{the secant modulus}) \quad (3)$$

The hoop strain, call it $e$, is assumed to vary linearly over the skin-ring module from the middle surface of the internal ring flange to the middle surface of the shell skin. For simplicity, it is assumed that the hoop strain is uniform in the ring flange and in the shell skin and equal to the values at the middle surfaces of those parts. The hoop strain is uniform over the thickness of the ring web but varies linearly over the height of the ring web.

As the flat, hogged out plate is bent into a cylindrical form, the neutral axis for circumferential bending, located the distance "d" from the skin middle surface as indicated in the sketch above, shifts as plastic flow occurs over growing portions of the skin-ring module cross section with increasing circumferential bending. Therefore, even though we are using deformation theory, we must simulate the cold-bending process incrementally. Iterations will be needed for each bending increment because we do not know at the
C start of increment i exactly where the neutral axis is for that
C increment. For the first iteration at cold-bending increment i
C we assume that the neutral axis is at the same location as it was
C at the end of increment i - 1. That assumption yields an initial
C estimate of the distribution of plastic modulus over the
C skin-ring cross section. From this initial estimate we compute
C a new location d of the neutral axis. That new value of d leads
C to a new and better estimate of the distribution of plastic
C modulus, etc. Iterations continue until d no longer changes
C a significant amount from that obtained in the previous
C iteration.

C The location, d, of the neutral axis is computed from:
C
C \[ d = \frac{E(\text{flange})A(\text{flange})[h+t(\text{skin})/2] + t(\text{web})\int E(\text{web})x \, dx}{E(\text{skin})A(\text{skin}) + E(\text{flange})A(\text{flange}) + t(\text{web})\int E(\text{web}) \, dx} \]  (4)

C in which d = distance from the shell skin middle surface to
C the location of the neutral axis for circumferential bending.
C E(\text{flange}), E(\text{web}), and E(\text{skin}) are the plastic moduli of the
C flange, web, and shell skin, respectively. A(\text{flange}) and
C A(\text{skin}) are the areas of the flange and skin cross sections, that
C is, A(\text{flange}) = t(\text{flange})w(\text{flange}) and A(\text{skin}) = t(\text{skin})b(\text{ring}),
C in which w(\text{flange}) is the width of the flange, b(\text{ring}) is the
C spacing between adjacent rings, and t(\text{flange}) and t(\text{skin}) are
C the flange and skin thicknesses, respectively. "int[ ]" means
C "integral", and the radial coordinate, x, is measured from
C the shell skin middle surface and increases radially inward,
C as is shown in the sketch above.
C
C Simpson's rule is used to perform the integrations.
C
C Once we have this new estimate of the location of the neutral
C axis, we can compute the distribution of incremental hoop strain
C from the ith cold-bending increment. At each location in the
C skin-ring module cross section the total hoop strain is given
C by the sum of the total strain from the previous converged cold-
C bending increment plus the incremental strain. From the new hoop
C strains we know where we are on the uniaxial stress-strain
C curve and therefore we can compute new values for the plastic
C moduli, E(\text{flange}), E(\text{web}), and E(\text{skin}), From Eqs. (1 - 3).
C
C The total hoop strain for the ith cold-bending increment is
C given by:
C \[ e(\text{total})(i) = e(\text{total})(i-1) - (x-d)\kappa(i) \]  (5)
C in which
C \[ \kappa(i) = 1/R(i) - 1/R(i-1) \]  (6)
where $R(i)$ is the known radius of curvature at the neutral axis at the $i$th cold-bending increment and $R(i-1)$ is the known radius of curvature at the neutral axis at the $(i-1)$st cold-bending increment. We use $e_{(\text{total})}(i)$ to obtain the stress, $\sigma$, from the known stress-strain curve. The secant modulus, $E_s$, which appears in Eq.(3), is simply given by

$$E_s = \frac{\sigma}{|e_{(\text{total})}(i)|} \quad (7)$$

Given $E_s$ we compute $g$ in Eq.(3). Then we compute the plastic modulus $E$ from Eq.(1) and (2). We have a value of $E$ for every value of $x$. The value of the plastic modulus $E$ corresponding to the flange is $E[x = h + t(skin)/2]$. The value of the plastic modulus $E$ corresponding to the skin is $E(x=0)$.

We keep iterating at the $i$th cold-bending increment until the position of the neutral axis does no longer changes. Then we go to the next cold-bending increment. We keep adding cold-bending increments until we reach $R_o$, the smallest radius of curvature used in the cold-bending process. Note that this radius may be significantly smaller than $R$, the design radius of the cylindrical shell attained after springback from the smaller radius, $R_o$.

We have the stress-strain "curve" as a table of $(\text{stress}, \text{strain})$ pairs. It is assumed that these tabular points are connected by straight line segments.

We compute the properties at $N$ points along the $x$-axis from $x = 0$ to $x = h + t(skin)/2$.

We start with the assumption that the location of $d$ is given by the value obtained assuming that no plastic flow has occurred. Assuming that the same material is used for the entire structure, we have, from Eq.(4):

$$d(\text{elastic}) = \frac{A(\text{flange})[h + t(skin)/2] + A(\text{web})[h + t(skin)]/2}{A(skin) + A(\text{web}) + A(\text{flange})} \quad (5)$$

The minimum radius to which the originally flat, "hogged out", plate is bent, called "RCOLD" here, is determined iteratively. We keep iterating until the radius after elastic spring-back is within 5 per cent of the design radius, $R_{CYL}$. $R_{CYL}$ is the input datum that the PANDA2 user gives in response to the prompt,

Radius of curvature (cyl. rad.) in the plane of screen, $R$

Once RCOLD has been determined all the elastic-plastic stiffnesses,
E11, E12, E22, G12 and the 6 x 6 matrices of integrated constitutive
coefficients, \(C_{X(i,j,k)}\), \(C_{Y(i,j,k)}\), \(C_{Y(i,j,ix)}\), and
\(C_{X(i,j,k)}\), and the prebuckling elastic-plastic stress resultants,
\(CN(i,k)\) and \(CN(i,k)\) can be determined. These quantities are
defined above. **They are to be used in the discretized "skin"-ring
model, for which critical buckling load factors are determined later**
in **SUBROUTINE STRUCT**.

At this point in the calculations, the "closed-form" buckling solution
can be obtained. The critical buckling load factor from this model is
called "Buckle" and the circumferential wavelength of the critical
buckling mode is called "WAVLEN". The length, WAVLEN, is used later
to determine the circumferential length of the discretized "skin"-ring
and skin-ring module models.

Notice that sometimes we refer to the "skin"-ring module model and
other times we refer to the skin-ring module model. The two models
are topologically identical. "skin" means "skin+smeared stringers".
The string, skin, without the quotes means "panel skin without
smeared stiffeners". (NOTE: The sub-stiffeners, if any, are **ALWAYS**
smeared in the cold-bending models.)

```plaintext
DIMENSION XCOORD(100),FMPROD(100),RHIST(20)
DIMENSION ETOTAL(100),EPLAST(100),PHOOP(100),SIGMA(100)
DIMENSION EPROD(100),EIM(20),SIM(20),ETOTL2(100),EIGVAL(100)
DIMENSION E11(100),E12(100),E22(100),G12(100),ASTF(3,3),BSTF(3,3)
DIMENSION A11(100),A12(100),A13(100),A22(100),A23(100),A33(100)
DIMENSION B11(100),B12(100),B13(100),B22(100),B23(100),B33(100)
DIMENSION EIGALT(10),WORKSP(10),EVECT(50),ASTFD(3,3),BSTFD(3,3)
DIMENSION PCWORD(*),CPLT(*),IADDCC(*),IPOINC(*),CONSTR(*)
DIMENSION WORDB(*),FSAFEP(*),ZCOORD(100),ETOTLZ(100)
DIMENSION CXCOLD(6,6,5),CYCOLD(6,6,5),CY3CLD(6,6,11),CXCLD0(6,6,5)
DIMENSION CNXVAR(23,8),CNYVAR(23,8),CXCLD1(6,6,5)
DOUBLE PRECISION ASTFD,BSTFD,EIGALT,WORKSP,EVECT
DOUBLE PRECISION CONST,CLIN,CQUAD,CCUBIK
COMMON/GEOM1/AXIAL,CIRC,RCYL
COMMON/GEOM2/B2(2),B2(2),HH(2),WW(2),W2(2)
COMMON/GEOM3/ISTIF(2),NLAYER(4,2),NSEG(2),INTEXT(2)
COMMON/LAYER/MATL(90),LTYPE(99,5,2),TT(90),ANGLE(90)
COMMON/MATER1/E1(20),E2(20),GGG(20),FFNU(20),DENS(20)
COMMON/COLBND/FLGCLD,FN1WEB,RCOLD
COMMON/RCLDSX/RCLDSV
COMMON/NCRITX/NCRIT1,NCRIT2,NCRIT3,NCRIT4,NCRIT5
COMMON/NCRITY/NCRIT6,NCRIT7,NCRIT8
COMMON/MATER2/STRAIN(20,20),STRESS(20,20)
COMMON/THICK/TX(5),TY(5)
```
**BEGIN SUBROUTINE COLDBD (COLD-BENDING RING BUCKLING) ****

1' See Item No. 790 of the file, ...panda2/doc/panda2.news',
1' Buckling from COLDBD ("closed-form" solution) is Model no. 1'
C
IF (NPRT.GE.0) WRITE(IFILE,'(/,A,/,A,/,A)')
1'*** BEGIN SUBROUTINE COLDBD (COLD-BENDING RING BUCKLING) ****',
1' See Item No. 790 of the file, ...panda2/doc/panda2.news',
1' Buckling from COLDBD ("closed-form" solution) is Model no. 1'
C
IF (RCOLD.GT.10.E+16) THEN
  WRITE(IFILE,'(A,/,A,/,A,/,A,1P,E12.4)')
1' SUBROUTINE COLDBD is used only if the shell is fabricated',
1' by cold-bending a "hogged out" flat plate into a cylindrical',
1' panel. In that case the cold-bending radius, RCOLD, must be',
1' less than 10.E+16. Your value of RCOLD=',RCOLD
  CALL ERREX
ENDIF
C
IF (ISTIF(2).NE.1.AND.ISTIF(2).NE.3) THEN
  WRITE(IFILE,'(A,/,A,/,A,/,A,I2)')
1' The ring cross section is neither Tee-shaped nor rectangular.',
1' You are allowed to simulate cold-bending only for "T" and "R",
1' ring cross sections. Therefore, ISTIF(2) must be either',
1' 1 or 3. In your case, ISTIF(2) =',ISTIF(2)
  CALL ERREX
ENDIF
H = HH(2)
WFLANG = 0.
IF (ISTIF(2).EQ.1.AND.FLGCLD.GT.0.1) WFLANG = WW(2)
B = BBB(2)
C
NLAY = NLAYER(1,2)
IF (NLAY.GT.1) THEN
  WRITE(IFILE,'(A,/,I2)')
1' Only one layer is allowed in the skin. NLAY=',NLAY
  CALL ERREX
ENDIF
K = LTYPE(1,1,1)
TSKIN = TT(K)
M = MATL(1)
C
WRITE(IFILE,'(A,/,2I6,1P,4E12.4)')
C    K,M,TSKIN,E1(M),E2(M),GGG(M)='K,M,TSKIN,E1(M),E2(M),GGG(M)
FN1 = FN1WEB
EAXIAL = E1(M)
EINPUT = E2(M)
GELAST = GGG(M)
C
CALL MOVER(STRAIN(1,M),1,EIM,1,20)
CALL MOVER(STRESS(1,M),1,SIM,1,20)
C
WRITE(IFILE,'(A,/,1P,6E12.4)')
C
1' EIM(1),SIM(1),EIM(2),SIM(2),EIM(3),SIM(3)=',
C
1 EIM(1),SIM(1),EIM(2),SIM(2),EIM(3),SIM(3)
NSS = 20
DO 10 I = 2,20
   IF (SIM(I).LT.0.000001) THEN
      NSS = I - 1
      GO TO 11
   ENDIF
10 CONTINUE
11 CONTINUE
C
DIFF = ABS(EAXIAL - EINPUT)/EAXIAL
IF (DIFF.GT.0.01) THEN
   WRITE(IFILE,'(A,/,A,/,A,1P,2E12.4,/,A)')
   1' You are allowed to simulate cold bending only for an',
   1' isotropic material. In your case E1 is not equal to E2.',
   1' In your case: E1, E2 =', E1,E2,
   1' Please correct your input data.'
   CALL ERREX
ENDIF
C
EELAST = SIM(2)/EIM(2)
DIFF = ABS(EINPUT - EELAST)/EINPUT
IF (DIFF.GT.0.01) THEN
   WRITE(IFILE,'(A,/,A,/,A,1P,2E12.4,/,A)')
   1' The second point on your stress-strain curve does not',
   1' match your elastic modulus, EINPUT, for this material:',
   1' EINPUT, SIM(2)/EIM(2) =',EINPUT,SIM(2)/EIM(2),
   1' Please correct either EINPUT or the stress-strain curve.'
   CALL ERREX
ENDIF
C
FNU    = FFNU(M)
C
GINPUT = EINPUT/(2.*(1.+FNU))
DIFF = ABS(GELAST - GINPUT)/EAXIAL
IF (DIFF.GT.0.01) THEN
   WRITE(IFILE,'(A,/,A,/,A,1P,3E12.4,/,A)')
C
You are allowed to simulate cold bending only for an isotropic material. In your case the shear modulus, $G$, is not equal to $E_2/[2.(1+\nu)]$. Your $G, E_2, \nu = G, E_2, FNU$.

Please correct your input data.

CALL ERREX

ENDIF

C

NLAY = NLAYER(3,2)
IF (NLAY.GT.1) THEN
    WRITE(IFILE,'(A,I2')
    ENDIF

K = LTYPE(1,3,2)
M = MATL(1)
EWEB = E2(M)
DIFF = ABS(EINPUT - EWEB)/EINPUT
IF (DIFF.GT.0.01) THEN
    WRITE(IFILE,'(A,/,A,/,A,1P,2E12.4,/,A)')
    ENDIF

The elastic modulus of the ring web must be the same as that for the panel skin in order to simulate cold-bending.

In your case the moduli for skin and web are:

EINPUT, EWEB = EINPUT, EWEB,

Please correct your input data.

CALL ERREX

C

TWEB = TT(K)
TWEB = TY(3)
WRITE(IFILE,'(A,/,2I6,1P,4E12.4)')

NSS, K, TWEB, E1(M), E2(M), GGG(M) = NSS, K, TWEB, E1(M), E2(M), GGG(M)

TFLANG = 0.
IF (ISTIF(2).EQ.1.AND.FLGCLD.GT.0.1) THEN
    NLAY = NLAYER(4,2)
    K = LTYPE(1,4,2)
    M = MATL(1)
    EFLANG = E2(M)
    DIFF = ABS(EINPUT - EFLANG)/EINPUT
    IF (DIFF.GT.0.01) THEN
        WRITE(IFILE,'(A,/,A,/,A,1P,2E12.4,/,A)')
        ENDIF

The elastic modulus of the ring flange must be the same as that for the panel skin in order to simulate cold-bending.

CALL ERREX
In your case the moduli for skin and flange are:
'EINPUT,EFLANG=','EINPUT,EFLANG,' Please correct your input data.'
CALL ERREX
ENDIF
C TFLANG = TT(K)
TFLANG = TY(4)
C WRITE(IFILE,'(A,,2I6,1P,4E12.4)')
C 1' K,M,TFLANG,E1(M),E2(M),GGG(M)=','K,M,TFLANG,E1(M),E2(M),GGG(M)
ENDIF
C
AFLANG= WFLANG*TFLANG
AWEB  = H*TWEB
ASKIN = B*TSKIN
ISUB = 0
IF (KLLAYER.GT.0.AND.ISTFSB(2).NE.0) THEN
ASKIN =ASKIN + HSUB(2,1,1)*TSUB(2,1,1)*BSUB(2,1,1)/B
IF (INTEXT(1).EQ.1.AND.INTXSB(2).EQ.0) ISUB = 1
IF (INTEXT(1).EQ.0.AND.INTXSB(2).EQ.1) ISUB = 1
IF (NPRT.EQ.2) THEN
  IF (ISUB.EQ.0) WRITE(IFILE,'(A)')
    ' Sub-rings are external. Hence, no cold-bending buckling.'
  IF (ISUB.NE.0) WRITE(IFILE,'(A)')
    ' Sub-rings are internal. Hence, yes cold-bending buckling.'
ENDIF
C If ISUB = 1 the sub-ring is internal.
ENDIF
ATOTAL = AFLANG + AWEB + ASKIN
C
DELAST = ATOTAL
C
DELAST = (AFLANG*(H+TSKIN/2.) +AWEB*(H+TSKIN+TSKIN**2/(4.*H))/2.)/ATOTAL
1' Location of neutral axis for elastic material, DELAST=','DELAST
D = DELAST
NBEND = 2
NX = 11
C
NOTE: NX must be the same as the number of nodal points used
in the discretized "skin"-ring and skin-ring module models.
C
DX = (H + TSKIN/2.)/FLOAT(NX - 1)
IBACK = 0
IF (IMOD.EQ.0) THEN
C
Starting value for the cold-bending radius before springback: RCOLD
C
RCOLD = 2.*ABS(RCYL)/3.
ELSE
RCOLD = RCLDSV
ENDIF

KOUNT = 0
15 CONTINUE

BEGINNING OF THE CONVERGENCE LOOP FOR COLD-BENDING RADIUS, RCOLD

KOUNT = KOUNT + 1
RHIST(IKOUNT) = RCOLD
DO 20 IX = 1,NX
   ETOTL2(IX) = 0.
20 CONTINUE
CURTOT = 1./RCOLD
DCURV = CURTOT/FLOAT(NBEND - 1)

DO 500 IBEND = 2,NBEND
   IF (NPRT.GE.2) WRITE(IFILE,'(/,A,I4)') ' IBEND=',IBEND
   ITER = 0
30 CONTINUE

BEGINNING OF THE CONVERGENCE LOOP FOR THE LOCATION, d, OF THE NEUTRAL AXIS FOR CIRCUMFERENTIAL BENDING. "d" IS THE RADIAL DISTANCE FROM THE PANEL SKIN MIDDLE SURFACE TO THE LOCATION OF THIS NEUTRAL AXIS. d IS D IN THIS SUBROUTINE.

ITER = ITER + 1
X = -DX

IX IS THE NODAL POINT NUMBER ON THE RING WEB MIDDLE SURFACE. IN THIS MODEL THE RING WEB IS ASSUMED TO EXTEND FROM THE MIDDLE SURFACE OF THE SHELL SKIN TO THE MIDDLE SURFACE OF THE RING OUTSTANDING FLANGE.

DO 200 IX = 1,NX
   X = X + DX
   ETOTAL(IX) = ETOTL2(IX) -(X-D)*DCURV
200 CONTINUE

ETOTAL(IX) IS THE HOOP STRAIN AT X = (IX-1)*DX
GIVEN ETOTAL, FIND SIG2 (NSS = NUMBER OF POINTS IN SS CURVE, INCLUDING THE ORIGIN.)

DO 50 I = 2,NSS
IF (ABS(ETOTAL(IX)).LT.EIM(I)) THEN
  II = I
  II1 = II - 1
  GO TO 51
ENDIF

50 CONTINUE
51 CONTINUE

SIDIFF = SIM(II) - SIM(II1)
EIDIFF = EIM(II) - EIM(II1)

C SIG2 is the uniaxial stress corresponding to the strain, ETOTAL(IX)
C We obtain SIG2 from the stress-strain curve.
C
SIG2 = SIM(II1) + SIDIFF * (ABS(ETOTAL(IX)) - EIM(II1)) / EIDIFF

C The secant modulus is ES:
ES = SIG2 / ABS(ETOTAL(IX))
EELAST = SIM(2) / EIM(2)

C NOTE: The first non-zero point on your stress-strain curve MUST agree
C with the elastic modulus of the isotropic material, that is,
C EELAST must equal to Young's modulus, E. (called EINPUT here).
C
C The quantities, g, aa, bb, del, EPLAST, are from Eqs.(34) and (33) of
C the paper;
C Bushnell, David, "Theoretical basis of the panda computer
C program for preliminary design of stiffened panels under combined
C in-plane loads", Computers & Structures, Vol. 27, No. 4,
C
C EPLAST(IX) is the hoop plastic modulus at the IXth nodal point in the
C ring web.

  g = 1.5 * (EELAST / ES - 1.0)
  aa = (1.0 + 2. * g / 3.) / EELAST
  bb = (FNU + g / 3.) / EELAST
  del = aa**2 - bb**2
  EPLAST(IX) = aa / del
  EPROD(IX) = EPLAST(IX) * X

  C WRITE(IFILE,'(A,I3,/(1P,5E12.4))')
  C 1  ' IX,ETOTAL(IX),ES,X,g,aa,bb,del,EPLAST(IX),EPROD(IX)='
  C 1  ' TOTAL(IX),ES,X,g,aa,bb,del,EPLAST(IX),EPROD(IX)

  C 200 CONTINUE
  C
  C Numerical integration of int[E(web)dx]
  CALL SIMPSN(IFILE,NX,DX,EPLAST,EPINT)
  C Numerical integration of int[E(web)xdx]
  CALL SIMPSN(IFILE,NX,DX,EPROD,EPXINT)

  C
  C WRITE(IFILE,'(A,I3,1P,3E12.4)')
  C 1  ' NX,DX,EPINT,EPXINT='NX,DX,EPINT,EPXINT
DPAST = D

D = location of the neutral axis including plasticity.

\[
D = \frac{(EPLAST(NX) \times AFLANG \times (H + TSKIN/2.) + TWEB \times EPXINT)}{(EPLAST(1) \times ASKIN + EPLAST(NX) \times AFLANG + TWEB \times EPINT)}
\]

1 ' Iteration=',ITER,' Location of neutral axis, d=',D
   IF (NPRT.GE.2) WRITE(IFILE,'(A,I3,A,1P,E12.4)')
   DIFF = ABS((D - DPAST)/D)
   IF (ITER.GT.30) THEN
      WRITE(IFILE,'(A)')
      CALL ERREX
   ENDIF
   IF (ITER.LT.2) GO TO 30
   IF (DIFF.GT.0.01) GO TO 30

   Location, D, of the neutral axis for circumferential elastic-
   plastic bending has converged.

250    CONTINUE
   DO 300 IX = 1,NX
      ETOTL2(IX) = ETOTAL(IX)
   300    CONTINUE
   500 CONTINUE

The following "instantaneous" moduli are needed for
the buckling analysis:
- E11 instantaneous modulus in the plane of the web in
  the x-direction
- E12 instantaneous "Poisson-type" modulus
- E22 instantaneous modulus in the plane of the web in
  the y-direction (circumferential direction)
- G in-plane shear modulus

First, find the instantaneous moduli for the web:

SIG12 = 0.
SIG1 = FN1/TWEB

DO 600 IX = 1,NX
   XCOORD(IX) = DX*FLOAT(IX - 1)
   DO 550 I = 2,NSS
      IF (ABS(ETOTAL(IX)).LT.EIM(I)) THEN
         II = I
   550    CONTINUE
II1 = II - 1
GO TO 551
ENDIF

550 CONTINUE
551 CONTINUE
SIDIFF = SIM(II) - SIM(II1)
EIDIFF = EIM(II) - EIM(II)
SIG2 = SIM(II1) + SIDIFF*(ABS(ETOTAL(IX))) - EIM(II1))/EIDIFF
ES = SIG2/ABS(ETOTAL(IX))
ETT = SIDIFF/EIDIFF
CALL SMOOTH(NSS,ABS(ETOTAL(IX)),II,EIM,SIM,EIM,ETT)
ET = ETT

C ET is the tangent modulus. The same "smoothing" technique for ET
C is used here as that used in the original PANDA computer program
C (1987 paper, "Theoretical basis..." cited above). SUBROUTINE
C SMOOTH was taken from the PANDA software.
C
FMULT = 1.0
IF (ETOTAL(IX).LT.0.) FMULT = -1.0
SIG2 = FMULT*SIG2
SIGMA(IX) = SIG2

C SIGMA(IX) is the hoop stress at nodal point IX in the ring web.
C
C NOTE: Since the material of panel skin, ring web, and ring
C outstanding flange is the same, and since the ring web is
C assumed to extend from the middle surface of the panel skin
C to the middle surface of the outstanding ring flange, the
C stress, SIGMA(1), is the stress in the panel skin (assumed
C to be uniform in the axial direction), and the stress, SIGMA(NX),
C is the stress in the outstanding flange of the ring (assumed
C to be uniform along the width of the flange). The axial
C stress in the panel skin and ring outstanding flange is assumed to
C remain zero during the cold-bending process. It is assumed that
C the hoop stress does not vary through the thickness of the
C panel skin or through the thickness of the ring outstanding flange.
C The assumption of uniform hoop stress through the thickness of
C the panel "skin" is a questionable assumption in the case of a
C panel skin reinforced by sub-rings. Can one really assume that
C the hoop stress during cold bending is uniform over the panel
C skin as well as over the height of the sub-rings? Only for very
C stubby sub-rings! None-the-less, that is the assumption we
C make here.
C
C FHOOP(IX) is the hoop resultant in the ring web at nodal point IX
C
FHOOP(IX) = SIG2*TWEB
C SBAR is the "effective" (VonMises) stress:

\[
SBAR = \sqrt{\text{SIG1} \times \text{SIG1} + \text{SIG2} \times \text{SIG2} - \text{SIG1} \times \text{SIG2} + 3. \times \text{SIG12}^2}
\]

C The quantities, gprime, g, s1, s2, aa, bb, del, E11, E12, E22, G12, 
C are from Eqs.(45), (44), (43), and (42) of the paper: 
C Bushnell, David, "Theoretical basis of the panda computer 
C program for preliminary design of stiffened panels under combined 
C in-plane loads", Computers & Structures, Vol. 27, No. 4, 
C These are the "instantaneous" quantities used in the stability 
C equations:

\[
gprime = 2.25 \times \text{EELAST} \times (1./\text{ET} - 1./\text{ES}) / SBAR^2
\]
\[
g = 1.5 \times (\text{EELAST}/\text{ES} - 1.0)
\]
\[
s1 = (2. \times \text{SIG1} - \text{SIG2}) / 3.
\]
\[
s2 = (2. \times \text{SIG2} - \text{SIG1}) / 3.
\]
\[
ga = (1. + 2. \times g/3. + gprime \times s2^2) / \text{EELAST}
\]
\[
bb = (\text{FNU} + g/3. - gprime \times s1 \times s2) / \text{EELAST}
\]
\[
cc = (1. + 2. \times g/3. + gprime \times s1^2) / \text{EELAST}
\]
\[
del = aa \times cc - bb^2
\]
\[
E11(\text{IX}) = aa / del
\]
\[
E12(\text{IX}) = bb / del
\]
\[
E22(\text{IX}) = cc / del
\]
\[
G12(\text{IX}) = \text{GELAST} \times (1. + \text{FNU}) / (1. + \text{FNU} + g + 2. \times gprime \times \text{SIG12}^2)
\]
\[
\text{FMPROD}(\text{IX}) = \text{FHOOP}(\text{IX}) \times (D - XCOORD(\text{IX}))
\]

600 CONTINUE
C end of the loop over IX, the number of nodal points on the ring web.
C
CALL SIMPSN(IFILE,NX,DX,FHOOP,FWEB)
CALL SIMPSN(IFILE,NX,DX,FMPROD,FMWEB)
C
C23456789012345678901234567890123456789012345678901234567890123456789012
FCESKN = SIGMA(1) * TSKIN * B
FCEFLG = SIGMA(NX) * TFLANG * WFLANG
FORCE = FWEB + FCESKN + FCEFLG
FMOMNT = FMWEB + FCESKN * D + FCEFLG * (D - XCOORD(NX))
C
C get curvature change, CURBCK, due to spring-back
C assumption: the spring-back process is entirely elastic.
C
C First, compute the elastic bending stiffness of a single module 
C about the neutral axis for elastic bending:
FMULT = 1
IF (FLGCLD.LT.0.1.OR.ISTIF(2).NE.1) FMULT = 0.00001
C
C NOTE: FMULT is very small if the ring outstanding flange is 
C welded to the web tip AFTER completion of cold-bending.
C
C EICLD is the circumferential bending stiffness, "EI".
C
C  EICLD =       EELAST*TSKIN**3*B/12.
1 +EELAST*(H+TSKIN/2.)**3*TWEB/12.
1 +FMULT*EELAST*TFLANG**3*WFLANG/12.
1 +EELAST*TSKIN*B*DELAST**2
1 +EELAST*H*TWEB*((H+TSKIN/2.)/2.-DELAST)**2
1 +FMULT*EELAST*WFLANG*TFLANG*(H+TSKIN/2.-DELAST)**2
C
ARCYL = ABS(RCYL)
ARCOLD = ABS(RCOLD)
C CURBCK is the curvature change due to elastic spring-back.
C
C  CURBCK = ABS((FMOMNT/B)/C55N)
CURBCK = ABS(FMOMNT/EICLD)
CUREND = 1./ARCOLD - CURBCK
RADEND = 1./CUREND
C RAEND is the radius of the cold-bent panel after elastic
C spring-back.
C
FACTR = 0.5
C NOTE: with FACTR = 1.0 the process often did not converge.
DIFF = (RADEND - ARCYL)/RADEND
IF (NPRT.GE.2) WRITE(IFILE,'(A,I3,1P4E12.4)')
1' KOUNT,RCOLD,RAEND,ARCYL,DIFF=', KOUNT,RCOLD,RAEND,ARCYL,DIFF
IF (IBACK.EQ.0.AND.KOUNT.LT.10.AND.EELAST/SIM(2).GT.10.0
1 .AND.ABS(DIFF).GT.0.05) THEN
   IF (DIFF.LT.0.0) RCOLD = MIN((1.-FACTR*DIFF)*ARCOLD,ARCYL)
   IF (DIFF.GT.0.0) RCOLD = (1.-FACTR*DIFF)*ARCOLD
C Iterate again to obtain a better value of RCOLD...
GO TO 15
ENDIF
C Either RCOLD converged or KOUNT reached its maximum value, 10:
C
IF (KOUNT.EQ.10) THEN
  WRITE(IFILE,'(/,A,/,A,/,A,/,1P,(5E12.4),/,A)')
1' ****************** CONVERGENCE FAILURE **********************
1' Cold-bending radius, RCOLD, fails to converge. Run abort.'
1' History of RCOLD =',(RHIST(I),I=1,KOUNT),
1' *************************************************************
  CALL ERREX
ENDIF
C We now have a satisfactory value for the cold-bending
C radius, RCOLD, before elastic springback.
Iterations for RCOLD have converged to within 5 per cent, which means that the radius after elastic spring-back is within 5 per cent of the design radius, ABS(RCYL). (ABS(RCYL) is an input datum provided by the PANDA2 user).

IF (IBACK.EQ.0) THEN

In our determination of RCOLD we used the smallest possible value of NBEND: NBEND = 2 (cold bending from flat to RCOLD in just one step). Now we increase NBEND from 2 to 3 and recompute the elastic-plastic properties, E11(IX), E12(IX), E22(IX), G12(IX), SIGMA(IX), and FHOOP(IX), IX = 1,NX, at RCOLD a bit more accurately. NOTE: We do not iterate on RCOLD in this step. We first save RCOLD (RCLDSV) for use with the perturbed design (when IMOD = 1).

IF (IMOD.EQ.0) RCLDSV = RCOLD
NBEND = 3
IBACK = 1
GO TO 15
ENDIF

IF (NPRT.GE.2) THEN
WRITE(IFILE,'(/,A,I3,/A)')
1' Number of points on stress-strain curve, NSS =',NSS,
1' STRAIN STRESS'
DO 610 I = 1,NSS
WRITE(IFILE,'(1P,2E12.4)') EIM(I),SIM(I)
610 CONTINUE
WRITE(IFILE,'(/,A,1P,E12.4,/A,1P,E12.4)')
1' ring spacing, B(RNG)= ',B,
1' skin thickness, TSKIN= ',TSKIN
WRITE(IFILE,'(A,1P,E12.4,/A,1P,E12.4)')
1' ring web height, H(RNG)= ',H,
1' ring web thickness, TWEB= ',TWEB
WRITE(IFILE,'(A,1P,E12.4,/A,1P,E12.4)')
1' cold-bending radius, RCOLD= ',RCOLD,
1' radius after springback, RADEND= ',RADEND
WRITE(IFILE,'(A,1P,E12.4,/A,1P,E12.4)')
1' design radius of cylinder, RCYL= ',ABS(RCYL),
1' At R = RCOLD: location d of the ring neutral axis=',D
WRITE(IFILE,'(/,A,1P,E12.4,/A,1P,E12.4,/A,1P,E12.4)')
1' ring web hoop force, FWEB= ',FWEB,
1' skin hoop force, SIGMA(1)*TSKIN*B= ',FCESK,
1' flange hoop force, SIGMA(NX)*TFLANG*W(RNG)= ',FCEFLG
WRITE(IFILE,'(/,A,1P,E12.4,/,A,1P,E12.4)')
1' force integrated over the ring cross section= ',FORCE,
1' moment integrated over the ring cross section= ',FMOMNT
C Print out stress and strain distribution over the ring module cross
C section and compute the bending moment that creates this distribution.
C
WRITE(IFILE,'(/,A,I3,A,/(1P,5E12.4))')
1' x-coordinates for ',NX,' radial locations along the ring web:','
 WRITE(IFILE,'(/,A,I3,A,/(1P,5E12.4))')
1' hoop strain for ',NX,' radial locations along the ring web:','
 WRITE(IFILE,'(/,A,I3,A,/(1P,5E12.4))')
1' hoop stress for ',NX,' radial locations along the ring web:','
 WRITE(IFILE,'(/,A,1P,E12.4,/,A,1P,E12.4)')
1' uniform hoop resultant in the shell skin=','SIGMA(1)*TSKIN,
1' uniform hoop resultant in the outstanding flange=','
ENDIF
C
C Next, obtain the C(i,j) and the CNXVAR and CNYVAR for the
C converged cold-bent state:
C
CALL MOVER(CX,1,CXCOLD,1,180)
CALL MOVER(CX,1,CXCLD1,1,180)
CALL MOVER(CX(1,1,1),1,CXCLD1(1,1,5),1,36)
CALL MOVER(CY,1,CYCOLD,1,180)
CALL MOVER(CX,1,CXCLD0,1,180)
CALL MOVER(CSKIN,1,CXCLD0(1,1,1),1,36)
CALL MOVER(CSKIN,1,CXCLD0(1,1,5),1,36)
C
E22LIN = EELAST/(1.-FNU**2)
E11LIN = E22LIN
E12LIN = FNU*E22LIN
IF (NPRT.GE.2) WRITE(IFILE,'(/,A,1P,3E12.4)')
1' E11LIN, E12LIN, E22LIN=',E11LIN, E12LIN, E22LIN
C
C First, do the ring web:
DO 700 IX = 1,NX
CALL MOVER(CY(1,1,3),1,CY3CLD(1,1,IX),1,36)
DIFF = ABS(E22LIN - E22(IX))/E22LIN
C If the cold-bending process remains entirely elastic, DIFF=0.0
IF (NPRT.GE.2)
  WRITE(IFILE,'(A,I3,1P,E12.4)') 'Ring Web: IX,DIFF=',IX,DIFF
ENDIF
CY3CLD(1,1,IX) = E11(IX)*CY(1,1,3)/E11LIN
CY3CLD(1,2,IX) = E12(IX)*CY(1,2,3)/E12LIN
CY3CLD(2,1,IX) = CY3CLD(1,2,IX)
CY3CLD(2,2,IX) = E22(IX)*CY(2,2,3)/E22LIN
CY3CLD(3,3,IX) = G12(IX)*CY(3,3,3)/GELAST
CY3CLD(4,4,IX) = E11(IX)*CY(4,4,3)/E11LIN
CY3CLD(4,5,IX) = E12(IX)*CY(4,5,3)/E12LIN
CY3CLD(5,4,IX) = CY3CLD(4,5,IX)
CY3CLD(5,5,IX) = E22(IX)*CY(5,5,3)/E22LIN
CY3CLD(6,6,IX) = G12(IX)*CY(6,6,3)/GELAST
ENDIF
CNXVAR(IX,3) = FN1
CNYVAR(IX,3) = FHOOP(IX)
700 CONTINUE
C end of the loop over IX, the nodal points along the ring web.
C
C Next, do the shell skin and ring outstanding flange:
C
C First, do the shell skin:
C NOTE: We assume that the elastic-plastic moduli in the
C skin are the same as those at Node Point 1 in the
C ring web.
DIFF = ABS(E22LIN - E22(1))/E22LIN
C If the cold-bending process remains entirely elastic, DIFF=0.0
IF (NPRT.GE.2)
  WRITE(IFILE,'(A,1P,E12.4)') 'Shell skin: DIFF=',DIFF
ENDIF
CXCOLD(1,1,1) = E11(1)*CX(1,1,1)/E11LIN
CXCOLD(1,2,1) = E12(1)*CX(1,2,1)/E12LIN
CXCOLD(2,1,1) = CXCOLD(1,2,1)
CXCOLD(2,2,1) = E22(1)*CX(2,2,1)/E22LIN
CXCOLD(3,3,1) = G12(1)*CX(3,3,1)/GELAST
CXCOLD(4,4,1) = E11(1)*CX(4,4,1)/E11LIN
CXCOLD(4,5,1) = E12(1)*CX(4,5,1)/E12LIN
CXCOLD(5,4,1) = CXCOLD(4,5,1)
CXCOLD(5,5,1) = E22(1)*CX(5,5,1)/E22LIN
CXCOLD(6,6,1) = G12(1)*CX(6,6,1)/GELAST
CXCOLD(1,1,5) = E11(1)*CX(1,1,5)/E11LIN
CXCOLD(1,2,5) = E12(1)*CX(1,2,5)/E12LIN
CXCOLD(2,1,5) = CXCOLD(1,2,5)
CXCOLD(2,2,5) = E22(1)*CX(2,2,5)/E22LIN
CXCOLD(3,3,5) = G12(1)*CX(3,3,5)/GELAST
CXCOLD(4,4,5) = E11(1)*CX(4,4,5)/E11LIN
CXCOLD(4,5,5) = E12(1)*CX(4,5,5)/E12LIN
CXCOLD(5,4,5) = CXCOLD(4,5,5)
CXCOLD(5,5,5) = E22(1)*CX(5,5,5)/E22LIN
CXCOLD(6,6,5) = G12(1)*CX(6,6,5)/GELAST
CALL MOVER(CXCOLD(1,1,1),1,CXCLD1(1,1,1),1,36)
CALL MOVER(CXCLD1(1,1,1),1,CXCLD1(1,1,5),1,36)

C CSKIN(i,j) are the integrated constitutive quantities for the panel skin without any smeared stiffeners or sub-stiffeners. As of this writing, the stiffness matrix, CXCLD0(i,j) is not used anywhere to compute ring buckling under the cold-bending process.

CALL MOVER(CXCLD0(1,1,1),1,CXCLD0(1,1,5),1,36)

CNOTE: If there are sub-rings SIGMA(1)*TSKIN is a conservative value for CNYVAR because it leaves out the hoop tension in the subrings. Therefore, in this model there is less hoop tension than would be in the actual structure with subrings.

CALL MOVER(SIGMA(1)*TSKIN,0,CNYVAR(1,1),1,23)
CALL MOVER(SIGMA(1)*TSKIN,0,CNYVAR(1,2),1,23)
CALL MOVER(SIGMA(1)*TSKIN,0,CNYVAR(1,NSEGX),1,23)
CALL MOVER(0.,0,CNXVAR(1,1),1,23)
CALL MOVER(0.,0,CNXVAR(1,2),1,23)
CALL MOVER(0.,0.,CNXVAR(1,NSEGX),1,23)

CB ISTIF(2) = 1 means the ring is Tee shaped. NOTE:
CB no faying flange is accounted for in this application!
CB
IF (ISTIF(2).NE.1) GO TO 730

C Next, do the ring outstanding flange of the Tee-shaped ring:
FMULT = 1.0
IF (FLGCLD.LT.0.1) THEN

FMULT = 0.00001
DO 720  I = 1,6
    DO 710  J = 1,6
        CYCOLD(I,J,4) = FMULT*CYCOLD(I,J,4)
    710 CONTINUE
720 CONTINUE
ENDIF

DIFF = ABS(E22LIN - E22(NX))/E22LIN
C If the cold-bending process remains entirely elastic, DIFF=0.0
IF (NPRT.GE.2) WRITE(IFILE,'(A,I3,1P,E12.4)')
1' Ring outstanding flange: NX,DIFF='NX,DIFF

C NOTE: We assume that the elastic-plastic moduli in the outstanding
C ring flange are the same as those at Node Point NX in the
C ring web.
C
IF (DIFF.GT.0.05) THEN
    CYCOLD(1,1,4) = FMULT*E11(NX)*CY(1,1,4)/E11LIN
    CYCOLD(1,2,4) = FMULT*E12(NX)*CY(1,2,4)/E12LIN
    CYCOLD(2,1,4) = CYCOLD(1,2,4)
    CYCOLD(2,2,4) = FMULT*E22(NX)*CY(2,2,4)/E22LIN
    CYCOLD(3,3,4) = FMULT*G12(NX)*CY(3,3,4)/GELAST
    CYCOLD(4,4,4) = FMULT*E11(NX)*CY(4,4,4)/E11LIN
    CYCOLD(4,5,4) = FMULT*E12(NX)*CY(4,5,4)/E12LIN
    CYCOLD(5,4,4) = CYCOLD(4,5,4)
    CYCOLD(5,5,4) = FMULT*E22(NX)*CY(5,5,4)/E22LIN
    CYCOLD(6,6,4) = FMULT*G12(NX)*CY(6,6,4)/GELAST
ENDIF

CALL MOVER(SIGMA(1)*TFLANG,0,CNYVAR(1,4),1,23)
IF (FLGCLD.LT.0.1) CALL MOVER(0.,0.,CNYVAR(1,4),1,23)
CALL MOVER(0.,0.,CNXVAR(1,4),1,23)

730 CONTINUE
STABILITY EQUATIONS FOR THE "CLOSED FORM" SOLUTION

Find the elements of the stiffness and load-geometric matrices before integration over the web height corresponding to the assumed buckling modal displacement in the ring web. The buckling modal displacement in the ring web is assumed to be:

\[
w = \left[ a_3 H^3 S^2 (S - 3.) + a_4 H^4 S^2 (S^2 - 6.) + a_5 H^5 S^2 (S^3 - 10.) \right] \sin(N \pi y / L)
\]  \quad (8)

in which \( w \) is the normal displacement (rolling) of the web, \( H \) is the height of the web, and \( s = x / H \), with \( x \) being the coordinate in the plane of the web from the web root (\( x = 0 \)) to the web tip (\( x = h \)). \( L \) is the length of the web in the circumferential coordinate direction, \( y \). \( N \) is the number of circumferential half waves over the circumferential length, \( L \).

It is assumed that the panel skin experiences no out-of-plane deformation in the buckling mode. Only the web and the outstanding flange, if any, participate in the buckling mode.

\( a_3, a_4, a_5 \) are undetermined coefficients with the following units:
\( a_3 \) has units \( 1/\text{in}^2 \); \( a_4 \) has units \( 1/\text{in}^3 \); \( a_5 \) has units \( 1/\text{in}^4 \)

NOTE: The assumed displacement pattern, Eq. (8), was originally planned for a ring that has no outstanding flange. In that case the curvature \( w_{xx} \) at the web tip should be zero because the web tip is free. Indeed, \( w_{xx} \) is zero at the web tip even if there exists an outstanding flange. Therefore, Eq. (8) may be a poor choice for a ring buckling mode. The prediction of buckling may be either unconservative or conservative when an outstanding flange exists. (NOTE: This dilemma does not exist in the case of the discretized "skin"-ring and skin-ring single module models processed later in SUBROUTINE STRUCT. Therefore, the PANDA2 user does not have to worry about generating unconservative designs.)

Also, the analysis is based on the assumption that the loading of the web is uniaxial: loading only in the hoop direction (\( y \) direction). If there is a significant \( FN_1 \) (in-plane loading of the web normal to the shell skin surface), the theory used here may be inadequate.

The cold-bending ring web buckling problem is an eigenvalue problem of the following form:

\[
\begin{bmatrix} A \end{bmatrix} - \lambda \begin{bmatrix} B \end{bmatrix} q = 0.
\]  \quad (9)

in which \( [A] \) is the 3 x 3 stiffness matrix and \( [B] \) is the 3 x 3
load-geometric matrix. The system of rank 3 represented by Eq.(9) is obtained by minimizing the total potential energy, $U - W$, with respect to the undetermined coefficients, $a_3$, $a_4$, and $a_5$, in Eq.(8).

It is assumed that the web buckling mode can be captured by only normal deflections $w$ that vary along the $s$-coordinate as given in Eq.(8) and that vary in the circumferential direction $y$ trigonometrically as $\sin(n^\pi y/l)$. The strain energy of buckling is given by

$$\frac{H}{2}\int (4,4)w_{ss}^2 + 2(4,5)w_{ss}w_{yy} + (5,5)w_{yy}^2 + 4C6,6)w_{sy}^2 \, dyds \quad (10)$$

in which $w_{ss}$, $w_{sy}$, $w_{yy}$ represent second partial derivatives of $w$ with respect to the coordinate directions $s$ and $y$, and the $C(i,j)$ are the bending stiffnesses of the cold-bent ring web.

The work done by the prebuckling resultants, $N_1$ and $N_2$, during the buckling displacements $w$ is given by

$$\frac{H}{2}\int [N_1 w_s^2 + N_2 w_y^2] \, dyds \quad (11)$$

Minimization of $U - W$ with respect to the undetermined coefficients $a_3$, $a_4$, $a_5$ yields the coefficients $A(i,j)$ and $B(i,j)$ listed below.

Integration over $s$ is performed by Simpson's rule (in SUBROUTINE SIMPSN, which is used elsewhere in PANDA2 and which is located in the util.src library).

Integration over $s$ is performed by Simpson's rule (in SUBROUTINE SIMPSN, which is used elsewhere in PANDA2 and which is located in the util.src library).

In the following several statements we compute a circumferential length of ring, $FL$, that usually represents only a small part of the entire shell. We do this so that we don't have to search over a large quantity of wave numbers to find the critical cold-bending ring web buckling mode.

```c
PI = 3.1415927
FL = CIRC
DS = 1./FLOAT(NX-1)

NWVMAX = FL/H
IF (NWVMAX.GT.10) THEN
    FL = 10.*CIRC/FLOAT(NWVMAX)
ENDIF
NWVMAX = FL/H
IF (NWVMAX.GT.10) THEN
    FL = 10.*FL/FLOAT(NWVMAX)
ENDIF
```
NWVMAX = 2.*FL/H

C
BUCKLE = 10.E+16

C
IF (IMOD.EQ.0) THEN
   NBEG = 1
   NEND = NWVMAX
ELSE
C
   For the perturbed design (IMOD = 1) we use only the critical
   number of circumferential halfwaves, NCRIT3, determined for
   the unperturbed (current) design.
C
   NBEG = NCRIT3
   NEND = NCRIT3
ENDIF
C
DO 1000 N WAVE = NBEG, NEND
C
   FN = N WAVE
   S = -DS
C
DO 800 IX = 1, NX
C
   Some frequently used combinations:
C
   C44 = E11(IX)*TWEB**3/12.
   C45 = E12(IX)*TWEB**3/12.
   C55 = E22(IX)*TWEB**3/12.
   C66 = G12(IX)*TWEB**3/12.
   FN2 = FHOOP(IX)
   S = S + DS
C
   A11(IX) = S1*S1*C44 + S3*S1*C45 + S1*S3*C45 + S3*S3*C55
   1 + S36*S36*4.*C66
   A12(IX) = S1*S21*C44 + S3*S21*C45 + S1*S26*C45 + S3*S26*C55
   1 + S36*S212*4.*C66
   A13(IX) = S1*S31*C44 + S3*S31*C45 + S1*S310*C45 + S3*S310*C55
   1 + S36*S3102*4.*C66

1
C A22(IX) = S21*S21*C44 + S26*S21*C45 + S21*S26*C45 + S26*S26*C55
      1 +S212*S212*4.*C66
      1
C A23(IX) = S21*S31*C44 + S26*S31*C45 + S21*S310*C45 + S26*S310*C55
      1 +S212*S320*4.*C66
      1
C A33(IX) = S31*S31*C44 + S310*S31*C45 + S31*S310*C45 + S310*S310*C55
      1 +S320*S320*4.*C66

C B11(IX) = S3*S3*FN2/C**2 + S36*S36*FN1/C**2
C B12(IX) = S3*S26*FN2/C**2 + S36*S212*FN1/C**2
C B13(IX) = S3*S310*FN2/C**2 + S36*S320*FN1/C**2

C B22(IX) = S26*S26*FN2/C**2 + S212*S212*FN1/C**2
C B23(IX) = S26*S310*FN2/C**2 + S212*S320*FN1/C**2
C B33(IX) = S310*S310*FN2/C**2 + S320*S320*FN1/C**2

800 CONTINUE
C end of the loop over IX
C The Aij and Bij have yet to be integrated over the height of the web.
C
C Next, integrate the Aij and Bij using Simpson's rule:
C
CALL SIMPSN(IFILE,NX,DS,A11,ASTF(1,1))
CALL SIMPSN(IFILE,NX,DS,A12,ASTF(1,2))
CALL SIMPSN(IFILE,NX,DS,A13,ASTF(1,3))
CALL SIMPSN(IFILE,NX,DS,A22,ASTF(2,2))
CALL SIMPSN(IFILE,NX,DS,A23,ASTF(2,3))
CALL SIMPSN(IFILE,NX,DS,A33,ASTF(3,3))

CALL SIMPSN(IFILE,NX,DS,B11,BSTF(1,1))
CALL SIMPSN(IFILE,NX,DS,B12,BSTF(1,2))
CALL SIMPSN(IFILE,NX,DS,B13,BSTF(1,3))
CALL SIMPSN(IFILE,NX,DS,B22,BSTF(2,2))
CALL SIMPSN(IFILE,NX,DS,B23,BSTF(2,3))
CALL SIMPSN(IFILE,NX,DS,B33,BSTF(3,3))

IF (ISTIF(2).EQ.1) THEN

C Next, find the contributions of the outstanding flange, if any, to
C the stiffness and load-geometric matrices. Note that the following
C quantities INCLUDE integration over the width of the outstanding
C ring flange. We can do the integration in "closed form" because it
C is assumed that the prebuckled state of the outstanding flange is
C uniform both along the width of the flange and through the thickness
of the flange.

The following formulas are based on the assumption that the flange cross section does not deform in the buckling mode. The flange centroid experiences a rotation equal to $dw/dx$ of the web at the tip of the web and an axial displacement equal to $w$ of the web at the tip of the web.

The following "instantaneous" moduli are needed for the buckling analysis:

- $E_{11}$ instantaneous modulus in the plane of the flange in the vertical direction (along the flange width)
- $E_{12}$ instantaneous "Poisson-type" modulus
- $E_{22}$ instantaneous modulus in the plane of the flange in the $y$-direction (circumferential direction)
- $G$ in-plane shear modulus

First, find the instantaneous moduli for the flange:

$$\sigma_{12} = 0.$$  
$$\sigma_1 = 0.$$  

DO 750 I = 2,NSS  
    IF (ABS(ETOTAL(NX)).LT.EIM(I)) THEN  
        II = I  
        II1 = II - 1  
        GO TO 751  
    ENDIF  
ENDF  

750 CONTINUE  
751 CONTINUE  
SIDIFF = SIM(II) - SIM(II1)  
EIDIFF = EIM(II) - EIM(II1)  
SIG2 = SIM(II1) + SIDIFF*(ABS(ETOTAL(NX)) - EIM(II1))/EIDIFF  
FMULT = 1.0  
IF (ETOTAL(NX).LT.0.) FMULT = -1.0  
FN2F = FMULT*SIG2*TFLANG  
ES = SIG2/ABS(ETOTAL(NX))  
ETT = SIDIFF/EIDIFF  
CALL SMOOTH(NSS,ETOTAL(NX),II,EIM,SIM,EIM,ETT)  
ET = ETT  
SBAR = SQRT(SIG1*SIG1 + SIG2*SIG2 - SIG1*SIG2 + 3.*SIG12**2)  
gprime = 2.25*EELAST*(1./ET - 1./ES)/SBAR**2  
g = 1.5*(EELAST/ES - 1.0)  
s1 = (2.*SIG1 - SIG2)/3.  
s2 = (2.*SIG2 - SIG1)/3.  
bb = (FNU + g/3. - gprime*s1*s2)/EELAST  
cc = (1.+2.*g/3. + gprime*s1**2)/EELAST
del = aa*cc - bb**2
E22(NX) = cc/del
EI  = E22(NX)*TFLANG*WFLANG**3/12.

Frequently used quantities:

WF = WFLANG
RWEB = RCOLD
S3 = -C**2*H**3*2.
S26 = -C**2*H**3*H*5.
S36 = -H**2*3.
S212 = -H**2*H*8.
S320 = -H**2*H**2*15.

ASTF(1,1) = ASTF(1,1) + S3*S3*EI + S36*S36*EI/RWEB**2
ASTF(1,2) = ASTF(1,2) + S3*S26*EI + S36*S212*EI/RWEB**2
ASTF(1,3) = ASTF(1,3) + S3*S310*EI + S36*S320*EI/RWEB**2

ASTF(2,2) = ASTF(2,2) + S26*S26*EI + S212*S212*EI/RWEB**2
ASTF(2,3) = ASTF(2,3) + S26*S310*EI + S212*S320*EI/RWEB**2

ASTF(3,3) = ASTF(3,3) + S310*S310*EI + S320*S320*EI/RWEB**2

BSTF(1,1) = BSTF(1,1) + S3*S3*FN2F*WF/C**2 + C**2*S36*S36*FN2F*WF**3/12.
BSTF(1,2) = BSTF(1,2) + S3*S26*FN2F*WF/C**2 + C**2*S36*S212*FN2F*WF**3/12.
BSTF(1,3) = BSTF(1,3) + S3*S310*FN2F*WF/C**2 + C**2*S36*S320*FN2F*WF**3/12.

BSTF(2,2) = BSTF(2,2) + S26*S26*FN2F*WF/C**2 + C**2*S212*S212*FN2F*WF**3/12.
BSTF(2,3) = BSTF(2,3) + S26*S310*FN2F*WF/C**2 + C**2*S212*S320*FN2F*WF**3/12.

BSTF(3,3) = BSTF(3,3) + S310*S310*FN2F*WF/C**2 + C**2*S320*S320*FN2F*WF**3/12.

ENDIF

ASTFD(1,1) = ASTF(1,1)
ASTFD(1,2) = ASTF(1,2)
ASTFD(1,3) = ASTF(1,3)
ASTFD(2,2) = ASTF(2,2)
ASTFD(2,3) = ASTF(2,3)
ASTFD(3,3) = ASTF(3,3)

BSTFD(1,1) = BSTF(1,1)
The cold-bending ring web buckling problem is an eigenvalue problem of the following form:

\[ \{ [A] - \lambda [B] \} q = 0 \]

in which \([A]\) is the 3 x 3 stiffness matrix and \([B]\) is the 3 x 3 load-geometric matrix. A cubic equation in the eigenvalue, \(\lambda\), is obtained by setting the determinant of the matrix \([A - \lambda B]\) equal to zero:

\[ CCUBIK \lambda^3 + CQUAD \lambda^2 + CLIN \lambda + CONST = 0 \]  

Next, we set up the coefficients:

- **CONST** = constant term
- **CLIN** = term linear in \(\lambda\)
- **CQUAD** = term quadratic in \(\lambda\)
- **CCUBIK** = term cubic in \(\lambda\)

The cubic equation is solved for \(\lambda\), the ring web buckling load factor (eigenvalue). Here the output from SUBROUTINE CUBIC is the lowest positive eigenvalue, EIGVAL(NWAVE), in which NWAVE is the number
C of circumferential half-waves over the web length, FL.

CALL CUBIC(CONST,CLIN,CQUAD,CCUBIK,EIGVAL(NWAVE),
1           IMOD,ICUBIC,JCUBIC,1,1,0)

C
IF (NPRT.GE.2) WRITE(IFILE,'(A,I3,1P,E12.4)')
1 ' From "CUBIC": NWAVE, EIGVAL(NWAVE)=' NWAVE, EIGVAL(NWAVE)

C Here's what we tried before we decided to use SUBROUTINE CUBIC:
C
Next, solve for the three eigenvalues. Use SUBROUTINE GSEIG,
which is also used for extracting eigenvalues for the alternative
(double trig. series) buckling solution in PANDA2. This eigensystem
is much, much smaller than that for the alternative buckling
solution in PANDA2. Here we have only three roots (system
rank = 3, that is, MNTOT = 3).

Use generalized Jacobi iteration for current design computations...

MNTOT = 3
CALL GSEIG (MNTOT, ASTFD, BSTFD, EIGALT, EVECT, WORKSP, IPRINT)
IF (MNTOT.LT.0) THEN
  WRITE(IFILE,'(A,,A,I4,/,A)')
1 ' ***** WARNING ********** WARNING ********** WARNING *****',
1 ' SUB. GSEIG FAILED TO OBTAIN EIGENVALUE: MNTOT=',MNTOT,
1 ' ***** WARNING ********** WARNING ********** WARNING *****'

The lowest positive eigenvalue from SUBROUTINE GSEIG always
agreed with that from SUBROUTINE CUBIC, that is, when GSEIG
did not fail because of complex eigenvalues. The failure of
GSEIG caused run abortion during SUPEROPT runs, which is
annoying to the PANDA2 user. That is why we decide to use
SUBROUTINE CUBIC instead of SUBROUTINE GSEIG.

BUCKLE = MIN(EIGVAL(NWAVE),BUCKLE)

NWAVE1 = NWAVE-1
IF (NWAVE.GT.1.AND.EIGVAL(NWAVE).GT.EIGVAL(NWAVE1)) GO TO 1050

C End of the loop over the number of circumferential halfwaves,
C NWAVE
1050 CONTINUE

C
IF (IMOD.EQ.0) THEN
  DO 1100 NWAVE = NBEG,NEND

1100 CONTINUE
DIFF = ABS(BUCKLE - EIGVAL(NWAVE))/ABS(BUCKLE)
IF (DIFF.LT.0.0001) THEN
   NCRIT3 = NWAVE
   GO TO 1200
ENDIF
1100 CONTINUE
1200 CONTINUE
ENDIF
C
WAVLEN = FL/FLOAT(NCRIT3)
IF (NPRT.GE.0) WRITE(IFILE,'(/,A,1P,E12.4,A,1P,E12.4,A,I5)')
   ' BUCKLE=',BUCKLE, ' WAVLEN=',WAVLEN, ' NCRIT3=',NCRIT3
C2345678901234567890123456789012345678901234567890123456789012345678901234567890123
1' circumferential length of web used for buckling analysis, FL=',
1 FL,
1' number of circumferential halfwaves over the length CIRC=',
1 NCRITC
C
IF (IMOD.EQ.0) NCRITC = CIRC/WAVLEN
IF (NPRT.GE.2) WRITE(IFILE,'(A,1P,E12.4,/,A,I5)')
   ' circumferential length of web used for buckling analysis, FL=',
   FL,
   ' number of circumferential halfwaves over the length CIRC=',
   NCRITC
ENDIF
C
C Constraint condition...
INUMTT = INUMTT + 1
FSAFTY = 1.1
CALL CONVRF(FSAFTY,CCN2)
CALL CONVRT(NCRITC,CCN)
IF (IMOD.EQ.0.AND.(BUCKLE/FSAFTY).LT.MAXCON) THEN
   ICAR = ICAR + 1
   PCWORD(ICAR) =
   1 'Cold-bending ring buckling, closed form soln'
   CPLOT(ICAR) = BUCKLE/FSAFTY - 1.
   IADDC(CAR) = 0
   FSAFEP(ICAR) = FSAFTY
ENDIF
IF (IMOD.EQ.0.AND.(BUCKLE/FSAFTY).GT.CONMAX) GO TO 1300
IF (IMOD.EQ.1.AND.IPOINC(INUMTT).EQ.0) GO TO 1300
ICONST = ICONST + 1
IF (IMOD.EQ.0) IPOINC(INUMTT) = 1
CONSTR(ICONST) = BUCKLE/FSAFTY
WORD(B(ICONST)) =
   1'Cold-bending ring buckling, closed form soln; N='//CCN//
IF (NPRT.GE.0) WRITE(IFILE,'(A,1P,E12.4,2X,A)')
   ' Margin=',CONSTR(ICONST)-1.,WORDB(ICONST)
IF (NPRT.GE.2) WRITE(IFILE,'(A,,A,,A,I3,A,I2,A,I2)')
   ' *** THE FOLLOWING COLD-BENDING BUCKLING MARGIN JUST COMPUTED:',
   WORDB(ICONST),
   ' ******** CONSTRAINT NO.',ICONST,', LOAD SET NO.',ILOADS,
   ' SUBCASE NO.',ICASE
1300 CONTINUE
   IF (ITYPE.EQ.1.AND.NPRT.GE.2)
      WRITE(IFILE,*)' AFTER 4440 C: IMOD,INUMTT,ICONST=',
      IMOD,INUMTT,ICONST
C
   IF (NPRT.GE.0) WRITE(IFILE,'(A,,A,,A,,/)')
   ' End of computation of cold-bending ring buckling load factor',
   ' in SUBROUTINE COLBD.',
   ' ***********************************************************'
C
C   Next, find out if the sub-ring buckles under cold bending
C
C   ISUB = 0 means either that there is no sub-ring or the
C         sub-ring is external and therefore experiences no
C         destabilizing hoop compression during cold bending.
C
C
   IF (ISUB.EQ.0) GO TO 3000
C
NZ = NX
DZ = (HSUB(2,1,1) +TSKIN/2.)/FLOAT(NZ-1)
Z = -DZ
DO 1400 IZ = 1,NZ
   Z = Z + DZ
   DO 1350 I = 1,NX
      IF (Z.LT.XCOORD(I)) THEN
         II = I
         I1I = I - 1
         GO TO 1351
      ENDIF
      IF (Z.GE.XCOORD(NX)) ETOTLZ(IZ) = ETOTAL(I)
   1350   CONTINUE
   1351   CONTINUE
      EDIFF = ETOTAL(II) - ETOTAL(I1I)
      XDIFF = DX
      ETOTLZ(IZ) = ETOTAL(I1I) +EDIFF*(Z - XCOORD(I1I))/XDIFF
1400 CONTINUE
C
C First, find the instantaneous moduli for the web:
SIG12 = 0.
SIG1 = 0.
DO 1600 IZ = 1,NZ
    ZCOORD(IZ) = DZ*FLOAT(IZ - 1)
DO 1550 I = 2,NSS
    IF (ABS(ETOTLZ(IZ)).LT.EIM(I)) THEN
        II = I
        II1 = II - 1
        GO TO 1551
ENDIF
1550 CONTINUE
1551 CONTINUE
SIDIFF = SIM(II) - SIM(II1)
EIDIFF = EIM(II) - EIM(II1)
SIG2 = SIM(II1) + SIDIFF*(ABS(ETOTLZ(IZ)) - EIM(II1))/EIDIFF
ES = SIG2/ABS(ETOTLZ(IZ))
ETT = SIDIFF/EIDIFF
CALL SMOOTH(NSS,ABS(ETOTLZ(IZ)),II,EIM,SIM,EIM,ETT)
ET = ETT
FMULT = 1.0
IF (ETOTLZ(IZ).LT.0.) FMULT = -1.0
SIG2 = FMULT*SIG2
SIGMA(IZ) = SIG2
FHOOP(IZ) = SIG2*TSUB(2,1,1)
SBAR = SQRT(SIG1*SIG1 + SIG2*SIG2 - SIG1*SIG2 + 3.*SIG12**2)
gprime = 2.25*EELAST*(1./ET - 1./ES)/SBAR**2
h = 1.5*(EELAST/ES - 1.0)
s1 = (2.*SIG1 - SIG2)/3.
s2 = (2.*SIG2 - SIG1)/3.
aa = (1.+2.*g/3. + gprime*s2**2)/EELAST
bb = (FNU + g/3. - gprime*s1*s2)/EELAST
cc = (1.+2.*g/3. + gprime*s1**2)/EELAST
del = aa*cc - bb**2
E11(IZ) = aa/del
E12(IZ) = bb/del
E22(IZ) = cc/del
G12(IZ) = GELAST*(1.+FNU)/(1.+FNU + 2.*gprime*SIG12**2)
FMPROD(IZ) = FHOOP(IZ)*(D-ZCOORD(IZ))
1600 CONTINUE
CALL SIMPSN(IFILE,NZ,DZ,FHOOP,FWEB)
CALL SIMPSN(IFILE,NZ,DZ,FMPROD,FMWEB)
C
C23456789012345678901234567890123456789012345678901234567890123456789012
IF (NPRT.GE.2) THEN
    WRITE(IFILE,'(/,A,1P,E12.4,/,A,1P,E12.4)')
1' sub-ring spacing,       BSUB=  ',BSUB(2,1,1),
1' skin thickness,         TSKIN=  ',TSKIN
WRITE(IFILE,'((A,1P,E12.4,/,A,1P,E12.4))')
1' sub-ring web height,     HSUB(2,1,1)=  ',HSUB(2,1,1),
1' sub-ring web thickness,   TSUB(2,1,1)=  ',TSUB(2,1,1)
WRITE(IFILE,'((A,1P,E12.4,/,A,1P,E12.4))')
1' cold-bending radius,     RCOLD=  ',RCOLD,
1' radius after springback, RADEND=  ',RADEND
WRITE(IFILE,'((A,1P,E12.4,/,A,1P,E12.4))')
1' design radius of cylinder, RCYL=  ',ABS(RCYL),
1' At R = RCOLD: location d of the ring neutral axis= ',D
WRITE(IFILE,'(/,A,1P,E12.4)')
1' sub-ring web hoop force,  FWEB=  ',FWEB

C Print out stress and strain distribution over the ring module cross
C section and compute the bending moment that creates this distribution.
C
WRITE(IFILE,'(/,A,I3,A,/1P,5E12.4))')
1' z-coords. for ',NZ,' radial locations along the subring web:',
1 (ZCOORD(I), I=1,NZ)
WRITE(IFILE,'(/,A,I3,A,/1P,5E12.4))')
1' hoop strain for ',NZ,' radial locations along the subring web:',
1 (ETOTLZ(I), I=1,NZ)
WRITE(IFILE,'(/,A,I3,A,/1P,5E12.4))')
1' hoop stress for ',NZ,' radial locations along the subring web:',
1 (SIGMA(I), I=1,NZ)
WRITE(IFILE,'(/,A,I3,A,A,/1P,5E12.4))')
1' hoop resultant for ',NZ,' radial locations along the subring',
1' web:', (FHOOP(I), I=1,NZ)
ENDIF

C STABILITY EQUATIONS FOR THE "CLOSED FORM" SOLUTION
C
PI = 3.1415927
FL = CIRC
DS = 1./FLOAT(NZ-1)
H = HSUB(2,1,1)
TWEB = TSUB(2,1,1)
FN1 = 0.

NWVMAX = FL/H
IF (NWVMAX.GT.10) THEN
   FL = 10.*CIRC/FLOAT(NWVMAX)
ENDIF
NWVMAX = FL/H
IF (NWVMAX.GT.10) THEN
   FL = 10.*FL/FLOAT(NWVMAX)
ENDIF  
NWVMAX = 2.*FL/H  
C  
BUCKLE = 10.E+16  
C  
IF (IMOD.EQ.0) THEN  
  NBEG = 1  
  NEND = NWVMAX  
ELSE  
  NBEG = NCRIT4  
  NEND = NCRIT4  
ENDIF  
C  
DO 2000 NWAVE = NBEG,NEND  
C  
FN = NWAVE  
S = -DS  
C  
DO 1800 IZ = 1,NZ  
C  
C Some frequently used combinations:  
C  
C44 = E11(IZ)*TWEB**3/12.  
C45 = E12(IZ)*TWEB**3/12.  
C55 = E22(IZ)*TWEB**3/12.  
C66 = G12(IZ)*TWEB**3/12.  
FN2 = FHOOP(IZ)  
S = S + DS  
C = FN*PI/FL  
S1 =  H*6.*(S - 1.)  
S3 =  C**2*H**3*S**2*(S - 3.)  
S36 = C*H**2*S*(3.*S - 6.)  
S21 = H*12.*H*(S**2 - 1.)  
S26 = C**2*H**3*S**2*H*(S**2 - 6.)  
S31 = H*20.*H**2*(S**3 - 1.)  
S310= C**2*H**3*S**2*H**2*(S**3 - 10.)  
S320= C*H**2*S**2*H**2*(5.*S**3 - 20.)  
S212= C*H**2*S**2*H*(4.*S**2 - 12.)  
C  
A11(IZ) = S1*S1*C44 +S3*S1*C45 +S1*S3*C45 +S3*S3*C55  
1 +S36*S36*4.*C66  
A12(IZ) = S1*S21*C44 +S3*S21*C45 +S1*S26*C45 +S3*S26*C55  
1 +S36*S212*4.*C66  
A13(IZ) = S1*S31*C44 +S3*S31*C45 +S1*S310*C45 +S3*S310*C55  
1 +S36*S320*4.*C66  
C  
A22(IZ) = S21*S21*C44 +S26*S21*C45 +S21*S26*C45 +S26*S26*C55
A23(IZ) = S21*S31*C44 + S26*S31*C45 + S21*S310*C45 + S26*S310*C55
+ S212*S320*C66

A33(IZ) = S31*S31*C44 + S310*S31*C45 + S31*S310*C45 + S310*S310*C55
+ S320*S320*C66

B11(IZ) = S3*S3*FN2/C**2 + S36*S36*FN1/C**2
B12(IZ) = S3*S26*FN2/C**2 + S36*S212*FN1/C**2
B13(IZ) = S3*S310*FN2/C**2 + S36*S320*FN1/C**2

B22(IZ) = S26*S26*FN2/C**2 + S212*S212*FN1/C**2
B23(IZ) = S26*S310*FN2/C**2 + S212*S320*FN1/C**2

B33(IZ) = S310*S310*FN2/C**2 + S320*S320*FN1/C**2

1800 CONTINUE
C end of the loop over IZ
C
C The Aij and Bij have yet to be integrated over the height of the web.
C
C Next, integrate the Aij and Bij using Simpson's rule:
C
CALL SIMPSN(IFILE,NZ,DS,A11,ASTF(1,1))
CALL SIMPSN(IFILE,NZ,DS,A12,ASTF(1,2))
CALL SIMPSN(IFILE,NZ,DS,A13,ASTF(1,3))
CALL SIMPSN(IFILE,NZ,DS,A22,ASTF(2,2))
CALL SIMPSN(IFILE,NZ,DS,A23,ASTF(2,3))
CALL SIMPSN(IFILE,NZ,DS,A33,ASTF(3,3))

CALL SIMPSN(IFILE,NZ,DS,B11,BSTF(1,1))
CALL SIMPSN(IFILE,NZ,DS,B12,BSTF(1,2))
CALL SIMPSN(IFILE,NZ,DS,B13,BSTF(1,3))
CALL SIMPSN(IFILE,NZ,DS,B22,BSTF(2,2))
CALL SIMPSN(IFILE,NZ,DS,B23,BSTF(2,3))
CALL SIMPSN(IFILE,NZ,DS,B33,BSTF(3,3))

ASTFD(1,1) = ASTF(1,1)
ASTFD(1,2) = ASTF(1,2)
ASTFD(1,3) = ASTF(1,3)
ASTFD(2,2) = ASTF(2,2)
ASTFD(2,3) = ASTF(2,3)
ASTFD(3,3) = ASTF(3,3)

BSTFD(1,1) = BSTF(1,1)
BSTFD(1,2) = BSTF(1,2)
BSTFD(1,3) = BSTF(1,3)
BSTFD(2,2) = BSTF(2,2)
BSTFD(2,3) = BSTF(2,3)
BSTFD(3,3) = BSTF(3,3)

CALL CUBICC(ASTFD(1,1),ASTFD(1,2),ASTFD(1,3),ASTFD(2,2),
          1 ASTFD(2,3),ASTFD(3,3),
          1 BSTFD(1,1),BSTFD(1,2),BSTFD(1,3),BSTFD(2,2),
          1 BSTFD(2,3),BSTFD(3,3),
          1 CONST,CLIN,CQUAD,CCUBIK)

CALL CUBIC(CONST,CLIN,CQUAD,CCUBIK,EIGVAL(NWAVE),
          1 IMOD,ICUBIC,JCUBIC,1,1,0)

BUCKLE = MIN(EIGVAL(NWAVE),BUCKLE)

NWAVE1 = NWAVE - 1
IF (NWAVE.GT.1.AND.EIGVAL(NWAVE).GT.EIGVAL(NWAVE1)) GO TO 2050

2000 CONTINUE
C End of the loop over the number of circumferential waves, NWAVE
2050 CONTINUE
C
IF (IMOD.EQ.0) THEN
  DO 2100 NWAVE = NBEG,NEND
    DIFF = ABS(BUCKLE - EIGVAL(NWAVE))/ABS(BUCKLE)
    IF (DIFF.LT.0.0001) THEN
      NCRIT4 = NWAVE
      GO TO 2200
    ENDIF
  2100 CONTINUE
  2200 CONTINUE
ENDIF
C
WAVLN2 = FL/FLOAT(NCRIT4)
IF (NPRT.GE.0) WRITE(IFILE,'(/,A,1P,E12.4,A,1P,E12.4,A,I5)')
1' BUCKLE=',BUCKLE, ' WAVLN2=',WAVLN2, ' NCRIT4=',NCRIT4
C23456789012345678901234567890123456789012345678901234567890123456789012
C
IF (IMOD.EQ.0) NCRITD = CIRC/WAVLN2
IF (NPRT.GE.2) WRITE(IFILE,'(A,1P,E12.4,/,A,I5)')
1 circumferential length of web used for buckling analysis, FL=',
1 FL,
1 number of circumferential halfwaves over the length CIRC=',
1 NCRITD

C
IF (ISTIF(1).NE.0.AND.IMOD.EQ.0.AND.NPRT.GE.0) THEN
  WRITE(IFILE,'(/,A,1P,E12.4,/,A,1P,E12.4)')
 1 circumferential spacing of the stringers=',
 1 BBB(1),
 1 circumferential half-wavelength of the critical buckling mode=',
 1 WAVLN2
ENDIF

C Constraint condition for sub-ring cold-bending buckling...
INUMTT = INUMTT + 1
FSAFTY = 1.1
CALL CONVRF(FSAFTY,CCN2)
CALL CONVRT(NCRITD,CCN)
IF (IMOD.EQ.0.AND.(BUCKLE/FSAFTY).LT.MAXCON) THEN
  ICAR = ICAR + 1
  PCWORD(ICAR) =
  'Cold-bending subring buckling, closed form soln'
  IADDCC(ICAR) = 0
  FSAFEP(ICAR) = FSAFTY
ENDIF
IF (IMOD.EQ.0.AND.(BUCKLE/FSAFTY).GT.CONMAX) GO TO 2300
IF (IMOD.EQ.1.AND.IPOINC(INUMTT).EQ.0) GO TO 2300
ICONST = ICONST + 1
IF (IMOD.EQ.0) IPOINC(INUMTT) = 1
CONSTR(ICONST) = BUCKLE/FSAFTY
WORDB(ICONST)=
1 'Cold-bending subring buckling, closed form soln; N='//CCN//
1';FS='//CCN2
IF (NPRT.GE.0) WRITE(IFILE,'(A,1P,E12.4,2X,A)')
1 ' Margin=',CONSTR(ICONST)-1.,WORDB(ICONST)
ELSE IF (NPRT.GE.2) WRITE(IFILE,'(A,1P,E12.4,/,A,I3,A,I2,A,I2)')
1 ' *** THE FOLLOWING COLD-BENDING BUCKLING MARGIN JUST COMPUTED:',
1 ' WORDB(ICONST),
1 ' ******** CONSTRAINT NO.','ICONST,'; LOAD SET NO.','ILOADS,
1 '; SUBCASE NO.','ICASE
2300 CONTINUE
IF (ITYPE.EQ.1.AND.NPRT.GE.2)
1 WRITE(IFILE,*') AFTER 4440 C: IMOD,INUMTT,ICONST=','
1 IMOD,INUMTT,ICONST

C
IF (NPRT.GE.0) WRITE(IFILE,'(A,,,A,,,A,,,/)' )
1' End of computation of cold-bending subring buckling load factor',
1' in SUBROUTINE COLDBD.',
1' ***************************************************************************'

C 3000 CONTINUE
C23456789012345678901234567890123456789012345678901234567890123456789012
C
C RETURN
END
C
C C=DECK
SMOOTH

SUBROUTINE SMOOTH(NP,EBAR,JJ,ETOT,SIG,EPS,ET)

C PURPOSE IS TO SMOOTH THE TANGENT MODULUS IN THE NEIGHBORHOOD OF A
C CORNER IN THE STRESS-STRAIN CURVE...
C
DIMENSION ETOT(20),SIG(20),EPS(20)

C JJ1 = JJ - 1
IF (JJ1.LE.1) RETURN
IF (JJ.GE.(NP-1)) RETURN
EJ = ETOT(JJ)
EJ1 = ETOT(JJ1)
DTOT = EJ - EJ1
ELOC = EBAR - EJ1
ETMID = ET
JJM = JJ1 - 1
ETM = (SIG(JJ1) -SIG(JJM))/(EPS(JJ1) -EPS(JJM))

C JJP = JJ + 1
ETP = (SIG(JJP) -SIG(JJ))/(EPS(JJP) -EPS(JJ))

C EMINUS = (ETM + ETMID)/2.
IF (JJ.EQ.3) EMINUS = ETM
EPLUS = (ETP + ETMID)/2.
IF (JJ.EQ.(NP-2)) EPLUS = ETP

C 1000 CONTINUE
C
RETURN
END
C=DECK CUBICC

SUBROUTINE CUBICC(a11,a12,a13,a22,a23,a33,
1                  b11,b12,b13,b22,b23,b33,
1                  CONST,CLIN,CQUAD,CCUBIK)

C Purpose is to get the coefficients, 
C CONST,CLIN,CQUAD,CCUBIK 
C of the cubic equation for the eigenvalues 
C of the cold-bending ring buckling problem. 
C The cubic equation is:
C determinant = 0
C
C Input: 
C a11,a12,a13,a22,a23,a33 = strain energy coefficients 
C b11,b12,b13,b22,b23,b33 = "work done" coefficients 
C
C Output: 
C CONST,CLIN,CQUAD,CCUBIK = coefficients of cubic equation
C
C DOUBLE PRECISION a11,a12,a13,a22,a23,a33 
C DOUBLE PRECISION b11,b12,b13,b22,b23,b33 
C DOUBLE PRECISION CONST,CLIN,CQUAD,CCUBIK
C
C CONST = a33*(a11*a22 -a12**2) +a23*(a12*a13 -a11*a23) 
1              +a13*(a12*a23 -a13*a22)
C
C CLIN = a33*(b11*a22 +b22*a11 -2.*a12*b12)
1    +b33*(a11*a22 -a12**2)
1    +a23*(b12*a13 +b13*a23 -a12*b11 -b23*a11) 
1    +a23*(a12*a13 -a11*a23) 
1    +b23*(b12*a13 +b13*a12 -b11*a23 -b23*a11) 
1    +a13*(b12*a13 +b13*a23 -b11*a23 -b23*a11) 
1    +b13*(a12*a23 -a13*a22) 
C
C CQUAD = a33*(b11*b22 -b12**2)
1    +b33*(a11*b22 +a22*b11 -2.*a12*b12)
1    +a23*(b12*b13 -b11*b23) 
1    +b23*(b12*a13 +a12*b13 -a11*a23 -a11*b23) 
1    +a13*(b12*b23 -b13*b22) 
1    +b13*(b12*a23 +a12*b23 -b13*a22 -a13*b22) 
C
C CCUBIK= b33*(b11*b22 -b12**2) +b23*(b12*b13 -b11*b23)
1              +b13*(b12*b23 -b13*b22)
C
RETURN
END
What is going on in the new coding is described there.

**Typical new output in the *.OPM file from SUBROUTINE COLDBD and from SUBROUTINE STRUCT is listed below. This output is obtained with the print index, NPRINT = 2 in the *.OPT file:**

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**CHAPTER 26(b)**

************************************************************
************************************************************
************************************************************
************         CHAPTER 26(b)           *****************
************************************************************
************************************************************
************************************************************

** CHAPTER 26b: DESIGN PERTURBATION INDEX, IMOD= 0 ****

************************************************************

CHAPTER 26b Compute the ring web buckling load factor and circumferential wavelength from cold-bending a flat "hogged out" plate into a cylindrical panel with cold-bending radius RCOLD from iterations. This analysis is performed only for cylindrical shells with INTERNAL rings with rectangular or Tee-shaped cross sections. The entire shell must be fabricated of the same isotropic material. See Item No.790 in ...panda2/doc/panda2.news .

*** BEGIN SUBROUTINE COLDBD (COLD-BENDING RING BUCKLING) ****

See Item No. 790 of the file, ...panda2/doc/panda2.news

Buckling from COLDBD ("closed-form" solution) is Model no. 1
Location of neutral axis for elastic material, DELAST= 9.3628E-02

IBEND= 2
Iteration= 1 Location of neutral axis, d= 6.0412E-02
Iteration= 2 Location of neutral axis, d= 5.8127E-02
Iteration= 3 Location of neutral axis, d= 5.7979E-02
KOUNT,RCOLD,RADEND,ARCYL,DIFF= 1 3.2000E+01 6.6653E+01 4.8000E+01 2.7985E-01

IBEND= 2
Iteration= 1 Location of neutral axis, d= 5.2824E-02
Iteration= 2 Location of neutral axis, d= 5.2521E-02
KOUNT,RCOLD,RADEND,ARCYL,DIFF= 2 2.7522E+01 5.0708E+01 4.8000E+01
IBEND=   2
Iteration=  1 Location of neutral axis, d=  5.1601E-02
Iteration=  2 Location of neutral axis, d=  5.1545E-02
KOUNT,RCOLD,RADEND,ARCYL,DIFF=  3  2.6787E+01  4.8382E+01  4.8000E+01  7.8914E-03

IBEND=   2
Iteration=  1 Location of neutral axis, d=  7.4343E-02
Iteration=  2 Location of neutral axis, d=  7.5316E-02
Iteration=  3 Location of neutral axis, d=  7.5357E-02

IBEND=   3
Iteration=  1 Location of neutral axis, d=  5.2983E-02
Iteration=  2 Location of neutral axis, d=  5.2302E-02
Iteration=  3 Location of neutral axis, d=  5.2282E-02
KOUNT,RCOLD,RADEND,ARCYL,DIFF=  4  2.6787E+01  4.9094E+01  4.8000E+01  2.2293E-02

Number of points on stress-strain curve, NSS = 5

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<th>STRESS</th>
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<td>7.0000E+04</td>
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<tr>
<td>1.2727E-02</td>
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<td>1.1000E+05</td>
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<tr>
<td>1.9091E+01</td>
<td>1.2100E+05</td>
</tr>
</tbody>
</table>

ring spacing,        B(RNG)= 1.3974E+01
skin thickness,      TSKIN= 5.8919E-02
ring web height,     H(RNG)= 9.7173E-01
ring web thickness,  TWEB= 1.8576E-01
ring flange width,   W(RNG)= 0.0000E+00
ring flange thickness, TFLANG= 0.0000E+00
cold-bending radius, RCOLD= 2.6787E+01
radius after springback, RADEND= 4.9094E+01
design radius of cylinder, RCYL= 4.8000E+01
At R = RCOLD: location d of the ring neutral axis= 5.2282E-02

ring web hoop force,  FWEB= -1.5634E+04
skin hoop force, SIGMA(1)*TSKIN*B= 2.1307E+04
flange hoop force, SIGMA(NX)*TFLANG*W(RNG)= 0.0000E+00

force integrated over the ring cross section= 5.6728E+03
moment integrated over the ring cross section= 9.8416E+03

x-coordinates for 11 radial locations along the ring web:
   0.0000E+00  1.0008E-01  2.0017E-01  3.0025E-01  4.0033E-01
5.0042E-01  6.0050E-01  7.0058E-01  8.0066E-01  9.0075E-01
1.0008E+00

hoop strain for 11 radial locations along the ring web:
2.3820E-03 -1.3541E-03 -5.0903E-03 -8.8265E-03 -1.2563E-02
-1.6299E-02 -2.0035E-02 -2.3771E-02 -2.7507E-02 -3.1244E-02
-3.4980E-02

hoop stress for 11 radial locations along the ring web:
2.6203E+04 -1.4896E+04 -5.5994E+04 -8.1611E+04 -9.9224E+04
-1.0561E+05 -1.1000E+05 -1.1000E+05 -1.1000E+05 -1.1000E+05
-1.1001E+05

hoop resultant for 11 radial locations along the ring web:
4.8674E+03 -2.7670E+03 -1.0401E+04 -1.5160E+04 -1.8432E+04
-1.9619E+04 -2.0434E+04 -2.0434E+04 -2.0435E+04 -2.0435E+04
-2.0435E+04

uniform hoop resultant in the shell skin= 1.5248E+03
uniform hoop resultant in the outstanding flange= 0.0000E+00

E11LIN, E12LIN, E22LIN= 1.2088E+07  3.6264E+06  1.2088E+07
Ring Web: IX,DIFF= 1  0.0000E+00
Ring Web: IX,DIFF= 2  0.0000E+00
Ring Web: IX,DIFF= 3  0.0000E+00
Ring Web: IX,DIFF= 4  2.7726E-01
Ring Web: IX,DIFF= 5  6.8951E-01
Ring Web: IX,DIFF= 6  7.3218E-01
Ring Web: IX,DIFF= 7  8.6633E-01
Ring Web: IX,DIFF= 8  8.8521E-01
Ring Web: IX,DIFF= 9  8.9942E-01
Ring Web: IX,DIFF= 10 9.1051E-01
Ring Web: IX,DIFF= 11 9.1937E-01
Shell skin: DIFF= 0.0000E+00
From "CUBIC": NWAVE, EIGVAL(NWAVE)= 1  3.9232E+01
From "CUBIC": NWAVE, EIGVAL(NWAVE)= 2  1.0792E+01
From "CUBIC": NWAVE, EIGVAL(NWAVE)= 3  5.5201E+00
From "CUBIC": NWAVE, EIGVAL(NWAVE)= 4  3.6823E+00
From "CUBIC": NWAVE, EIGVAL(NWAVE)= 5  2.8401E+00
From "CUBIC": NWAVE, EIGVAL(NWAVE)= 6  2.3936E+00
From "CUBIC": NWAVE, EIGVAL(NWAVE)= 7  2.1381E+00
From "CUBIC": NWAVE, EIGVAL(NWAVE)= 8  1.9900E+00
From "CUBIC": NWAVE, EIGVAL(NWAVE)= 9  1.9087E+00
From "CUBIC": NWAVE, EIGVAL(NWAVE)= 10 1.8758E+00
From "CUBIC": NWAVE, EIGVAL(NWAVE)= 11 1.8761E+00

BUCKLE= 1.8758E+00 WAVLEN= 9.7288E-01 NCRIT3= 10
circumferential length of web used for buckling analysis, FL= 9.7288E+00
number of circumferential halfwaves over the length CIRC=  155

circumferential spacing of the stringers= 2.2773E+00
circumferential half-wavelength of the critical buckling mode= 9.7288E-01
Margin= 7.0525E-01 Cold-bending ring buckling, closed form soln; N=155;FS=1.1

*** THE FOLLOWING COLD-BENDING BUCKLING MARGIN JUST COMPUTED:
Cold-bending ring buckling, closed form soln; N=155;FS=1.1

******** CONSTRAINT NO. 17; LOAD SET NO. 1; SUBCASE NO. 1
End of computation of cold-bending ring buckling load factor in SUBROUTINE COLDDB.

********************************************************************

***** BEGIN DISCRETIZED "SKIN"-RING MODULE MODEL OF *****
*** COLD-BENDING BUCKLING OF RING (smeared stringers) ***
***** This is Model No. 2 of cold-bending ring buckling *****
circumferential waves over the circ.length,  4.8644E+00=  1
ring cold buckling load factor from a discretized module=  6.4476E+00
(smeared stringers)
circumferential waves over the circ.length,  4.8644E+00=  2
ring cold buckling load factor from a discretized module=  2.4048E+00
(smeared stringers)
circumferential waves over the circ.length,  4.8644E+00=  3
ring cold buckling load factor from a discretized module=  1.8210E+00
(smeared stringers)
circumferential waves over the circ.length,  4.8644E+00=  4
ring cold buckling load factor from a discretized module=  1.8404E+00
(smeared stringers)

 circumferential spacing of the stringers= 2.2773E+00
circumferential half-wavelength of the critical buckling mode= 1.6215E+00

**** BEGIN SUB. MODE ("SKIN"-RING MODULE COLD BENDING 1) ****

(lines skipped to save space)

Margin=  6.5544E-01 Cold-bending ring buckling, "skin"-ring module; N=93
;FS=1.1

*** THE FOLLOWING COLD-BENDING BUCKLING MARGIN JUST COMPUTED:
Cold-bending ring buckling, "skin"-ring module; N=93 ;FS=1.1

******** CONSTRAINT NO. 18; LOAD SET NO. 1; SUBCASE NO. 1
End of computation of cold-bending ring buckling load factor
in SUBROUTINE STRUCT from "skin"-ring discretized module.

********************************************************************


***** BEGIN DISCRETIZED SKIN-RING MODULE MODEL OF *****
*** COLD-BENDING BUCKLING OF RING (no stringers) ***
***** This is Model No. 3 of cold-bending ring buckling *****
circumferential waves over the circ.length, 2.2773E+00= 1
ring cold buckling load factor from a discretized module= 1.1032E+00 (no stringers)
circumferential waves over the circ.length, 2.2773E+00= 2
ring cold buckling load factor from a discretized module= 1.6198E+00 (no stringers)
circumferential spacing of the stringers= 2.2773E+00
circumferential half-wavelength of the critical buckling mode= 2.2848E+00

**** BEGIN SUB. MODE (SKIN-RING MODULE COLD BENDING 2) ****

(lines skipped to save space)

*********** LOAD SET NO. 1 **************
ICASE = 1 (ICASE=1 MEANS PANEL MIDLENGTH)
          (ICASE=2 MEANS AT RINGS )

APPLIED LOADS IN LOAD SET A ("eigenvalue" loads):
  Applied axial stress resultant, Nx=  -2.2190E+03
  Applied circumferential stress resultant, Ny=  -2.2190E-03
  Applied in-plane shear resultant, Nxy=  1.1095E+01
  Applied axial moment resultant, Mx=  0.0000E+00
  Applied circumferential moment resultant, My=  0.0000E+00
  Applied pressure (positive for upward), p =  4.6229E-05

APPLIED LOADS IN LOAD SET B ( fixed uniform loads):
  Applied axial stress resultant, Nx0=  0.0000E+00
  Applied circumferential stress resultant, Ny0=  0.0000E+00
  Applied in-plane shear resultant, Nxy0=  0.0000E+00

NOTE: "F.S." means "Factor of Safety";
"DONL" means "Donnell shell theory used." panda2.news ITEM 128
"SAND" means "Sanders shell theory used" in discretized model
"Dseg" means "Segment numbering used in input data." ITEM 272

MARGINS FOR CURRENT DESIGN: LOAD CASE NO. 1, SUBCASE NO. 1
MAR. MARGIN
NO.  VALUE                DEFINITION
1  1.45E-01 Local buckling from discrete model-1.,M=5  axial halfwaves;FS=0.99


The new "cold-bending buckling" margins appear near the end of the list of margins for Load Set 1, Subcase 1, as follows:

<table>
<thead>
<tr>
<th>MAR.</th>
<th>MARGIN NO.</th>
<th>VALUE</th>
<th>DEFINITION</th>
</tr>
</thead>
<tbody>
<tr>
<td>17</td>
<td>7.05E-01</td>
<td>Cold-bending ring buckling, closed form soln; N=155;FS=1.1</td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>6.55E-01</td>
<td>Cold-bending ring buckling, &quot;skin&quot;-ring module; N=93 ;FS=1.1</td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>2.94E-03</td>
<td>Cold-bending ring buckling, skin-ring module; N=66 ;FS=1.1</td>
<td></td>
</tr>
</tbody>
</table>

*********************************
SOME COMMENTS ABOUT THE RESULTS:
*********************************

The case to which the results listed above correspond is the "nasaortho" case described in the report:

"Optimum designs from PANDA2 of a uniformly axially compressed cylindrical shell with internal stringers and internal rings both with rectangular cross sections, and verification of the designs by BIGBOSOR4 and STAGS", David Bushnell, 31 March, 2009
In that report the internally stiffened shell was optimized before the cold-bending simulation capability existed. The shell (with imperfection amplitude, Wimp = plus and minus 0.125 inch) was re-optimized (2 SUPEROPTs) with the cold-bending simulation included. The results listed above correspond to the re-optimized "nasaortho" design.

The new optimum design is as follows:

<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>CURRENT</th>
<th>DEFINITION</th>
</tr>
</thead>
<tbody>
<tr>
<td>NUMBER</td>
<td>VALUE</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>2.2773E+00</td>
<td>B(STR):stiffener spacing, b: STR seg=NA, layer=NA</td>
</tr>
<tr>
<td>2</td>
<td>7.5902E-01</td>
<td>B2(STR):width of stringer base, b2 (must be &gt; 0, see</td>
</tr>
<tr>
<td>3</td>
<td>9.8977E-01</td>
<td>H(STR):height of stiffener (type H for sketch), h: S</td>
</tr>
<tr>
<td>4</td>
<td>5.8191E-02</td>
<td>T(1 )(SKN):thickness for layer index no.(1 ): SKN seg=1</td>
</tr>
<tr>
<td>5</td>
<td>8.6462E-02</td>
<td>T(2 )(STR):thickness for layer index no.(2 ): STR seg=3</td>
</tr>
<tr>
<td>6</td>
<td>1.3974E+01</td>
<td>B(RNG):stiffener spacing, b: RNG seg=NA, layer=NA</td>
</tr>
<tr>
<td>7</td>
<td>0.0000E+00</td>
<td>B2(RNG):width of ring base, b2 (zero is allowed): RNG</td>
</tr>
<tr>
<td>8</td>
<td>9.7137E-01</td>
<td>H(RNG):height of stiffener (type H for sketch), h: R</td>
</tr>
<tr>
<td>9</td>
<td>1.8576E-01</td>
<td>T(3 )(RNG):thickness for layer index no.(3 ): RNG seg=3</td>
</tr>
</tbody>
</table>

The old optimum design (before cold-bending simulation) is as follows:

<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>CURRENT</th>
<th>DEFINITION</th>
</tr>
</thead>
<tbody>
<tr>
<td>NUMBER</td>
<td>VALUE</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1.8594E+00</td>
<td>B(STR):stiffener spacing, b: STR seg=NA, layer=NA</td>
</tr>
<tr>
<td>2</td>
<td>6.1973E-01</td>
<td>B2(STR):width of stringer base, b2 (must be &gt; 0, see</td>
</tr>
<tr>
<td>3</td>
<td>8.8811E-01</td>
<td>H(STR):height of stiffener (type H for sketch), h: S</td>
</tr>
<tr>
<td>4</td>
<td>4.9565E-02</td>
<td>T(1 )(SKN):thickness for layer index no.(1 ): SKN seg=1</td>
</tr>
<tr>
<td>5</td>
<td>8.0641E-02</td>
<td>T(2 )(STR):thickness for layer index no.(2 ): STR seg=3</td>
</tr>
<tr>
<td>6</td>
<td>1.1772E+01</td>
<td>B(RNG):stiffener spacing, b: RNG seg=NA, layer=NA</td>
</tr>
<tr>
<td>7</td>
<td>0.0000E+00</td>
<td>B2(RNG):width of ring base, b2 (zero is allowed): RNG</td>
</tr>
<tr>
<td>8</td>
<td>1.7544E+00</td>
<td>H(RNG):height of stiffener (type H for sketch), h: R</td>
</tr>
<tr>
<td>9</td>
<td>8.9081E-02</td>
<td>T(3 )(RNG):thickness for layer index no.(3 ): RNG seg=3</td>
</tr>
</tbody>
</table>

The new optimum design is as follows:

The old optimum design (before cold-bending simulation) is as follows:
The new optimum design is heavier, of course, WEIGHT = 107.0 lb vs 99.83 lb. The ring is much "stockier" in the new design than in the old design:

**new design:**

<table>
<thead>
<tr>
<th>No.</th>
<th>Value</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>9.7173E-01</td>
<td>H(RNG): height of stiffener (type H for sketch), h: R</td>
</tr>
<tr>
<td>9</td>
<td>1.8576E-01</td>
<td>T(3)(RNG): thickness for layer index no. (3): RNG seg=3</td>
</tr>
</tbody>
</table>

**old design:**

<table>
<thead>
<tr>
<th>No.</th>
<th>Value</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>1.7544E+00</td>
<td>H(RNG): height of stiffener (type H for sketch), h: R</td>
</tr>
<tr>
<td>9</td>
<td>8.9081E-02</td>
<td>T(3)(RNG): thickness for layer index no. (3): RNG seg=3</td>
</tr>
</tbody>
</table>

**COMMENTS 2**

Comparing the cold-bending ring buckling modes from the two discretized single module models is of interest:

Segment 2 is the base under the ring and Segment 3 is the ring web.

**buckling mode in Segments 2 & 3 from the cold-bending "skin"-ring module model**

(includes smeared stringers):

<table>
<thead>
<tr>
<th>NODE</th>
<th>Z</th>
<th>W</th>
<th>WD</th>
<th>WDD</th>
<th>U</th>
<th>V</th>
<th>WDDD</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.00E+00</td>
<td>4.92E-02</td>
<td>-7.05E-03</td>
<td>-1.89E-02</td>
<td>-4.54E-03</td>
<td>-3.57E-04</td>
<td>-1.03E-02</td>
</tr>
<tr>
<td>2</td>
<td>0.00E+00</td>
<td>4.57E-02</td>
<td>-1.10E-02</td>
<td>-2.25E-02</td>
<td>-5.96E-03</td>
<td>-1.02E-03</td>
<td>-1.03E-02</td>
</tr>
<tr>
<td>3</td>
<td>0.00E+00</td>
<td>3.95E-02</td>
<td>-2.25E-02</td>
<td>-2.61E-02</td>
<td>-7.70E-03</td>
<td>-1.55E-03</td>
<td>-1.03E-02</td>
</tr>
<tr>
<td>4</td>
<td>0.00E+00</td>
<td>3.00E-02</td>
<td>-3.21E-02</td>
<td>-2.93E-02</td>
<td>-1.01E-02</td>
<td>-2.00E-03</td>
<td>-9.21E-03</td>
</tr>
<tr>
<td>5</td>
<td>0.00E+00</td>
<td>1.70E-02</td>
<td>-4.30E-02</td>
<td>-3.28E-02</td>
<td>-1.34E-02</td>
<td>-1.63E-03</td>
<td>-9.92E-03</td>
</tr>
<tr>
<td>6</td>
<td>0.00E+00</td>
<td>5.33E-18</td>
<td>-4.87E-02</td>
<td>-8.74E-17</td>
<td>-1.53E-02</td>
<td>0.00E+00</td>
<td>9.38E-02</td>
</tr>
<tr>
<td>7</td>
<td>0.00E+00</td>
<td>-1.70E-02</td>
<td>-4.30E-02</td>
<td>3.28E-02</td>
<td>-1.34E-02</td>
<td>1.63E-03</td>
<td>9.38E-02</td>
</tr>
<tr>
<td>8</td>
<td>0.00E+00</td>
<td>-3.00E-02</td>
<td>-3.21E-02</td>
<td>2.93E-02</td>
<td>-1.01E-02</td>
<td>2.00E-03</td>
<td>-9.92E-03</td>
</tr>
<tr>
<td>9</td>
<td>0.00E+00</td>
<td>-3.95E-02</td>
<td>-2.25E-02</td>
<td>2.61E-02</td>
<td>-7.70E-03</td>
<td>1.55E-03</td>
<td>-9.21E-03</td>
</tr>
<tr>
<td>10</td>
<td>0.00E+00</td>
<td>-4.57E-02</td>
<td>-1.10E-02</td>
<td>2.25E-02</td>
<td>-5.96E-03</td>
<td>1.02E-03</td>
<td>-1.03E-02</td>
</tr>
<tr>
<td>11</td>
<td>0.00E+00</td>
<td>-4.92E-02</td>
<td>-7.05E-03</td>
<td>-1.89E-02</td>
<td>-4.54E-03</td>
<td>3.57E-04</td>
<td>-1.03E-02</td>
</tr>
</tbody>
</table>
Segment 2 is the base under the ring and Segment 3 is the ring web.

buckling mode in Segments 2 & 3 from the cold-bending skin-ring module model (stringers not present):

<table>
<thead>
<tr>
<th>NODE</th>
<th>Z</th>
<th>W</th>
<th>WD</th>
<th>WDD</th>
<th>U</th>
<th>V</th>
<th>WDDD</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.00E+00</td>
<td>-4.32E-02</td>
<td>-9.43E-02</td>
<td>-3.29E-01</td>
<td>2.92E-03</td>
<td>3.95E-04</td>
<td>4.24E-01</td>
</tr>
<tr>
<td>2</td>
<td>0.00E+00</td>
<td>-8.96E-02</td>
<td>-1.64E-01</td>
<td>-1.80E-01</td>
<td>4.10E-03</td>
<td>3.64E-04</td>
<td>4.24E-01</td>
</tr>
<tr>
<td>3</td>
<td>0.00E+00</td>
<td>-1.58E-01</td>
<td>-2.01E-01</td>
<td>-3.22E-02</td>
<td>5.66E-03</td>
<td>2.23E-04</td>
<td>4.24E-01</td>
</tr>
<tr>
<td>4</td>
<td>0.00E+00</td>
<td>-2.30E-01</td>
<td>-1.14E-01</td>
<td>5.34E-01</td>
<td>7.82E-03</td>
<td>1.85E-04</td>
<td>1.62E+00</td>
</tr>
<tr>
<td>5</td>
<td>0.00E+00</td>
<td>-2.37E-01</td>
<td>3.30E-01</td>
<td>2.00E+00</td>
<td>1.07E-02</td>
<td>1.72E-04</td>
<td>4.21E+00</td>
</tr>
<tr>
<td>6</td>
<td>0.00E+00</td>
<td>-2.80E-17</td>
<td>6.80E-01</td>
<td>6.08E-06</td>
<td>1.24E-02</td>
<td>0.00E+00</td>
<td>-5.74E+00</td>
</tr>
<tr>
<td>7</td>
<td>0.00E+00</td>
<td>2.37E-01</td>
<td>3.30E-01</td>
<td>-2.00E+00</td>
<td>1.07E-02</td>
<td>-1.72E-04</td>
<td>-5.74E+00</td>
</tr>
<tr>
<td>8</td>
<td>0.00E+00</td>
<td>2.30E-01</td>
<td>-1.14E-01</td>
<td>-5.34E-01</td>
<td>7.82E-03</td>
<td>-1.85E-04</td>
<td>4.21E+00</td>
</tr>
<tr>
<td>9</td>
<td>0.00E+00</td>
<td>1.58E-01</td>
<td>-2.01E-01</td>
<td>3.22E-02</td>
<td>5.66E-03</td>
<td>-2.23E-04</td>
<td>1.62E+00</td>
</tr>
<tr>
<td>10</td>
<td>0.00E+00</td>
<td>8.96E-02</td>
<td>-1.64E-01</td>
<td>1.80E-01</td>
<td>4.10E-03</td>
<td>-3.64E-04</td>
<td>4.24E-01</td>
</tr>
<tr>
<td>11</td>
<td>0.00E+00</td>
<td>4.32E-02</td>
<td>-9.43E-02</td>
<td>3.29E-01</td>
<td>2.92E-03</td>
<td>-3.95E-04</td>
<td>4.24E-01</td>
</tr>
</tbody>
</table>

MODAL DISPLACEMENTS FOR SEGMENT NO. 2

<table>
<thead>
<tr>
<th>NODE</th>
<th>Z</th>
<th>W</th>
<th>WD</th>
<th>WDD</th>
<th>U</th>
<th>V</th>
<th>WDDD</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-2.91E-02</td>
<td>3.21E-02</td>
<td>6.80E-01</td>
<td>2.39E-01</td>
<td>1.13E-17</td>
<td>4.82E-19</td>
<td>5.75E-02</td>
</tr>
<tr>
<td>2</td>
<td>-1.26E-01</td>
<td>9.97E-02</td>
<td>7.08E-01</td>
<td>2.44E-01</td>
<td>-4.14E-14</td>
<td>1.89E-12</td>
<td>5.75E-02</td>
</tr>
<tr>
<td>3</td>
<td>-2.23E-01</td>
<td>1.70E-01</td>
<td>7.32E-01</td>
<td>2.50E-01</td>
<td>6.43E-14</td>
<td>3.85E-12</td>
<td>5.75E-02</td>
</tr>
<tr>
<td>4</td>
<td>-3.21E-01</td>
<td>2.42E-01</td>
<td>7.61E-01</td>
<td>3.51E-01</td>
<td>4.34E-13</td>
<td>6.16E-12</td>
<td>1.04E+00</td>
</tr>
<tr>
<td>5</td>
<td>-4.18E-01</td>
<td>3.18E-01</td>
<td>8.18E-01</td>
<td>8.31E-01</td>
<td>1.76E-12</td>
<td>8.96E-12</td>
<td>4.94E+00</td>
</tr>
<tr>
<td>6</td>
<td>-5.15E-01</td>
<td>4.01E-01</td>
<td>8.97E-01</td>
<td>7.89E-01</td>
<td>4.31E-12</td>
<td>1.21E-11</td>
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</tr>
<tr>
<td>7</td>
<td>-6.12E-01</td>
<td>4.92E-01</td>
<td>1.00E+00</td>
<td>1.37E+00</td>
<td>9.29E-12</td>
<td>1.51E-11</td>
<td>5.99E+00</td>
</tr>
<tr>
<td>8</td>
<td>-7.09E-01</td>
<td>5.96E-01</td>
<td>1.14E+00</td>
<td>1.38E+00</td>
<td>1.68E-11</td>
<td>1.69E-11</td>
<td>9.01E-02</td>
</tr>
</tbody>
</table>
Notice that in the mode shape for the discretized skin-ring module model (stringers neglected) the critical buckling mode is characterized mostly by the ring web cross section rotating at its root, producing significant antisymmetric normal deflections $W$ in the ring base (Segment 2). The mode shape for the discretized "skin"-ring module model (smeared stringers included) exhibits much less deformation of Segment 2 and much more deformation of the ring web (Segment 3). From the list of margins above we see that the model in which the stringers are neglected produces the critical margin, which is to be expected.

---

**COMMENTS 3**

These comments pertain to the output from SUBROUTINE COLDBD.

**a. The following output from SUBROUTINE COLDBD:**

```
Location of neutral axis for elastic material, DELAST=  9.3628E-02

IBEND=  2
Iteration=  1 Location of neutral axis, d=  6.0412E-02
Iteration=  2 Location of neutral axis, d=  5.8127E-02
Iteration=  3 Location of neutral axis, d=  5.7979E-02
KOUNT,RCOLD,RADEND,ARCYL,DIFF=  1  3.2000E+01  6.6653E+01  4.8000E+01 2.7985E+01

IBEND=  2
Iteration=  1 Location of neutral axis, d=  5.2824E-02
Iteration=  2 Location of neutral axis, d=  5.2521E-02
Iteration=  3 Location of neutral axis, d=  5.1545E-02
KOUNT,RCOLD,RADEND,ARCYL,DIFF=  2  2.7522E+01  5.0708E+01  4.8000E+01 5.3403E-02

IBEND=  2
Iteration=  1 Location of neutral axis, d=  5.1601E-02
Iteration=  2 Location of neutral axis, d=  5.1545E-02
Iteration=  3 Location of neutral axis, d=  7.4343E-02
Iteration=  4 Location of neutral axis, d=  7.5316E-02
Iteration=  5 Location of neutral axis, d=  7.5357E-02
```

demonstrates the nested iteration loops for the determination of RCOLD in the outer iteration loop and d in the inner iteration loop.

In particular, we want RCOLD to have a value such that, upon elastic spring-back, RADEND is close to ARCYL (ARCYL = ABS(RCYL), with ABS(RCYL) being an input datum provided by the PANDA2 user: RCYL = the radius of the fabricated cylindrical shell.

b. The following input from SUBROUTINE COLDBD:

\[ E11LIN, E12LIN, E22LIN = 1.2088E+07 \quad 3.6264E+06 \quad 1.2088E+07 \]

Ring Web: IX, DIFF = 1 0.0000E+00
Ring Web: IX, DIFF = 2 0.0000E+00
Ring Web: IX, DIFF = 3 0.0000E+00
Ring Web: IX, DIFF = 4 2.7726E-01
Ring Web: IX, DIFF = 5 6.8951E-01
Ring Web: IX, DIFF = 6 7.3218E-01
Ring Web: IX, DIFF = 7 8.6633E-01
Ring Web: IX, DIFF = 8 8.8521E-01
Ring Web: IX, DIFF = 9 8.9942E-01
Ring Web: IX, DIFF = 10 9.1051E-01
Ring Web: IX, DIFF = 11 9.1937E-01
Shell skin: DIFF = 0.0000E+00

lists the ring web nodal point number, IX, and a quantity called "DIFF". "DIFF" is a measure of how far we are into the nonlinear (plastic) portion of the PANDA2-user-supplied stress-strain curve. DIFF = 0 signifies loading below the proportional limit of the material. IX increases from the panel skin (IX = 1) to the tip of the ring web (IX = NX = 11). The tip of the ring web coincides with the middle surface of the outstanding flange, if any.

c. The following output from SUBROUTINE COLDBD:

\[ \text{From "CUBIC": NWAVE, EIGVAL(NWAVE) = 1 3.9232E+01} \]
\[ \text{From "CUBIC": NWAVE, EIGVAL(NWAVE) = 2 1.0792E+01} \]
\[ \text{From "CUBIC": NWAVE, EIGVAL(NWAVE) = 3 5.5201E+00} \]
From "CUBIC": NWAVE, EIGVAL(NWAVE)= 4 3.6823E+00
From "CUBIC": NWAVE, EIGVAL(NWAVE)= 5 2.8401E+00
From "CUBIC": NWAVE, EIGVAL(NWAVE)= 6 2.3936E+00
From "CUBIC": NWAVE, EIGVAL(NWAVE)= 7 2.1381E+00
From "CUBIC": NWAVE, EIGVAL(NWAVE)= 8 1.9900E+00
From "CUBIC": NWAVE, EIGVAL(NWAVE)= 9 1.9087E+00
From "CUBIC": NWAVE, EIGVAL(NWAVE)= 10 1.8758E+00 <---critical value
From "CUBIC": NWAVE, EIGVAL(NWAVE)= 11 1.8761E+00
---------------------------------------------------------
lists the eigenvalue for each circumferential wave number, NWAVE, generated by SUBROUTINE CUBIC.

d. The following output from SUBROUTINE COLDBD:
------------------------------------------------------------------------
BUCKLE= 1.8758E+00 WAVLEN= 9.7288E-01 NCRIT3= 10
circumferential length of web used for buckling analysis, FL= 9.7288E+00
number of circumferential halfwaves over the length CIRC= 155
circumferential spacing of the stringers= 2.2773E+00
circumferential half-wavelength of the critical buckling mode= 9.7288E-01
Margin= 7.0525E-01 Cold-bending ring buckling, closed form soln; N=155;FS=1.1
****** THE FOLLOWING COLD-BENDING BUCKLING MARGIN JUST COMPUTED:
Cold-bending ring buckling, closed form soln; N=155;FS=1.1
End of computation of cold-bending ring buckling load factor in SUBROUTINE COLDBD.
------------------------------------------------------------------------
lists the critical buckling load factor, BUCKLE, and the number of circumferential half-waves over the circumferential length, CIRC.

With NPRINT = 0 in the *.OPT file, the output from the new cold-bending buckling part of PANDA2 is as follows:
------------------------------------------------------------------------
**** CHAPTER 26b: DESIGN PERTURBATION INDEX, IMOD= 0 ****

*** BEGIN SUBROUTINE COLDBD (COLD-BENDING RING BUCKLING) ****
See Item No. 790 of the file, ...panda2/doc/panda2.news
Buckling from COLDBD ("closed-form" solution) is Model no. 1
BUCKLE= 1.8758E+00 WAVLEN= 9.7288E-01 NCRIT3= 10
------------------------------------------------------------------------
circumferential spacing of the stringers = 2.2773E+00
circumferential half-wavelength of the critical buckling mode = 9.7288E-01
Margin = 7.0525E-01 Cold-bending ring buckling, closed form soln;
N=155; FS=1.1
End of computation of cold-bending ring buckling load factor
in SUBROUTINE COLDBD.

***********************************************************
***** BEGIN DISCRETIZED "SKIN"-RING MODULE MODEL OF *****
*** COLD-BENDING BUCKLING OF RING (smeared stringers)  ***
***** This is Model No. 2 of cold-bending ring buckling *****
circumferential waves over the circ.length, 4.8644E+00= 1
ring cold buckling load factor from a discretized module = 6.4476E+00
(smeared stringers)
circumferential waves over the circ.length, 4.8644E+00= 2
ring cold buckling load factor from a discretized module = 2.4048E+00
(smeared stringers)
circumferential waves over the circ.length, 4.8644E+00= 3
ring cold buckling load factor from a discretized module = 1.8210E+00
(smeared stringers)
circumferential waves over the circ.length, 4.8644E+00= 4
ring cold buckling load factor from a discretized module = 1.8404E+00
(smeared stringers)
circumferential spacing of the stringers = 2.2773E+00
circumferential half-wavelength of the critical buckling mode = 1.6215E+00

**** BEGIN SUB. MODE ("SKIN"-RING MODULE COLD BENDING 1) ****
Margin = 6.5544E-01 Cold-bending ring buckling, "skin"-ring module; N=93;
FS=1.1
End of computation of cold-bending ring buckling load factor
in SUBROUTINE STRUCT from "skin"-ring discretized module.
***********************************************************

***** BEGIN DISCRETIZED SKIN-RING MODULE MODEL OF *****
*** COLD-BENDING BUCKLING OF RING (no stringers)  ***
***** This is Model No. 3 of cold-bending ring buckling *****
circumferential waves over the circ.length, 2.2773E+00= 1
ring cold buckling load factor from a discretized module = 1.1032E+00 (no
strings)
circumferential waves over the circ.length, 2.2773E+00= 2
ring cold buckling load factor from a discretized module= 1.6198E+00 (no stringers)
circumferential spacing of the stringers= 2.2773E+00
circumferential half-wavelength of the critical buckling mode= 2.2848E+00

**** BEGIN SUB. MODE (SKIN-RING MODULE COLD BENDING 2) ****
Margin= 2.9428E-03 Cold-bending ring buckling, skin-ring module; N=66;FS=1.1
End of computation of cold-bending ring buckling load factor in SUBROUTINE STRUCT from skin-ring discretized module.
***********************************************************
-------------------------------------------------------------------
There is a new sample case directory: ...panda2/case/nasacoldbend.
This directory contains the file, nasacoldbend.tar, which is the case described in this PANDA2.news item.